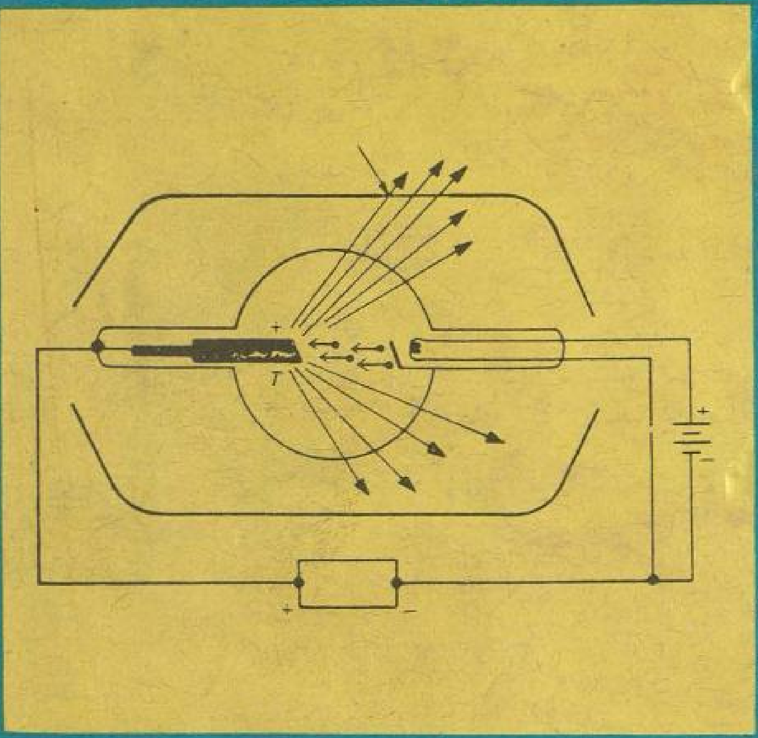
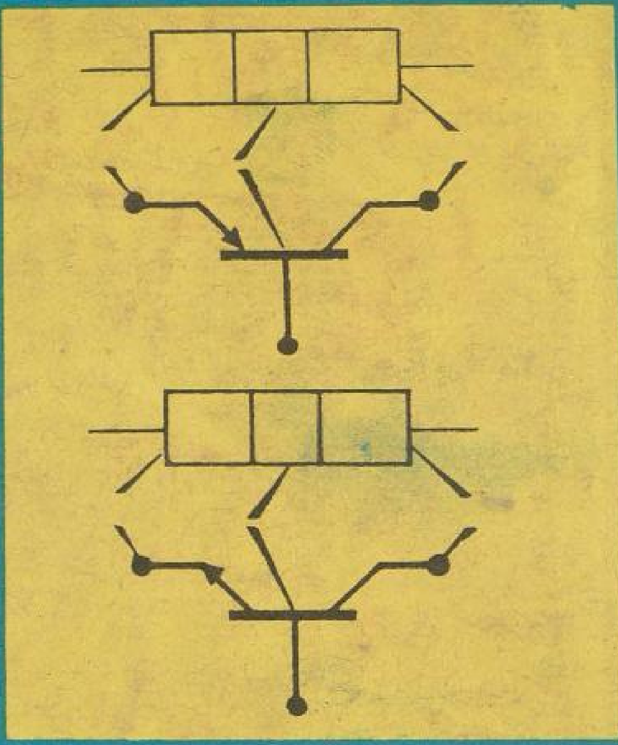
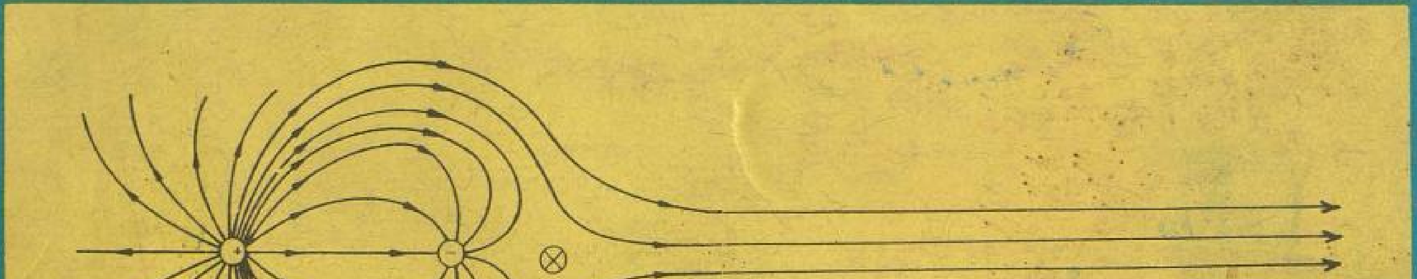
# A TEXTBOOK
# OF
# PHYSICS
## CLASS XII







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# A TEXTBOOK OF PHYSICS

## CLASS XII

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# ERRATA

| Page No | |
|---|---|
| 10 | 21st line read, $\frac{\sum_{i=1}^{N} v_{ix}^{2}}{N}$ instead of $\frac{\sum v_{ix}^{2}}{N}$ |
| 11 | Ist line read, $\sqrt{\frac{\sum_{i=1}^{N} v_{ix}^{2}}{N}}$ instead of $\sqrt{\frac{\sum v_{ix}}{N}}^{2}$ |
| 16 | Last line read, $m = \frac{\text{molecular mass of } H_2}{N}$ instead of $M_0 = \frac{\text{molecular wt of } H_2}{N_A}$ |
| 21 | Example 11.8 read, 35°C, −15°C instead of 35K and 15°C |
| 38 | Last line read, 0.05 x 0.50 = Re. 0.025 instead of 0.50 x 0.50 = Rs. 2.5 |
| 44 | Question 11.7, read changing instead of exchanging. |
| 45 | Problem 11.4, read 2.5°C instead of 25C |
| 46 | Problem 11.8 read, $C_v = 12470\ \text{J k mol}^{-1}\ \text{K}^{-1}$ instead of $C_v = \text{J mol}^{-1}\ \text{K}^{-1}$ |
| 46 | Problem 11.12, read 336000 $\text{Jkg}^{-1}$ and answer is − 36.92 $\text{JK}^{-1}$ instead of 336000 $\text{J K}^{-1}$ and 36.92 $\text{J K}^{-1}$ |
| 58 | Second line read, $10^{-6}$ instead of $10^{6}$ |
| 65 | Last line read, $A_3$ instead of $A_2$ |
| 91 | Problem 12.3 Ans: read, $d^2$ instead of $d_2$ |
| 92 | Problem No 12.4 Ans: read $\frac{Kq}{d^2}$ instead of $\frac{Kg}{d^2}$ |
| 92 | Problem No. 12.8 read $10^{-6}$ C instead of $10^{6}$ |
| 93 | Ans: 12.9 read, $10^{-4}$ N instead of $10^{-14}$ N |
| 114 | Line No. 26 read, R instead of *r* |
| 122 | Problem No. 13.15 read, R= 1000 Ω instead of 100 Ω |
| 169 | Ans. Problem No. 14.11, 14.12 read, *m* H instead of *m* Hz |
| 176 | Line No. 6 read, 12000 Ω instead of 1200 Ω |
| 221 | Include after last line: the object approaches that of light, its inertia increases indefinately, becoming ...... |
| 223 | Line No. 11 read $\text{ms}^{-1}$ instead of $\text{kms}^{-1}$ |
| 228 | Line No. 12, read $6\text{x}10^{-7}$ m instead of $60\text{x}10^{-7}$ n |
| 250 | Ans: 17.5:[ $4.5 \text{ x } 10^{15}$ Hz] |
| 250 | Ans 17.8: read $3.6\text{x}10^{18}$ instead of $3.6\text{x}10^{-18}$ |
| 294 | Line No. 22, read $10^{10}$ Bq instead of 1010 Bq |
| 311 | Ist line read A ∝ − particle instead of A-particle |
| 312 | Problem No. 19.8 and last line of the page may be considered as deleted. |

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## CLASS XII

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**PUNJAB TEXTBOOK BOARD, LAHORE.**

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# Physical Contants

| | | | |
|---|---|---|---|
| Gravitational constant | | $G$ | $6.672 \times 10^{-11}$ N-m$^2$/kg$^2$ |
| Acceleration of gravity | | $g_E$ | 9.80665 m/s$^2$ |
| Earth, mass | | $M_E$ | $5.98 \times 10^{24}$kg |
| | radius | $R_E$ | $6.37 \times 10^6$ m |
| | | | 3960 mi |
| | distance to moon | | $3.844 \times 10^8$ m |
| | | | $2.389 \times 10^5$ mi |
| | distance to sun | | $1.496 \times 10^{11}$ m |
| | | | $9.30 \times 10^7$ mi |
| Moon | | $g_M$ | 1.62 m/s$^2$ |
| | mass | $M_M$ | $7.35 \times 10^{22}$ kg |
| | radius | $R_M$ | $1.738 \times 10^6$ m |
| | period | | 27.32 days |
| Sun | mass | | $1.99 \times 10^{30}$ kg |
| | radius | | $6.96 \times 10^8$ m |
| Density of air (at STP) | | | 1.293 kg/m$^3$ |
| Molecular weight of air | | | 28.97 kg/kmol |
| Avogadro's number | | $N_A$ | $6.0220 \times 10^{26}$ particles/kmol |
| Gas constant | | $R$ | 8.314 J/mol-K |
| | | | $8.206 \times 10^{-2}$ l-atm/mol-K |
| Boltzmann's constant | | $k = R/N_A$ | $1.3807 \times 10^{-23}$ J/K |
| | | | $8.617 \times 10^{-5}$ eV/K |
| Standard temperature | | $T$ | 273.15 K |
| Standard pressure | | $P$ | 1.00 atm |
| Volume of ideal gas at STP | | $V$ | 22.415 l/mol |
| Mechanical equivalent of heat | | $J$ | $4.185 \times 10^3$ J/kcal |
| Heat of fusion of $H_2O$ | | | 79.7 cal/g |
| Heat of vaporization of $H_2O$ | | | 539.6 cal/g |
| Speed of sound in air at STP | | | 331 m/s |
| Electron charge | | $e$ | $1.60219 \times 10^{-19}$ C |
| Coulomb constant | | $k = 1/4\pi\epsilon_0$ | $8.98755 \times 10^9$ N-m$^2$/C$^2$ |
| Permittivity of free space | | $\epsilon_0$ | $8.85419 \times 10^{-12}$ C$^2$/N-m$^2$ |
| Magnetic constant | | $k = \mu_0/4\pi$ | $10^{-7}$ N/A$^2$(= Wb/A-m) |
| Permeability of free space | | $\mu_0$ | $4\pi \times 10^{-7}$ N/A$^2$(= Wb/A-m) |
| Speed of light | | $c$ | $2.997925 \times 10^8$ m/s |

# PREFACE

Like similar text-books prepared by various Universities of the world, this book has been written keeping the grade XII curriculum and the methodological and didactic requirement of our student in view. The curriculum contents of this book have been developed by the National Curriculum Committee consisting mostly of college teachers alongwith a few educationists and university teachers. The topics are nearly the same as in the previous book - but within this framework a substantial amount of material has been rearranged to provide a superior integration and a more effective teachable sequence of topics. Almost all the material has been rewritten and coverage of the topics has been brought up to date. Besides, some new developments in the field of laser physics, high temperature superconductivity, grand unification theory, depletion of ozone layer, discovery of monopole, etc., have been introduced.

The so-called 'heuristic' approach, which is based on the belief that students should be encouraged to discover things for themselves, has been adopted throughout. Experience shows that students are likely to become more interested when given a historical outline of events leading to the development of the subject. Great care has also been taken to adopt essentially a conceptual rather than a mathematical approach. Although simple equations have been used throughout the text, they have been kept to a minimum and are used for the precise statement of the principle being considered. Some of the concepts discussed in this book are complex, and a mere lack of mathematics would not have made them easy. Numerous solved examples have been included to illustrate the application of various theories. We have also incorporated some simple introductory questions at the end of each chapter, arranged in a sequence to give an almost programmed approach to the mastery of the material in each section.

A number of unsolved problems are also given in every chapter to set guidlines for class-room teaching. However, this should not prevent the teachers and the students from using other sources to improve their problem solving skills.

Summary of the contents has also been included at the end of each chapter to highlight the important things introduced in it. SI units (metre-kilogram-second-coloumb system of units) have been used throughout this book.

Our purpose have been to write a text that is clear, informative, interesting and teachable. We would welcome any suggestions for improvement from both teachers and students. It should be pointed out that the present work has been produced on an experimental basis and any suggestions for improvement will be considered for incorporation in the next edition.

Editors

بسم الله الرحمن الرحيم

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# 11

# HEAT

We are familiar with the notion of "hot" and "cold", which are primitively tied to our various sensory organs. They are also related to our many common every-day observations. Wet clothes get dried more slowly on cold days than on warm days. Many substances expand when heated and contract when cooled. A pot of cold water when placed on a hot burner or a stove becomes hot. We say that heat flows from the burner to the cold water and makes it hot, too. We use temperature as a parameter in order to scientifically describe whether an object is cold or hot. In the proceeding paragraph we shall try to make the concept of temperature precise and discuss the concept of heat.

## 11.1 TEMPERATURE AND THERMOMETRY

Hotness or coldness of an object is expressed in terms of a quantity called temperature. In order to measure temperature, we use some property of matter that changes uniformly with it. Such a property of a substance is called thermometric property. A device based on the thermometric property of the material is called, a thermometer. Let us have a brief over view of the effects of increase in temperature on different properties of matter.

As said earlier nearly all materials expand on heating and contract on cooling. An iron is longer when it is hot than when it is cold. Another property of matter that changes with temperature is colour. At very high temperatures solids become red. At still higher temperatures iron and some other solids turn orange.

Change of temperature also affects the physical "state" of a material. For instance, water at low temperature is ice (a solid), at a higher temperature it is liquid, and at still a higher temperature it is steam ( a gas). Other materials also pass through these three states as temperature varies. However, the temperature at which the change from one state to another state occurs is different for different materials. Resistance of a metallic wire is also affected by the change in temperature. Generally, the higher the temperature of the wire the greater is its resistance.

### 11.1.1 Thermometer and Scales of Temperature

Any of the above thermometric properties can be used to make a thermometer. In fact, most of these properties have been utilized for constructing thermometer of one type or another. Generally thermometers make use of the fact that most of the liquids expand on heating. The ordinary mercury thermometer uses the thermal expansion of mercury as a measure of temperature. Mercury is used as a thermometric substance because over a wide range of temperatures its expansion is quite a linear function of the temperature and that it does not stick to the walls of the capillary tube. As shown in Fig. 11.1 a bulb filled with the mercury is attached (fused) to a capillary tube. As the bulb is heated, the mercury expands and rises higher in the capillary. Hence the height of the mercury in the capillary can be used as a measure of the temperature of the bulb.

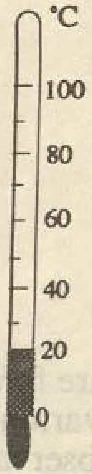


*Fig. 11.1 Mercury thermometer*

Three common methods for assigning numerical values to temperature by use of a thermometer are shown in Fig. 11.2. If the device of Fig. 11.1 is placed in a vessel containing melting ice, the mercury will reach a certain level in the capillary. A mark is made at that level. Now the same device is placed in boiling water (at an atmospheric pressure of 760 mm of mercury), and the capillary is marked at the new higher level of the mercury. There are now three common ways of placing numbers along the capillary tube so as to read the temperature. These give rise to three different temperature scales. They all are illustrated in Fig. 11.2.

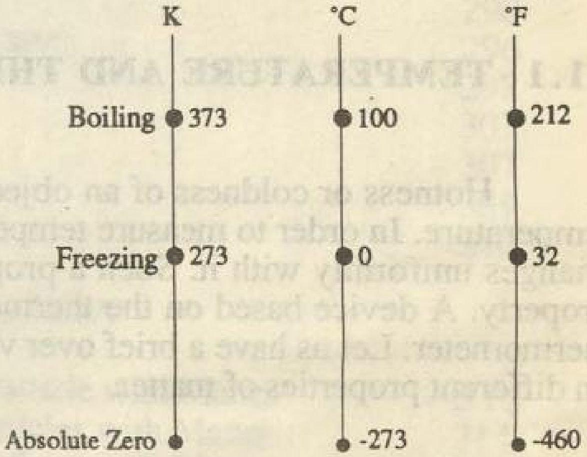


*Fig 11.2 Different temperature scales*

Here the two scale marks are shown together with three separate scales.

It should be noticed that both the Celsius (also called centigrade) and Kelvin scales split the region from freezing to boiling of water into 100 parts. Each part is called a degree. The Fahrenheit scale divides this region in 180 parts (degrees). A Celsius degree is larger than a Fahrenheit degree by 9/5. A simple equation for conversion between these scales is

$$°F = \frac{9}{5}(°C) + 32 \qquad (11.1)$$

Celsius scale and Fahrenheit scale both are arbitrary scales as there is no scientific reason to say that melting point of ice is 0°C or 32°F. Kelvin scale is approved by the international system. The zero of the Kelvin scale coincides with absolute zero. The Kelvin division however, is equal to the Celsius degree. Absoulte zero is that temperature at which the gases if they remain in gaseous state exert zero pressure and have zero volume in fact at temperature much above the absolute zero all gases undergo a change of gaseous state and become liquid.

## 11.2 HEAT AS ENERGY IN TRANSIT

Having established a consistent temperature scale in the preceeding section, we can examine the concept of heat as energy in transit. It is common to talk about the flow of heat. Heat flows from a stove burner to a tea-pot, from the sun to the earth and so on. It flows from an object at a higher temperature to a body at a lower temperature. Earlier heat was viewed as a fluid substance called caloric. According to this theory, any object contained a certain amount of caloric: if more caloric flows into the object, its temperature increases; and if caloric flows out, the temperature of the object decreases. When matter was broken apart, such as during burning, a great deal of caloric was believed to be released. However, the caloric fluid was never directly detected and this concept could not explain certain very important thermal phenomena. The caloric theory had to be abandoned.

One of the main problems with the caloric theory was its inability to account for the heat generated by friction. Indefinite amount of heat can be produced by rubbing two pieces of metals together for a long time. Count Rumford was actually aware of this problem when he supervised the boring-up of cannon barrels. Water was kept circulating during the cutting process. The "caloric" that caused the boiling of the water was assumed to be due to the breaking apart of the metal. But Rumford noticed that even when the cutting tools were so dull that they did not cut the metal, heat was still generated and the water boiled away. thus caloric was being released even though subdivision of matter was not occuring. Rumford also realized that this process could go on indefinitely and produce limitless amount of heat. This was not consistent with idea that heat is a substance and that only a finite amount of it could be contained within an object. Rumford, therefore, rejected the caloric theory and correctly guessed that heat was a form of energy that could be supplied by mechanical means. He, however, did not carry out any quantitive studies of this process. This idea was pursued by others, particularly by James Prescott Joule, who showed the connection between mechanical energy and heat energy.

Joule performed a number of experiments that were crucial in establishing our present view about heat. Accroding to this view heat, like work, represents energy in transit. Joule found that a given amount of work done on an object was always equivalent to a particular amount of heat. Thus 4180 joule of work raises the temperature of one litre of water through 1°C. The presently accepted value (sometimes called the mechanical equivalent of heat) is 4.184 Joule/calorie. This result is independent of whether the energy is supplied in the form of mechanical work through stirring of a liquid or rubbing of solids or whether the energy is supplied by electrical means. As a result of the experiments conducted by Joule and others, scientists came to interpret heat not as the flow of the substance but as a transfer of energy. Thus heat refers to energy that is transferred from one object to another due to a difference in the temperature of the two objects. The transfer of energy from an object at a higher temperature to an object at a lower temperature continues until the two objects come to a common temperature when this has occurred, the system is said to be in thermal equilibrium.

A microscopic view nicely explains the idea of heat as a transfer of energy. According to this view, the thermal kinetic energy of the molecules increases with increase in temperature; thus the molecules of the stove burner have much more kinetic energy on the average than those of the cold water in the pot. When the fast moving stove molecules collide with the slower moving molecules of pot some of their kinetic energy is transferred to the pot molecules, just as a fast moving billiard ball transfers some of its kinetic energy to a ball with which it collides. The molecules of the pot gain in kinetic energy. Now the fast moving pot molecules, in turn, transfer some of their kinetic energy by collision to slow moving water molecules. The temperature of the water and the pot consequently rises. Thus we see how heat flow is a transfer of energy.

When an object is heated, not all the energy it absorbs goes into increasing the velocity of the molecules. Some goes into rotational motion of the molecules and some into internal vibrational motion. This is illustrated in Fig. 11.3 (a and b).

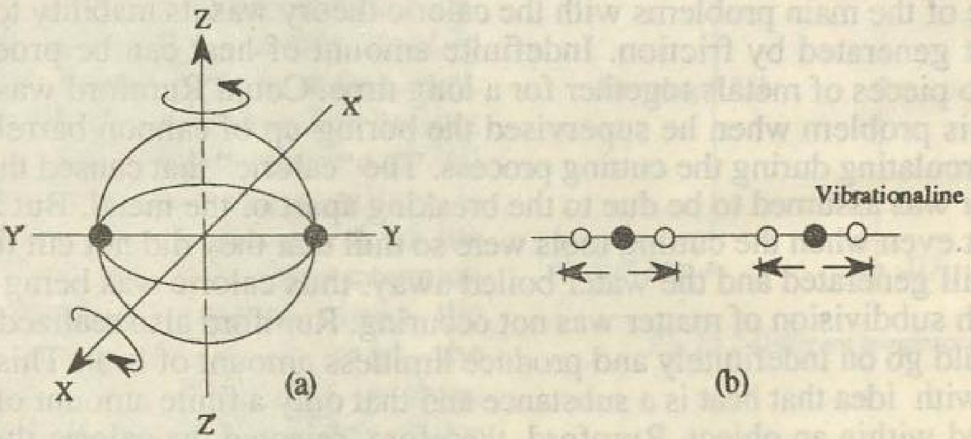


*Fig. 11.3 (a) Rotational energy (b) Virational energy*

Thus the total energy of the molecules in a substance is the sum of the translational kinetic energy, the rotational kinetic energy, the vibrational energy and the potential energy. The sum total of all the energy of all the molecules of atoms in an object is known as internal energy or thermal energy. Sometimes we use the term "heat content" of an object for expressing internal energy. This term is, however, misleading because it can be confused with heat itself. Heat, as we have seen, is not the energy that an object contains in it but it refers to the amount of energy transferred from a hot to a cold body. Once the heat energy is transferred to an object it is converted into the itnernal energy of the object.

## 11.3 THE GAS LAWS

As stated above, the temperature affects the properties of gases, liquids and solids. But the effect of temperature on gases is more pronounced than on liquids and solids. Let us examine this temperature dependent behaviour of gases in detail.

At low densities (or at very high temperatures) gases behave in a simple manner. As a result of experiments on gases, Robert Boyle discovered that temperature remaining constant the volume of a given mass of gas is inversely proportional to the pressure applied on it. The above statement is known as Boyle's law. If the pressure on a gas is doubled, the volume of the gas decreases to half its value. The Boyle's law can be written mathematically as

$$V \propto \frac{1}{p} \qquad \text{(constant temperature)} \qquad (11.2)$$

where $V$ is the volume and $p$ the pressure of the gas. The above expression can also be written as

$$pV = \text{constant} \qquad \text{(constant temperature)} \qquad (11.3)$$

Temperature also affects the volume of a gas. We know that a gas expands on heating. The quantitative relationship between the volume of a gas and its temperature was discovered by Jacques Charles nearly a century after the discovery of Boyle's law and is known as Charles' law. Charles' law can be stated as: "At constant pressure the volume of a gas is proportional to the absolute temperature". Mathematically, it can be written as

$$V \propto T \qquad \text{(constant pressure)} \qquad (11.4)$$

or $\quad V = \text{constant} \times T$

where $T$ is the absoulte temperature.

If we draw a graph between the volume and temperature of a gas, while keeping the pressure constant, a curve shown in Fig. 11.4 is obtained.

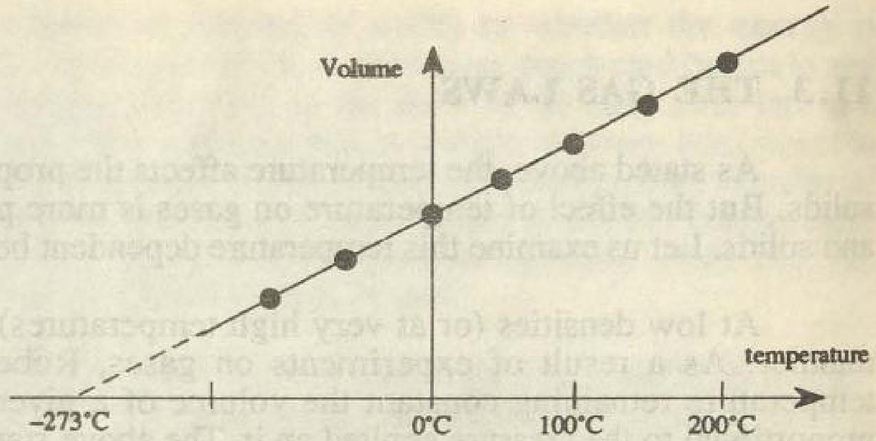


*Fig. 11.4 Volume as a function of temperature*

Because all the gases liquefy at very low temperatures, therefore, we cannot plot the graph below the liquefaction point.

When the straight line of the graph is extrapolated to lower and lower temperatures, as shown by the dotted line of Fig. 11.4, it crosses the axis at a temperature of –273°C. This seems to imply that if a gas could be cooled to –273°C, it would have no volume. Thus –273°C is called the absolute zero of the temperature.

### 11.3.1 The Ideal Gas and Gas Law

An ideal gas is one in which the molecules exert no force on each other. The only force they exert is when they physically collide. In other words, in an ideal gas the molecules have only kinetic energy and no potential energy.

The molecules behave as hard impenetrable spheres. Most of the gases with low density and well above the liquefication temperature behave as an ideal gas.

We have described above the relations among the three important variables; the pressure, the volume, and the temperature of a gas. Each one of these laws relates two of these variables while the third variable is kept constant. Though it seems convenient yet it is not practicable to hold one of the variables constant. We can combine the two laws into a single law that relates all the three variables. This single law can be stated in symbols as below:

$$pV \propto T \tag{11.5}$$

or

$$\frac{pV}{T} = \text{constant} \tag{11.6}$$

When any two of the variables are changed, this relation tells us how the third will vary.

In additon to volume, pressure and temperature, a gas has a fourth and equally important variable, which up to now has been assumed constant. This is the total mass of the gas or the total number of molecules in a gas. If we keep the temperature and pressure of the gas constant but change its volume by adding more molecules, we find that volume increases directly with the number of molecules $N$. That is, $V \propto N$, when pressure and temperature do not change. Similarly, if we keep the volume and temperature constant, we find that pressure is also directly proportional to the amount of gas present; i.e. $p \propto N$. In order to understand this, we must introduce the chemical measure of mass - the kilo mole (kmol). In the subject of chemistry mass is expressed in mole rather than kilomole. We use k mol to be consistant with SI unit of mass. The kilomole is the mass of a substance in kilograms and is numerically equal to the molecular mass of the substance. The number of kilomoles of mass $m$ of a substance (in kilogram) is given by

$$n = \frac{m \text{ kg}}{M\text{kg/kmole}} \qquad (11.7)$$

where $M$ is the molecular mass of the substance expressed in kg/k mole. The molecular mass is equal to the sum of the atomic masses of the elements in the substance, multiplied by the number of times each element occurs in the chemical formula. Thus the molecular mass of carbon dioxide, for example, will be :

$$M_{CO_2} = 12.00 + (2 \times 16.00) = 44.00$$

Knowing the molecular mass $M$, we can find the number of kilomoles of the given substance by using Eq. (11.7). Suppose we have 0.2 kg of $CO_2$ then

$$n = \frac{0.2 \text{ kg}}{44.00 \text{ kg/k mole}} = 4.545 \times 10^{-3} \text{ k mol}$$

If the mass is measured in kilomoles, another striking regularity occurs in the behaviour of low density gases, referred to as the Avogadro's hypothesis. It states "Under the same conditions of temperature and pressure equal volumes of gas contain the same number of kilomoles"

This implies that at constant pressure and temperature the number of kilomoles of any gas is proportional to the volume of the gas. Moreover, the constant of proportionality is the same for all gases. Eq.(11.6) combined with Avogadro's hypothesis gives us the ideal law:

$$pV = nRT \qquad (11.8)$$

where $R$ is a constant and its value is the same for all the gases and is thus known as universal gas constant. Its value is 8314 J/k.mol K.

**Example 11.1**

A closed pressure cooker with a volume of 0.01 $m^3$ contains 6.0 g of water. What is the pressure in the vessel at a temperature of 127°C assume that the water vapours can be treated as an ideal gas?

**Solution :**

The molecular mass of water ($H_2O$) is

$$M = M_{H_2} + M_O = (2 \times 1.0) + 16 = 18.0$$

knowing $M$, we can find $n$, the number of kilomoles

$$n = \frac{m}{M} = \frac{6.0 \times 10^{-3}\ \text{kg}}{18.0\ \text{kg/k mol}} = 3.33 \times 10^{-4}\ \text{k mol}$$

Using Eq. (11.8)

$$p = \frac{n\,R\,T}{V}$$

we get the required pressure

$$p = (3.33 \times 10^{-4}\ \text{k.mol}) \times \frac{8314\ \text{J} \times 400\ \text{K}}{\text{k mol K} \times 0.01\text{m}^3}$$

$$= 1.107 \times 10^5 \frac{\text{Nm}}{\text{m}^3} = 1.107 \times 10^5\ \text{N m}^{-2}$$

### 11.3.2 Kinetic Theory and Gas Laws

We know that matter is made up of molecules. One of the early successes of molecular theory of matter was an explanation of the behaviour of gases based on just a few simple assumptions involving a molecular model of gases. This theory is called the kinetic theory of gases. Some of the assumptions involved are that (a) the gases are composed of large number of molecules and on the average the molecules of a gas are rather far apart as compared to the size of the molecule (the average distance between the molecules is perhaps ten or more than ten times greater than the diameter of a molecule), (b) these molecules frequently make elastic collisions with one another and with the walls of the vessel in which the gas is kept. (c) because of these collisions, a particular molecule is assumed to be in continuous random motion, and (d) the molecules move freely between the collisions.

## Pressure of a Gas :

According to the kinetic theory, the pressure exerted by a gas is simply due to continuous impact of its molecules upon the walls of the container.

Let us consider a cubical container of side $L$ which has a large number of gas molecules moving in random fashion as shown in Fig. 11.5a. The area of each face of the container is $A = L^2$ and the volume of the container is $V = LA = L^3$. Let us confine our attention on a single molecule of mass $m$ moving with a velocity $v_1$ with components $v_{1x}$, $v_{1y}$ and $v_{1z}$. Let us calculate the number of times this molecule collides with the wall on the right (shown shaded) and bounces back in time $\Delta t$. In this time the molecule travels a distance $\Delta L = V_{1x}\,\Delta t$ along the X-axis. The distance the molecule travels between two successive collisions is $2L$. Thus the number of collisions in time $\Delta t$ with the right side wall will be

$$f = \frac{v_{1x}\,\Delta t}{2L}$$

Number of collision with this wall per unit time $= \dfrac{f}{\Delta t} = \dfrac{v_{1x}}{2L}$

The momentum of the molecule before collision is $mv_{1x}$ and after collision is $-mv_{1x}$ as shown in Fig. 11.5(b).

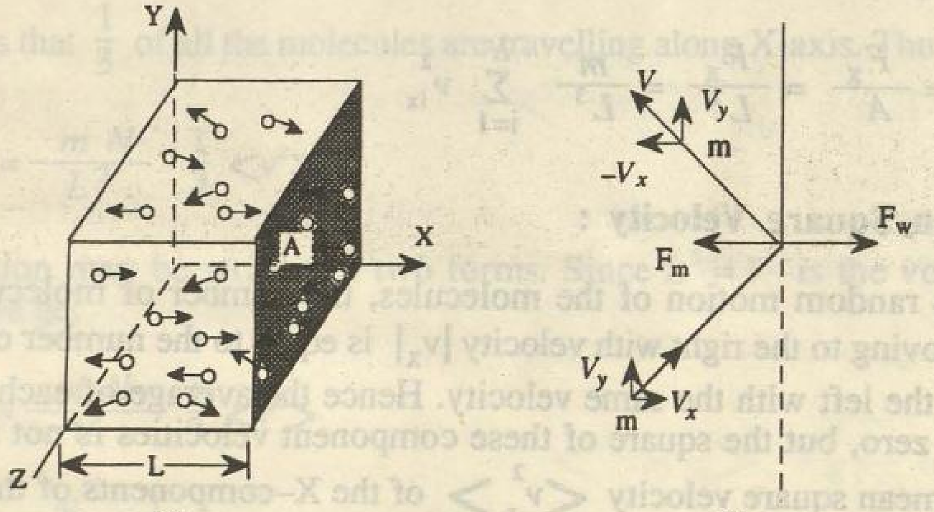


*Fig. 11.5 (a) one sixth of the molecules in the box are assumed to be travelling in the direction shown. They all strike the wall in time d/v. The number of molecules per unit volume is assumed to be $N_0$*
*(b) The momentum of the molecule before and after collision.*

Thus the change in the momentum of the molecule is

$$-mv_{1x} - (mv_{1x}) = -2\,mv_{1x} \quad \text{to the right}$$

$$= 2\,mv_{1x} \quad \text{to the left}$$

rate of change of momentum of molecule $= 2mv_{1x} \times \frac{v_{1x}}{2L} = \frac{mv_{1x}^2}{L}$

By Newton's law, the rate of change of momentum of the molecule caused by the wall is the force the wall exerts on the molecule. Hence, the force exerted on the wall by a molecule is $= \frac{mv_{ix}^2}{L}$

Suppose that there are $N$ molecules in the container moving with velocities $v_1, v_2, v_3 \ldots\ldots v_n$. The total force exerted on the right wall of the container by all the molecules will be

$$F_x = \frac{m}{L}\left(v_{1x}^2 + v_{2x}^2 + v_{3x}^2 \ldots\ldots\ldots + v_{nx}^2\right)$$

$$= \frac{m}{L}\sum_{i=1}^{N} v_{ix}^2$$

Because the pressure $p$ on the wall is defined to be the normal force per unit area, we may wirte

$$p = \frac{F_x}{A} = \frac{F_x}{L^2} = \frac{m}{L^3}\sum_{i=1}^{N} v_{ix}^2$$

**Root Mean Square Velocity :**

Due to the random motion of the molecules, the number of molecules, on the average, moving to the right with velocity $|v_x|$ is equal to the number of molecules moving to the left with the same velocity. Hence the average of each component velocity is zero, but the square of these component velocities is not zero. So we define the mean square velocity $\langle v_x^2 \rangle$ of the X–components of the molecular velocity as

$$\langle v_x^2 \rangle = \frac{v_{1x}^2 + v_{2x}^2 + \ldots\ldots + v_{Nx}^2}{N}$$

$$= \frac{\Sigma v_{1x}^2}{N}$$

and the root mean square velocity of the X–component $(v_x)_{rms}$ as

$$(v_x)_{\mathrm{rms}} = \sqrt{\langle v_x^2 \rangle} = \sqrt{\frac{\Sigma v_{1x}^2}{N}} \qquad (11.8a)$$

By root mean square velocity we mean the square root of the mean of squared velocities. Thus

$$p = \frac{m\,N}{L^3} \quad \langle v_x^2 \rangle$$

Since all component velocities are equally probable, we may write

$$\langle v_x \rangle^2 = \langle v_y \rangle^2 = \langle v_z \rangle^2 \text{ and } \langle v \rangle^2 = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle$$

$$= 3 \langle v_x^2 \rangle$$

that is

$$\langle v_x^2 \rangle = \frac{1}{3} \langle v \rangle^2$$

This means that $\frac{1}{3}$ of all the molecules are travelling along X-axis. Thus

$$p = \frac{m\,N}{L^3} \quad \frac{1}{3} \langle v^2 \rangle$$

This equation may be written in two forms. Since $L^3 = V$ is the volume of the container we get

$$p = \frac{m\,N}{V} \frac{1}{3} \langle v^2 \rangle \qquad (11.9)$$

$$= \frac{N}{V} \frac{1}{3} \langle m v^2 \rangle$$

$$pV = \frac{2}{3} N \langle \tfrac{1}{2} m v^2 \rangle \; \ldots\ldots \qquad (11.10)$$

$$p = \frac{2}{3} \frac{N}{V} \langle \tfrac{1}{2} m v^2 \rangle = \frac{2}{3} \frac{N}{V} \langle k \rangle$$

where <k> is the average kinetic energy (KE) of the molecule. This states that

$$\text{Pressure} = \frac{2}{3}\begin{bmatrix}\text{no of molecules}\\ \text{per unit volume}\end{bmatrix}\begin{bmatrix}\text{average translational kinetic}\\ \text{energy of the molecule}\end{bmatrix}$$

$$= \frac{2}{3}\ N_o \left\langle \frac{1}{2}\ mv^2 \right\rangle$$

where $N_o$ is the number of gas molecules in unit volume of the gas.

Thus the pressure exerted by the gas molecules is directly proportional to the average translational kinetic energy of the molecules of the gas. It may be noticed that the temperature of the gas does not appear in the equation.

On the other hand, the ideal gas law, obtained experimentally, is

$$pV = \mathrm{n\,R}\,T$$

or $$p = \frac{\mathrm{n\,R}\,T}{V}$$

Combining this equation with Eq. (11.10) we get

$$\mathrm{n}RT = \frac{2}{3}\ N\left\langle \frac{1}{2}mv^2 \right\rangle$$

$$T = \frac{2}{3}\ \frac{N}{\mathrm{nR}}\left\langle \frac{1}{2}mv^2 \right\rangle$$

where $N$ is the total number of molecules in the volume $V$. If $\mathrm{N_A}$ is the Avagadro's number, then total number of molecules $N = \mathrm{n\,N_A}$

Hence the expression for T becomes

$$T = \frac{2\,\mathrm{N_A}}{3R}\left\langle \frac{1}{2}mv^2 \right\rangle$$

From here we find that $T$ is directly proportional to the average translational K.E of the gas molecules. We, therefore, conclude that absolute temperature is a measure of the average translational K.E of the gas molecules - a fact of great importance.

From Eq. No. 11.10 we see that

$$pV = \frac{2}{3}\ \mathrm{N}\left\langle \frac{1}{2}\ mv^2 \right\rangle$$

Its average translational KE $\left(\frac{1}{2}\,mv^2\right)$

remains constant, the temperature of the gas will remain constant. Thus under the condition of constant temperature, the product $pV$ remains constant which is known as Boyle's law.

## Charles' Law

The above equation can also be written as

$$V = \frac{2}{3}\ \frac{N}{p}\ \left\langle \frac{1}{2}\ m v^2 \right\rangle$$

If the gas pressure remains constant, the volume varies directly with the average translational KE which is the measure of temperature. In other words, under constant pressure condition, the volume of a gas varies directly with its absolute temperature and it is known as Charles' law.

### Example 11.2

Estimate the average speed of nitrogen molecules in air under standard conditions of temperature and pressure.

### Solution

Under standard conditions, the temperature is 0°C or 273 K. Now according to the kinetic interpretation of the temperature,

$$T = \frac{2\,N_A}{3R}\ \left\langle \frac{1}{2}\ m v^2 \right\rangle$$

which on rearranging yields

$$<v^2> = \frac{3RT}{N_A\ m}$$

To find $m$ for nitrogen ($N_2$) molecules, we note that the molecular mass of $N_2$ is 28 kg/k mol. Hence

$$m = \frac{28\ \text{kg.}\ (\text{k mol})^{-1}}{N_A}$$

which on substitution gives

$$<v^2> = \frac{3\ R\ T\ \text{k mol}\ N_A}{N_A\ 28\ \text{kg}}$$

Because

$$R = 8341\ \text{J/k mol} \times \text{K} \quad \text{and} \quad T = 273\ \text{K}$$

$$<v^2> = \frac{3 \times 8314\ \text{J/k mol. K} \times 273\ \text{K}}{28\ \text{kg K}}$$

$$= 243184.5\ \text{m}^2\text{s}^{-2}$$

$$<v> = 493.1\ \text{ms}^{-1}$$

Hence the nitrogen molecules of the air have an average speed of 493.1 $\text{ms}^{-1}$ under standard conditions of temperature and pressure.

**Example 11.3**

A tank of oxygen at 0°C has a volume of 4 litres, and the pressure is 50 atm. What mass of oxygen is in the tank?

**Solution**

According to the ideal gas law, $pV = nRT$.
The data given is

$$p = 50 \times 1.01 \times 10^5\ \text{Nm}^{-2}$$

$$V = 4 \times 10^{-3}\ \text{m}^3$$

$$R = 8314\ \text{J (k mol)}^{-1}\ \text{K}^{-1}$$

$$T = 273\ \text{K}$$

In the above equation

$$n = \frac{pV}{RT}$$

Substituting the values of $p, V, R$ and $T$, we get

$$n = \frac{50 \times 1.01 \times 10^5\ (\text{Nm}^{-2}) \times 4 \times 10^{-3}\ (\text{m}^3)}{8314\ \text{J.(k mol)}^{-1}\ \text{K}^{-1} \times 273\ \text{K}}$$

$$= 8.9 \times 10^{-3}\ \text{k mol}$$

But one kmol of oxygen is 32 kg, so in tank the mass of oxygen $= 32 \times 8.9 \times 10^{-3}\ \text{kg} = 0.285\ \text{kg}$.

**Example 11.4**

In a diesel engine, the cylinder compresses air from approximately standard temperature and pressure (STP) to about one sixteenth of the original volume and a pressure of about 50 atm. What is temperature of the compressed air?

**Solution**

The ideal gas law can be expressed as

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

or $$T_2 = \frac{p_2 V_2}{p_1 V_1} T_1$$

Substituting the values

$p_1 = 1$ atm, $V_1 = V_1$ and $T_1 = 273$ K

$p_2 = 50$ atm, $V_2 = \frac{1}{16} V_1$ and $T_2 = ?$

$$T_2 = \frac{50\ \text{atm} \times V_1 \times 273\ \text{K}}{1\ \text{atm} \times 16\ \text{V}_1} = 853\ \text{K}$$

Hence the temperature of the compressed air is 853 K or

$(853 - 273) = 580^\circ\text{C}$.

**Example 11.5**

The temperature of the interior of the sun is approximately the same as that in a H-bomb, i.e., about $10^7$ K. Find the average translational KE of a proton at this temperature and its average speed. (At mass of hydrogen is 1 kg/k mol, and the same atomic mass applied approximately to a proton).

**Solution**

According to the kinetic interpretation of the temperature,

$$T = \left(\frac{2N_A}{3R}\right) \left\langle \frac{1}{2} mv^2 \right\rangle$$

Also, on rearranging

$$v^2 = \frac{3\,R\,T}{N_A\ \mathrm{m}}$$

Substituting the values of $R = 8314$ J/kmol K, $T = 10^7$ K and $\mathrm{N_A} = 6.02 \times 10^{26}$ atoms/k mol, we get

$$\mathrm{KE} = \frac{1}{2}\ mv^2 = \frac{3\,RT}{2\,N_A}$$

$$= \frac{3 \times 8314\ \mathrm{J.\ (kmol)^{-1}\ K^{-1}} \times 10^7\ \mathrm{K}}{2 \times 6.02 \times 10^{26}\ \mathrm{atom.\ (kmol)^{-1}}}$$

$$= 2071.56 \times 10^{-19}\ \mathrm{J}$$

and $m_o = \frac{M}{N_A} = \frac{1}{\mathrm{N_A}}$

$$<v^2> = \frac{3 \times 8314\ \mathrm{J\ (kmol)^{-1}\ K^{-1}} \times 10^7\ \mathrm{K}}{N_A\quad 1/N_A}$$

Hence

$$<v> = \sqrt{3 \times 8314 \times 10^7}\ \mathrm{m\ s^{-1}}$$

$$= 5 \times 10^5\ \mathrm{m\ s^{-1}}$$

**Example 11.6**

The escape velocity for a projectile on earth is about 11.2 kms$^{-1}$. At what temperature would hydrogen molecules have an average speed equal to this speed?

**Solution**

The temperature in terms of average speed is given as

$$T = \frac{\mathrm{N_A}\ mv^2}{3R}$$

*m* the mass no of hydrogen

$$M_o = \frac{\text{molecular wt. of } \mathrm{H_2}}{N_A} = \frac{2}{N_A}$$

Substituting the values in the equation for temperature

$$T = N_A \times \frac{2}{N_A} \times \frac{11.2 \times 10^3 \times 11.2 \times 10^3\ (\mathrm{ms}^{-1})^2}{3 \times 8314\ \mathrm{J.k\ (mol^{-1})K^{-1}}}$$

$$= 1.006 \times 10^4\ \mathrm{K}$$

**Example 11.7**

Show that the pressure of an ideal gas can be written as $p = \frac{1}{3} \rho V^2$, where $\rho$ is the density of the gas.

**Solution**

Form kinetic theory of gases we have the relation $p = \frac{1}{3}\ (N_o)\ (m_o v^2)$ where $N_o$ is the number of molecules in a unit volume and $m_0$ is the mass of one molecule. Therefore, $m_0 N_0$ is the mass of unit volume of the gas which by definition is the density of the gas.

$$p = \frac{1}{3}\ (N_o)\ (m_o v^2) = \frac{1}{3}\ \rho v^2$$

## 11.4 THERMAL EXPANSION

Most solid materials expand on heating. Suppose a rod of some material has a length $L_0$ at some initial temperature and its length increases by $\Delta L$ when heated through a temeprature $\Delta T$. Experiments have shown that if $\Delta T$ is not too large, $\Delta L$ is directly proportional to $\Delta T$. $\Delta L$ is also proportional to $L_o$. Thus

$$\Delta L \propto L_o \,.\, \Delta T$$

where

$$\Delta L = L_{\Delta T} - L_0$$

$$\Delta L = \alpha L_0\, \Delta T$$

Hence

$$L_{\Delta T} = L_0\,(1 + \alpha\, \Delta T) \qquad (11.11)$$

where $\alpha$ is the proportionality constant and is called the coefficient of linear expansion. It may be defined as the change in length per unit length per kelvin rise in temperature. It is different for different materials. It can be defined mathematically as:

$$\alpha = \frac{1}{L_0} \quad \frac{\Delta L}{\Delta T} \tag{11.12}$$

Increasing temperature causes an increase in volume as well. Experiments show that if the temperature change $\Delta T$ is not too large $\Delta V$ is approximately proportional to $\Delta T$ and to the original volume $V_0$. Thus

$$\Delta V \propto V_0 . \Delta T$$

$$\Delta V = \beta V_0 . \Delta T \tag{11.13}$$

The constant of proportionality is called the coefficient of volume expansion. Like $\alpha$, $\beta$ is also different for different materials. The coefficient of volume expansion for a solid is related to the coefficient of linear expansion. To obtain the relation we consider a rectangular parallelopiped with dimension $L_1$, $L_2$ and $L_3$. Then

$$V_0 = L_1 . L_2 . L_3$$

According to Eq. 11.11 when the temperatue increases by $\Delta T$, the length of each side changes and the new volume is

$$V_0 + \Delta V = L_1 (1 + \alpha \Delta T) \times L_2 (1 + \alpha \Delta T) \times L_3 (1 + \alpha \Delta T)$$

$$V_0 + \Delta V = L_1 L_2 L_3 \ (1 + \alpha\Delta T)^3$$

$$= L_1 L_2 L_3 \left[ 1 + 3\alpha\Delta T + 3\alpha^2 (\Delta T)^2 + \alpha^3 (\Delta T)^3 \right]$$

If $\alpha$ is small we can ignore terms containing $\alpha^2\Delta T^2$ and $\alpha^3\Delta T^3$. Dropping those and subtracting $V_0 = L_1 L_2 L_3$ from both sides we find on comparing it with the Eq. (11.13) that

$$\Delta V = (3\alpha) \ V_0 \Delta T$$

Comparing it with Eq. (11.13) we get

$$\beta = 3\alpha$$

**Thus the coefficient of cubical expansion is 3 times the coefficient of linear expansion. Given in Table 11.1 are the coefficients of thermal expansion of different materials.**

***TABLE 11.1: Coefficients of thermal expansion at 20 ℃***

| Material | $\alpha$ .K$^{-1}$ | $\beta$.K$^{-1}$ |
|---|---|---|
| Silver | $1.861 \times 10^{-5}$ | |
| Glass | $0.4 - 0.57 \times 10^{-5}$ | |
| Copper | $1.6 \times 10^{-5}$ | |
| Brick and Concrete | $1.0 - 1.4 \times 10^{-5}$ | |
| Steel | $1.2 \times 10^{-5}$ | |
| Brass | $2.0 \times 10^{-5}$ | |
| Aluminium | $2.6 \times 10^{-5}$ | |
| Mercury | | $18.2 \times 10^{-5}$ |
| Water | | $6.4 \times 10^{-5}$ |
| Glycerine | | $49.0 \times 10^{-5}$ |
| Petroleum | | $95.0 \times 10^{-5}$ |
| Methanol | | $113.4 \times 10^{-5}$ |
| Benzene | | $124.0 \times 10^{-5}$ |

Hence, in the design of any structure which is subject to changes in temperature, some provision must, in general, be made for expansion. Suppose that the ends of a rod are rigidly fixed so as to prevent expansion or contraction and the temperature of the rod is changed, tensile or compressive stresses called thermal stresses, will be set up in the rod. These stresses may become so large as to stress the rod beyond its breaking strength. It is due to this reason that gaps are left between the rails while railway lines are being laid. In bridges, one end is rigidly fastened to its abutment while the other end rests on rollers.

Thermal stresses can also be induced by non-unifrom heating. The breaking of thick glass container when very hot or very cold water is poured into it is a familiar phenomenon.

Thermal expansion can be put to many practical uses. Some of these are described below.

### 11.4.1 Bimetallic Thermometer

A useful thermometer can be made by bonding together two dissimilar metals whose coefficient of expansion is different (Fig. 11.6 a). When the temperature is increased, the different amounts of expansion cause the bimetallic strip to bend. Often the bimetallic strip is in the form of a coil. Its one end is fixed and the other end is attached to a pointer. This kind of thermometer (Fig. 11.6 b) is used for ordinary air thermometer, as an oven thermometers and in automobiles for the automatic choke. Bimetallic strips are also used in thermostates for heating system. Shown in Fig. 11.7 is a bimetallic strip used as a thermostat.

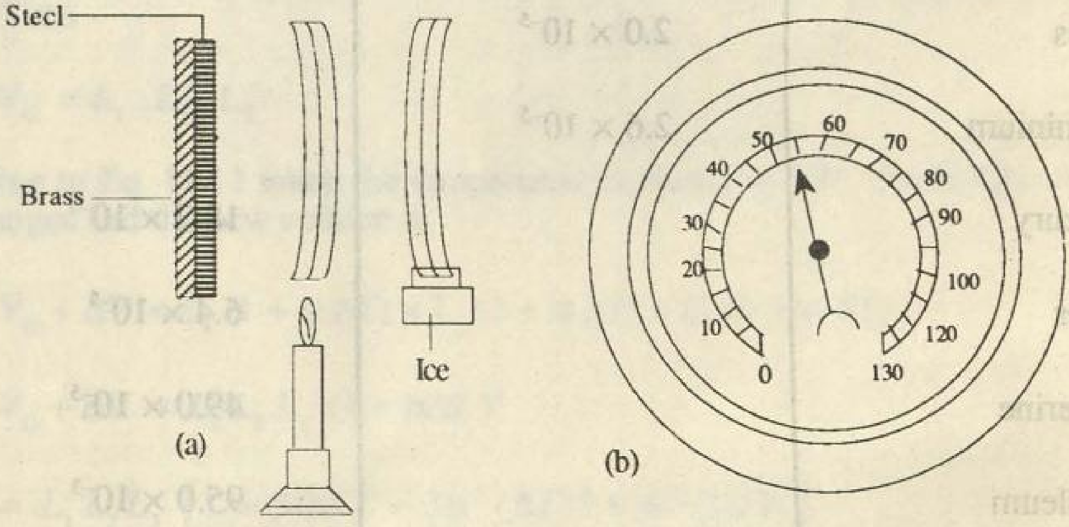


*Fig 11.6(a) Bimetallic Strip. One metal expands or contracts more than the other when heated or cooled.*
*(b) Common thermometer made from coiled strip.*

At a certain temperature the strip, due to uneven expansion of metals, bend sufficiently to break the contact that opens the electric circuit and stops the heater.

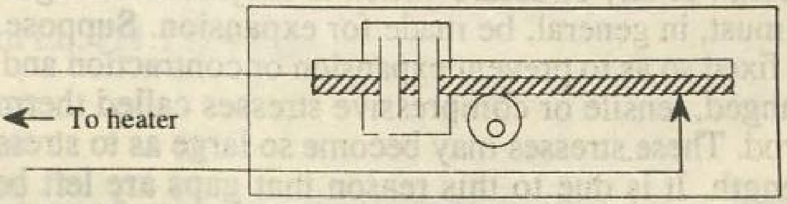


*Fig. 11.7 Bimetallic strip used in a thermostat*

**Example 11.8**

A concrete slab is 20 m long. How much longer will it be at 35K than it is at 15°C? $\alpha$ for concrete is $10 \times 10^{-6}\ \mathrm{K}^{-1}$.

**Solution**

We know that

$$L_t = L_o (1 + \alpha T)$$

Substituting the values of $\alpha$ and $T$ in the above equation and solving

$$L_t = (20\ \mathrm{m}) \left[1 + 10 \times 10^{-6}\ \mathrm{K}^{-1} (308\ \mathrm{K} - 258\ \mathrm{K})\right]$$

$$= 20\ \mathrm{m} + 20\ \mathrm{m} \times 10 \times 50 \times 10^{-6}$$

$$= 20\ \mathrm{m} + 01\ \mathrm{m} = 20.01\ \mathrm{m}$$

Hence, increase in length = 20.01 m – 20.0 m = 0.01 m

## 11.5 HEAT CAPACITY

When a body is heated, the change in its temperature is found to be proportional to the heat given to the body. It is also found that for a given temperature rise, the amount of heat required is proportional to the mass of the body. For example, if there is more water in a kettle then it will take more time to heat, say from 20°C to 100°C. This means it will absorb more heat. Furthermore, different materials required different amount of heat even when the masses of the bodies are the same and they are heated through the same temperature. Thus by heat capacity of a body we mean the amount of heat absorbed by a body when it is heated through 1 K

Heat absorbed $\propto$ the mass of the body, $m$

$\propto$ the temperature change $(T - T_o)$

or $Q \propto m (T - T_o)$

or $Q = cm (T - T_o) = cm\Delta T$ where $\Delta T = (T - T_o)$

or $$c = \frac{Q}{m\ (T - T_o)} = \frac{Q}{m\Delta T} \qquad (11.14)$$

where $c$ is a constant that depends upon the nature of the material of the body and is called the Specific Heat. If $T - T_0 = 1$ K then $Q = cm$ is called the heat capacity. It is defined as the amount of heat required to raise the temperature of 1.0 kg of substance through 1 K. The specific heat of water at 15°C is 4180 J $kg^{-1}$ ($K^{-1}$). The values of specific heat of some other substances are given in Table 11.2.

***TABLE 11.2 :*** *Specific heat of substances*

| Substance | Specific heat J/kg. K |
|---|---|
| Aluminium | 900 |
| Copper | 390 |
| Glass | 840 |
| Ice (–5°C) | 2100 |
| Iron or Steel | 450 |
| Marble | 860 |
| Silver | 250 |
| Alcohol (ethyl) | 2400 |
| Water | 4200 |
| Lead | 128 |

### 11.51 Molar Heat Capacity

It is often convenient to use the mole to describe the amount of a substance. Therefore, we define molar heat capacity as the amount of heat required to raise the temperature of one mole of a substance through 1 K. Mathematically, it can be writen as

$$c_m = \frac{1}{n} \times \frac{Q}{T_2 - T_1} = \frac{1}{n} \times \frac{\Delta Q}{\Delta T} \qquad (11.15)$$

where $C_m$ is the molar heat capacity. Hence,

$$C = C_m = \frac{1}{n} \; \frac{\Delta Q}{T_2 - T_1}$$

$$\Delta Q = nC_m \left(T_2 - T_1\right) \qquad (11.16)$$

The molar heat capacity of water is approximately 75300 J $kmol^{-1}$ ($K^{-1}$). Molar heat capacsity is also known as molar specific heat.

### 11.5.2 Measurement of Specific Heat

We know from our experience that when two bodies at different temperatures are placed in thermal contact, the temperature of the hotter body decreases while that of the colder body increases. When the two bodies reach thermal equilibrium, their final common temperature is intermediate between their original temperatures. We call the energy that is transferred between bodies because of their temperature difference as heat energy, or simply heat. As heat is a form of energy, no heat is lost when substances at different temperatures are mixed. Thus in any heat transfer the total energy lost by hot substances equals the heat gained by cold substances. This is known as the law of heat exchange. Thus the total number of heat energy units lost by hot body is equal to the total energy units gained by cold body. So

Heat lost = Heat gained

This equation provides us the method, known as method of mixture, for determining the specific heat of a substance. The hot substance of known mass and temperature but unknown specific heat is mixed with liquid of known mass and temperature contained in a vessel called calorimeter of known mass and temperature. The final temperature of the mixture is measured. All the data for the law of heat exchange equation are known except the specific heat of the substance which can be calculated as illustrated in the examples given below.

**Example 11.9**

A copper calorimeter of mass 300 g contains 500 g of water at a temperature of 15°C. A 560 g block of copper at a temperature of 100°C is dropped into the calorimeter. The temperature increases to 22.5°C. Neglect heat losses to surroundings. Find the specific heat of copper.

**Solution**

Mass of the calorimeter, $m_1$ = 300 g = 0.3 kg

Temperature of the calorimeter, $T_1$ = 15C° + 273 = 288 K

Sp. heat of copper calorimeter, $c$ = ?

Mass of water in the calorimeter, $m_2 = 0.5$ kg

Temperatue of the water, $T_1 = 288$ K

Sp. heat of the water = 4200 J/kg. K

Mass of the copper block, $m_3 = 0.56$ kg

Initial temperature of the copper block, $T_2 = 273 + 100\text{C}^\circ = 373$ K

Final temperature of the mixture, $T_3 = 273 + 22.5\text{C}^\circ = 295.5$ K

Sp. heat of the copper $c$ = ?

Heat absorbed = $cm\left(T_3 - T_1\right) + 4200\ m_2\left(\text{T}_3 - \text{T}_1\right)$

Heat given $= cm_3\left(T_2 - T_3\right)$

From the law of heat exchange

Heat gained = Heat lost

$$c \times 0.56\ \text{kg.}\ \ (373\ \text{K} - 295.5\ \text{K})$$

$$= c \times 0.3 \times (22.5 - 15) + 4200\ \frac{\text{J}}{\text{kgK}}$$

$$\times 0.5\ \text{kg} \times (2955.\ \text{K} - 288\ \text{K})$$

or $\quad c \times 0.56 \times 77.5 = 2.25\ \text{C} + 15750$

or $\quad 43.4\ \text{C} - 2.25\ \text{C} = 15750$

or $\quad c = 382.374\ \text{J kg}^{-1}\left(\text{K}^{-1}\right)$

## 11.6 THERMODYNAMICS

Thermodynamics is the study of the relationship between heat and other form of energy and is expressed by the laws of thermodynamics. There are four laws of thermodynamics. In this chapter we discuss only the first and second law of thermodynamics. Our consideration for the most part will remain restricted to the interaction of simple systems (solids, liquids and gases) and their surroundings. For instance, when we speak of a gas that may be compressed or expanded in cylinders by pistons, the gas is the "simple system" under consideration. Every thing other than the system in the universe is termed as surroundings. The systems and its surroundings are usually separated by walls of some kind. Walls that allow heat to flow through them are called diatherral walls. Walls that do not permit heat to flow are called adibatic walls. There are several kinds of systems. A closed system is one in which the mass of the system remains constant while in an open system mass and energy may enter or leave it. Many systems that we study in physics are closed, with respect to mass but there are other systems like plants and animals which are open systems because they exchange materials with the surroundings (food, oxygen, waste product etc).

A closed system is said to be an isolated system. For any meaningful thermodynamical discussion, the' systems must be defined in terms of some variables. Only then we can give a clear and unambigous description of it.

### Thermodynamic States

The thermodynamic state of a system refers to the Physical conditions of the systems and is usually described by physical paramenters like pressure, volume and temperature. When a vessel containing gas reaches thermal equilibrium the gas has a definite volume, pressure and temperature. For an ideal gas, the relation $pV = nRT$ represents this state. Thus if two of these variables are given the third can be found. This particular situation, where the gas has specified values of $p$, $V$, and $T$ is called a state of the system. Whenever the gas is returned to the same values of $p$, $V$ and $T$, its states will be the same.

Certain features of system are always the same when the system is in a given state. The variables that describe these features are called state variables. For a system consisting of gas, $p$, $V$, and $T$ are state variables. No matter how system reaches a particular state, the state itself is characterised by the same values for volume, temperature and pressure.

Since the energy of a closed system is conserved we can keep track of the changes that take place in the internal energy. If we know how much energy a system has at a particular time we can easily tell how much energy it will have at some later stage. Thus internal energy is a state variable. Heat on the other hand is not state variable, because there is no way to evaluate how much heat a system contains. Although we can calculate the amount of heat that must flow in a block of metal, for instance, to raise its temperature through certain range, yet the same temperature change could be brought by the action of friction without any heat flow into the block. Thus we see that we cannot associate a certain amount of heat with a given state of the system in a unambigous way.

### 11.6.1 First Law of Thermodynamics

One of the ways to change the internal energy of a system is by the way of losing or gaining heat from the surrourdings. Another is if the surroundings do work on the system or system does work on the surroundings. In either case, the change in the internal enrgy is governed by the principle of conservation of energy. The first law of thermodynamics is merely a statement of the law of conservation of energy when it is stated with reference to heat and work. That is "The heat energy supplied to a system is equal to the increase in the internal energy of the system from an initial value $u_i$, to the final value $U_f$ plus the work done by the system on its surroundings."

$$U_f - U_i = \Delta Q - \Delta W$$

Let us consider, as an example, that an amount of heat $\Delta Q$ is added to a system. This energy appears as an increase in the internal energy $\Delta U$ stored in the system and as the work done $\Delta W$ by the system. Mathematically, we can write the first law of thermodynamics as given below:

$$\Delta Q = \Delta U + \Delta W \tag{11.17}$$

or $$\Delta U = \Delta Q - \Delta W$$

note that be work W enters with a minus sign on the right hand side of Eq. (11.17). The minus sigh is present as matter of convention because the work done by the system is assigned a positive value and work done on the system is assigned a negative value. While applying Eq. (11.17) it must be remembered that all the quantities must be expressed in the same units. $\Delta Q$ is taken to be positive when heat enters the system and negative when it leaves the system.

Eq. (11.17) defines internal energy or more precisely the change in the internal energy of system. Thus the change in the internal energy is equal to the energy flowing in as heat energy minus the energy flowing out as work. Thus the change in the internal energy of a system is measured by the energy retained by the system in the form of the change in translational, vibrational and rotational kinetic energy due to intermolecular forces. Internal energy is similar to gravitational potential energy. We know from mechanics that when a body is moved from one position (A) to another position (B) in the gravitational field, the change in the gravitational potential energy depends only upon the two positions and not at all on the path through which the body is moved. Similarly, in thermodynamics when a system has its state changed from state (A) to state (B), the change in the internal energy $\Delta U$ ($= U_B - U_A = \Delta Q - \Delta W$) depends only upon the initial and final states and not at all on the path taken between these two states. However, just like potential energy it is the change in the internal energy and not its absolute value which is important.

## Applications

### *(i) Isobaric Process*

In many practical applications of thermodynamics, the work done by the system is carried out by means of change of volume. Let us then study the relation between work done and change in volume. Consider a system consisting of gas, as shown in (Fig. 11.8) enclosed in a cylinder by a frictionless piston of cross-sectional area $A$. Let us consider how much work the gas does as it expands pushing the piston slightly upward.

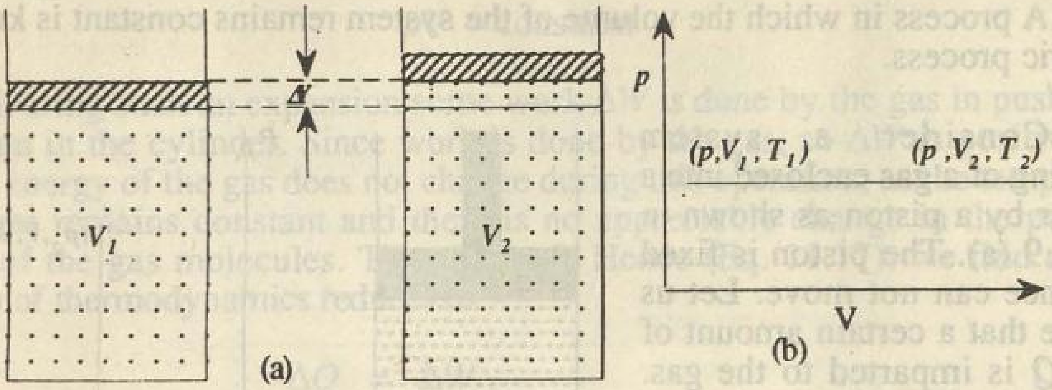


*Fig. 11.8 (a) If A is the area of the piston then $\Delta V = A\ \Delta Y$*
*(b) Heat supplied increases the volume of the gas*

Suppose the piston initiallly is in equilibrium under the action of the force $PA$ exerted by the gas and an external force $F$. Suppose now, that the gas expands and moves the piston outwards. If the displacement of the piston is kept very very small, the pressure of the gas will not change much and can be considered to be constant. Let this small displacement of the piston is $\Delta Y$ in the upward direction, the work done by the gas is

$$\Delta W = \text{Force. distance}$$

$$= pA\ .\ \Delta Y$$

But $A\Delta Y$ is $\Delta V$, the increase in the volume of the gas. Hence

$$\Delta W = p\Delta V = p\left(V_2 - V_1\right)$$

where $V_1$ is the initial volume and $V_2$ is the final volume of the gas as shown in the figure. Because the pressure exerted by the piston on the system is the same as the pressure exerted by the gas on the piston, therefore, the result can be summarized in the following way:

The work that the system does in expanding through $\Delta V$ against a constant pressure $p$ is $p\Delta V$. $\Delta V$ will be negative if the gas contracts and in this case the work done by the system is negative. Such processes in which pressure is kept constant are known as Isobaric processes.

Substituting the value of $\Delta W$ in Eq. (11.17) we get

$$\Delta Q = \Delta U + p\Delta V = \Delta U + p\left(\Delta_2 - V_1\right) \qquad (11.18)$$

which is another form of the first law of thermodynamics which is applied to those thermodynamic processes in which pressure of the system is kept constant.

### *(ii) Isochoric Process*

A process in which the volume of the system remains constant is known as Isochoric process.

Consider a system consisting of a gas enclosed into a cylinder by a piston as shown in Fig. 11.9 (a). The piston is fixed and hence can not move. Let us suppose that a certain amount of heat $\Delta Q$ is imparted to the gas. The gas is heated at constant volume and as a result its pressure increases from $p_1$ to $p_2$ as shown in Fig. 11.9 (b).

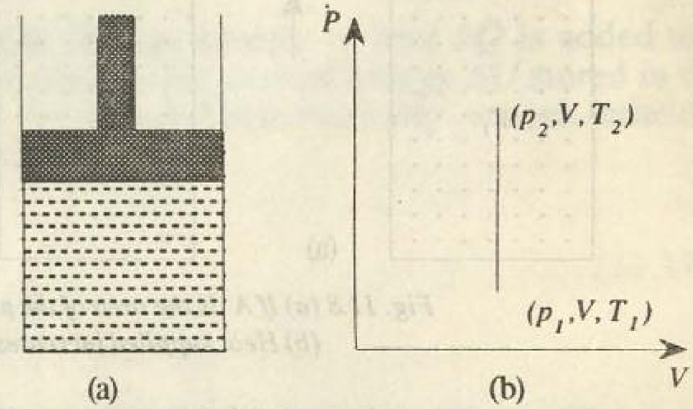


*Fig. 11.9 (a) Cylinder with fixed piston (b) the heat supplied increase P*

As position of the piston is fixed, hence no external work is done and the heat imparted to gas increases its internal energy only. That is to say, $\Delta W = 0$ in Eq. (11.16), and we get

$$\Delta Q = \Delta U \tag{11.19}$$

### *(iii) Isothermal Process*

An isothermal process is a process that takes place at constant temperature. For an isothermal process, we consider a gas contained in a cylinder having a perfectly heat conducting base and non-conducting walls and the piston. The cylinder is placed on a heat reservoir. (A heat reservoir is a body of large heat capacity at a temperature $T$. If a small quantity of heat enters or leaves its body, its temperature remains unchanged). In order to keep temperature of the gas constant the changes in its pressure and volume must be carried out very slowly. This is accomplished by slightly decreasing the weight on the piston. The tendency of the gas to cool down is compensated by the conduction of the heat from the heat reservior and thereby the temperature of the system remains constant, i.e., equal to the temperature of the reservoir. Such an expansion is called isothermal expansion. The continuous curve shown in Fig. 11.10 (b) is called an isotherm.

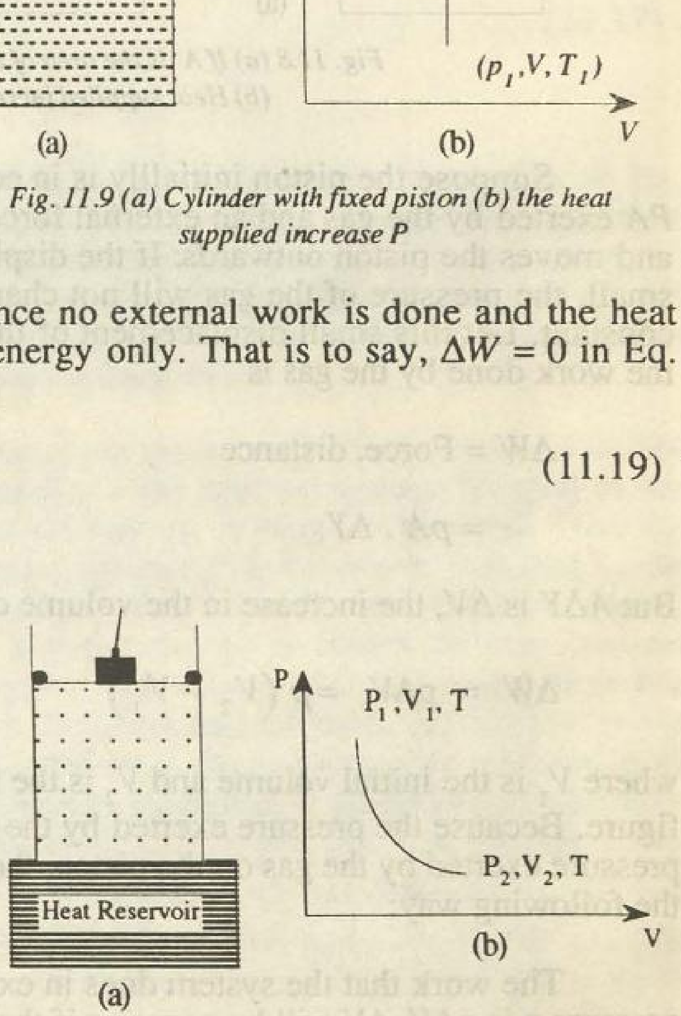


*Fig. 11.10 (a) In an isothermal process there is no change in the internal energy of the system. (b) an isotterm*

The isothermal curve or an isotherm is given by Boyle's law which is mathematically written as

$$pV = \text{constatnt}$$

During such an expansion some work $\Delta W$ is done by the gas in pushing up the piston in the cylinder. Since work is done by the gas, so $\Delta W$ is positive. The internal energy of the gas does not change during this operation, as the temperature of the gas remains constant and there is no appreciable change in the potential energy of the gas molecules. Thus $\Delta U = 0$. Hence (Eq. 11.17), we find that the first law of thermodynamics reduces to

$$\Delta Q = \Delta W \qquad (11.20)$$

This shows that if the gas expands and does external work $\Delta W$, an amount of heat $\Delta Q$ has to be supplied in order to maintain its temperature constant. Conversely, if the gas contracts, the work is being done on it and an amount of heat $\Delta Q$ has to be allowed to leave the gas.

### *(iv) Adiabatic Process*

A process in which no heat enters or leaves a system is called an adiabatic process. For every adiabatic process $\Delta Q = 0$. This prevention of heat flow may be achieved either by surrounding the system (say of a gas) with a thick layer of heat insulating material (such as cork, asbestos, firebrick or styrofoam) or by performing an expansion process quickly. Example is the rapid escape of air from a burst tyre. The flow of heat requires finite time, so any process performed quickly enough will be practically adiabatic.

Applying the first law of thermodynamics to an adiabatic process we get

$$O = \Delta U + \Delta W$$

or

$$\Delta U = -\Delta W \qquad (11.21)$$

Thus the change in the internal energy of the system in an adiabatic process is equal in magnitude to the work done by the system. If $\Delta W$ is negative, then the work is done at the cost of the internal energy of the gas. The curve shown in Fig. 11.11(b) is the graphic representation of an adiabatic expansion. Such a curve is known as adiabat. This curve is obtained by the following mathematical equation

$$pv^{\gamma} = \text{constant}$$

where $\gamma$ is explained in the following article.

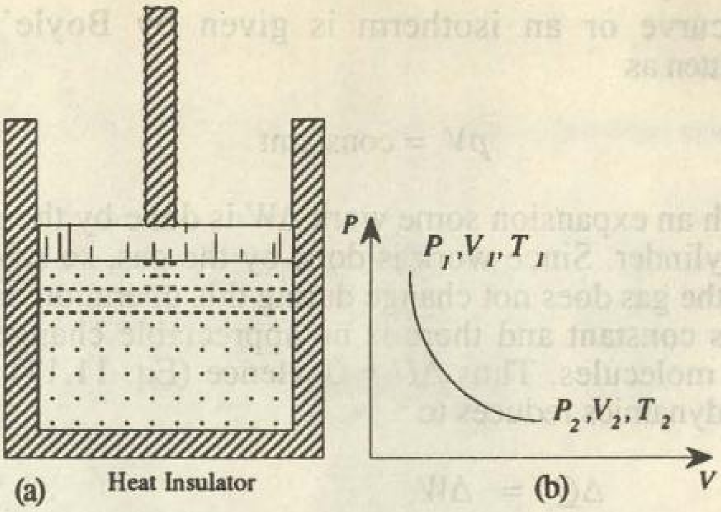


*Fig. 11.11 An adiabatic expansion (b) An adiabatic curve*

### (v) Heat Capacity of an Ideal Gas

The temperature of a gas may be changed under variety of conditions. The volume may be kept constant or the pressure may be kept constant or both may be allowed to vary in some definite manner. The amount of heat per mole necessary to cause unit rise of temperature is different in each case. Thus a gas has many different molar heat capacities. The molar heat capacity at constant volume and the molar heat capacity at constant pressure denoted by $C_v$ and $C_p$, respectively, are the two more important heat capacities, and are discussed below.

Consider *n* moles of an ideal gas contained in a cylinder fitted with a piston. If the piston is fixed and the gas is heated, its volume remains constant and all the heat supplied goes to increase the kinetic energy of the molecules due to which the temperature of the gas increases. If $\Delta Q_V$ is the amount of heat supplied and $\Delta T$ is the rise in temperature, then

$$\Delta Q_V = n\, C_V \Delta T \qquad (11.22)$$

The pressure of the gas increases during this process, but no work is done, because the volume is kept constant. Hence $\Delta W = 0$, and from first law of thermodynamics we get

$$\Delta Q_V = \Delta U$$

or

$$\Delta U = nC_V \Delta T \qquad (11.23)$$

If, however, the piston is free to move up or down, the gas may be allowed to expand at a constant pressure which is equal to the outside (or atmospheric) pressure, while it is being heated.

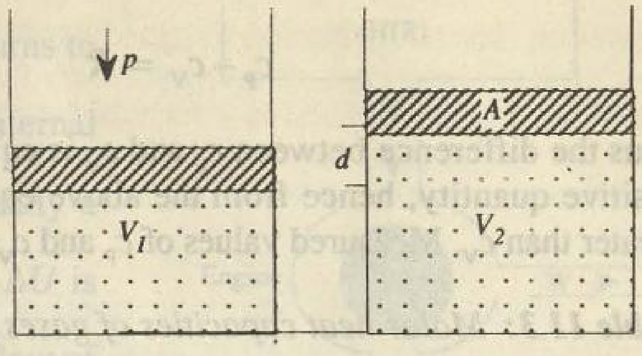


*Fig. 11.12 When a gas is at constant pressure a part of heat is used to increase the temperature of the gas and a part is used for expansion.*

When the gas is heated in this way its temperature rises and at the same time it expands by pushing, the piston upward against the constant external pressure, thus doing some work. By definition of the molar heat capacity at constant pressure $C_p$ the heat $\Delta Q_p$ flowing into the gas is

$$\Delta Q_p = nC_p\Delta T \qquad (11.24)$$

The work $\Delta W$ done by the gas against the external pressure can be calculated by taking the area of cross-section of the piston as $A$ and the distance through which the piston has moved as $d$ i.e.,

$$\Delta W = p\,Ad$$

But $Ad$ is the change in volume $\Delta V$, therefore

$$\Delta W = p\,\Delta V$$

Since p is constant, therfore it follows from the ideal gas equation that when a gas expands or contracts isobarically

$$p\Delta V = nR\Delta T \qquad (11.24a)$$

On comparing the above two equations, we find that

$$\Delta W = n\,R\,\Delta T$$

From first law of thermodynamics

$$\Delta U = \Delta Q - \Delta W$$

**On subtituting the values of $\Delta U$, $\Delta Q$ and $\Delta W$ from Eqs. (11.23), (11.24) and (11.24a), we get**

$$nC_v\,\Delta T = n\,C_p\Delta T - n\,R\,\Delta T$$

**On simplifying the above equation, we obtain**

$$c_P - c_V = R \tag{11.25}$$

Thus the difference between $c_p$ and $c_V$ is equal to molar gas constant. Since R is a positive quantity, hence from the above equation we find that the value of $C_p$ is greater than $c_V$. Measured values of $c_p$ and $c_V$ are listed below in Table 11.3

***Table 11.3:*** *Molar heat capacities of gases at low pressure*

| Type of gas | Gas | $C_P$ J.mol$^{-1}$K$^{-1}$ | $C_V$ J.mol$^{-1}$.K$^{-1}$ | $C_P - C_V$ | $\gamma = \frac{C_P}{C_V}$ |
|---|---|---|---|---|---|
| Monatomic | He | 20.78 | 12.47 | 8.31 | 1.67 |
| | A | 20.78 | 12.47 | 8.31 | 1.67 |
| Diatomic | $H_2$ | 28.74 | 20.42 | 8.32 | 1.41 |
| | $N_2$ | 29.07 | 20.76 | 8.31 | 1.40 |
| | $O_2$ | 29.41 | 21.10 | 8.31 | 1.40 |
| | CO | 29.16 | 20.85 | 8.31 | 1.40 |
| Polyatomic | $CO_2$ | 36.94 | 28.46 | 8.48 | 1.30 |
| | $SO_2$ | 40.37 | 31.39 | 8.89 | 1.29 |

In the last column of the Table 11.3 are given the values of the ratio $\frac{c_P}{c_V}$ denoted by Greek letter γ (gamma). It is seen that γ for monoatomic gases is 1.67 and is about 1.40 for diatomic gases.

## 11.7 HEAT ENGINE AND THE SECOND LAW OF THERMODYNAMICS

A heat engine is a device that accepts heat from a high temperature source of heat, called a reservoir; converts a part of the heat energy into work; and rejects the remaining part of the heat to a reservior at the lower temperature called "the sink". Here we consider heat engines that operates in a cycle. By cycle we mean: Any series of changes or operations performed by or on a system which brings it back to its original state.

Because a cyclic heat engine returns to its original state after each cycle, the internal energy of its "working substance", usually a gas, assumes its original value. Then $\Delta U$ is zero for a complete cycle, and the first law of thermodynamics relates that the heat $\Delta Q$ entering the system in each cycle equals the net work $\Delta W$ that the engine performs each cycle i.e. $\Delta Q = \Delta W$.

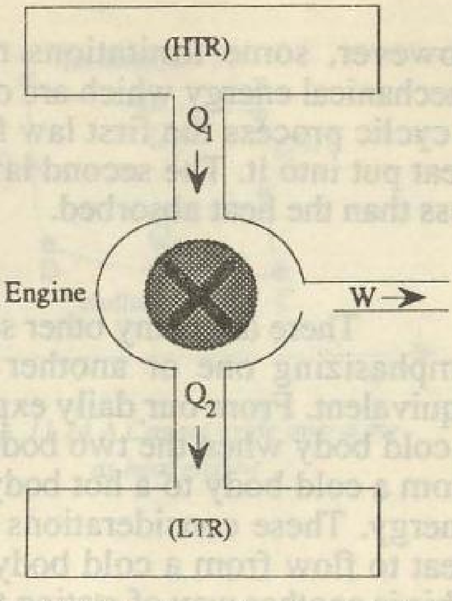


*Fig 11.13 A schematic representation of a heat engine.*

Thus, the net heat absorbed by a cyclic heat engine in one cycle is equal to the net work done by the engine during that cycle. Let $Q_1$ be the heat absorbed by an engine from a high temperature reservoir (HTR). Let $Q_2$ be the heat rejected by the engine to low temperature reservoir (LTR) or sink as shown in Fig. 11.13. The heat $Q_2$ rejected to sink is taken to be negative.

The object of an engine is to extract as much external work as possible per unit of heat energy supplied to it. We are interested in how effeciently heat engines do so. The thermal efficiency $\eta$ of a cyclic heat engine is defined to be the ratio of the net work $W$ done by the engine each cycle to the heat $Q_1$ absorbed each cycle, or

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \qquad (11.26)$$

where $Q_2$ is the heat rejected each cycle. From Eq. (11.26) it is obvious that a heat engine will have 100 per cent efficiency if $Q_2 = 0$, that is if no heat were exhausted by the engine so that all the heat absorbed were converted to work. It is an experimental fact that no cyclic engine can achieve an efficiency of 100 per cent and this fact is incorporated in the second law of thermodynamics as stated by Kelvin. "No heat engine, operating continously in a cycle, can extract heat from a heat reservoir and convert all of it into work." The first law of thermodynamics is a particular form of the law of conservation of energy and tells us of the equivalence of heat and work, the second law is concerned with the circumstances in which heat can be converted into work. It is based upon experimental results and cannot be proved directly. There is no known example that contradicts this law. There are, however, some limitations regarding the transfromation of heat energy into mechanical energy which are covered by the second law of the termodynamics. For a cyclic process the first law forbids getting more work out of an engine than the heat put into it. The second law, however, asserts that the work performed will be less than the heat absorbed.

There are many other statements of the second law of thermodynamics each emphasizing one or another aspect of the law but they can be proved to be equivalent. From our daily experience we know that heat flows from a hot body to a cold body when the two bodies are in thermal contact. Heat can be made to flow from a cold body to a hot body as in the case of refrigerator by the expenditure of energy. These considerations led Clausius to remark - "It is impossible to cause heat to flow from a cold body to a hot body without the expenditure of energy". This is another way of stating the second law of thermodynamics.

The two statements given above can be shown to be equivalent. For this we need to show that, if either statement is false the other statement must be false also. Suppose Clausius's statement were false so that we could have a refrigerator operating without doing any work on it. We could use an ordinary engine to remove heat from a hot body, to do work and to return part of the heat to a cold body. But by connecting our perfect refrigerator into the system, this heat would be returned to the hot body with the expenditure of any work and would become available again for use by the heat engine and the "perfect" refrigerator would become a heat engine that violates the Kelvin statement or we can reverse the reasoning. If the Kelvin statement were incorrect, we could have a heat engine which simply takes heat from a source and converts it completely into work. By connecting this 'perfect' heat engine to an ordinary refrigerator, we could extract heat from the hot body, convert it completely to work, use this work to run the ordinary refrigerator, extract heat from the cold body, and pass it plus the work converted to heat by the refrigerator to the hot body. The net result is a transfer of heat from cold to hot body without expenditure of work. This is the violation of Clausius statement.

### 11.7.1 Carnot Engine

The second law of thermodynamics forbids the existence of a heat engine with 100 percent efficiency. Then, what is the most efficient engine possible? Sadi Carnot, in 1824, proposed an ideal heat engine that has the maximum possible efficiency that can be obtained by any heat engine operating in a cycle between the two given reservoirs. The operating cycle of this hypothetical engine consists of isotherms and adiabats.

Consider an engine operating between the fixed temperatures of two given reservoirs. If this engine accepts heat $Q_1$ only from HTR (a process that proceeds) at a temperature $T_1$ along an isotherm and rejects heat $Q_2$ only to a LTR or sink (process that proceeds) at a temperature $T_2$ along an isotherm such that $T_2 < T_1$), the paths connecting the isotherms are adiabats. The Carnot cycle operating an engine is shown in Fig. 11.14. A Carnot cycle consists of the following steps.

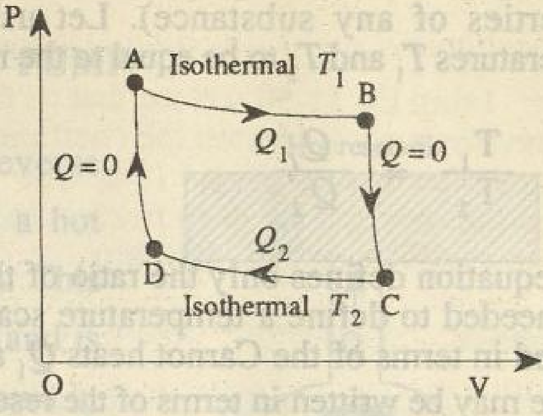


*Fig. 11.14 A Carnot cycle operating as heat engine*

1) Along isotherm A–B absorbing heat $Q_1$ at temperature $T_1$ while expanding.

2) Along adiabat B–C cooling the gas from temperature $T_1$ to $T_2$ while expanding.

3) Along isotherm C–D rejecting heat $Q_2$ at temperature $T_2$ while compressing.

4) Along adiabat D–A heating the gas from $T_1$ while compressing the shaded area ABCDA enclosed by the cyclic path represents the net work performed by the engine in one cycle. Then using Eq. (11.26) we find the efficiency of the Carnot engine to be

$$\eta_c = 1 - \frac{Q_2}{Q_1} \qquad (11.27)$$

An important proposition, proposed and proved by Carnot, is equivalent to the second law of thermodynamics. He said that: A Carnot engine is the most efficient engine that can operate between two given temperatures. Any engine that absorbs heat from a HTR rejects heat to a LTR and exchanges no heat in the intervenning processes necessarily oprates only along isotherms and adiabats. Such an engine is a Carnot engine. It also follows from the Carnot's proposition that the efficiencies of all Carnot engines operating between two given temepratures are the same (Otherwise some would be more efficient thereby violating the proposition). Thus, the efficiency of a Carnot engine is independent of the working substance between fixed temperatures $T_1$ and $T_2$. It has the same ratio of heat rejected to heat absorbed i.e., $\frac{Q_2}{Q_1}$, regardless of the working substance of the engine. Because $\frac{Q_2}{Q_1}$ is the same for any working substance of a Carnot engine, we can define an absolute temperature scale (it is absolute because it does not depend on the properties of any substance). Let us define the ratio of absolute (Kelvin) temperatures $T_1$ and $T_2$ to be equal to the ratio of $Q_1$ and $Q_2$ or

$$\frac{T_1}{T_2} = \frac{Q_1}{Q_2} \tag{11.28}$$

This equation defines only the ratio of the absolute temperatures. A fixed point is also needed to define a temperature scale. The absolute temperature can also be defined in terms of the Carnot heats $Q_1$ and $Q_2$. Thus the efficiency of any Carnot engine may be written in terms of the reservior temperatures:

$$\eta_C = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} \tag{11.29}$$

The above equation shows that the efficiency of a Carnot engine will be 100 percent, or $\eta_C = 1$, if $T_2 = 0$. This means when the LTR or sink is at absolute zero, when $T_2 = 0$, $Q_2 = 0$ and the work done by a Carnot engine is equal to $Q_1$. All the heat absorbed by the engine is converted into work, but an engine that converts all the heat that enters the engine in to work violets the second law of thermodynamics. Thus, achieving a temperature of absolute zero is violation of second law of thermodynamics.

### Example 11.10

Calculate the efficiency of a heat engine whose lower temperature or exhaust temperature is 17°C and the high temperature is 200°C.

### Solution

The data given is as follows:

$$T_1 = 200C^\circ + 273 = 473\text{ K}$$

$$T_2 = 17C^\circ + 273 = 290\text{ K}$$

$$\eta = 1 - \frac{T_2}{T_1}$$

Sustituting the values we get

$$\eta = 1 - \frac{290\text{K}}{473\text{K}} = 0.38 = 38\%$$

Thus the efficiency of heat engine is 38%. These calculations are based on the assumption of a Carnot cycle. The efficiency of an actual engine will be less.

## REFRIGERATOR AND THE HEAT PUMP

A heat engine operating in reverse transfers heat from a cold reservoir to a hot reservoir with the help of external work. This external work is also converted into heat and is given over to the hot reservoir. This is shown in Fig 11.15. A heat engine operating in reverse is called a refrigerator or a heat-pump. In a refmigerator the interest lies in the amount of heat that is removed from a cold reservoir, while in a heat pump the interest is the amount of heat rejected to the hot reservoir. Both these devices cause heat to flow from a low temperature reservoir to a HTR at the cost of the external mechanical work that must be supplied to the system.

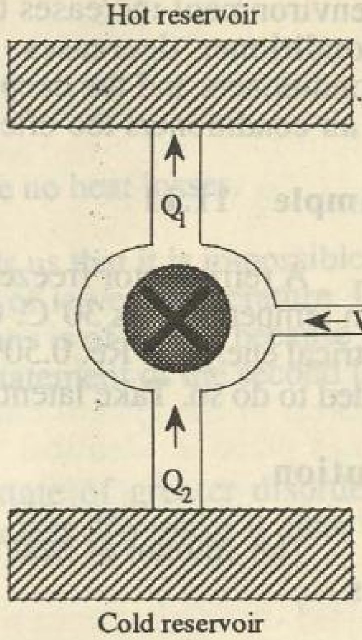


*Fig 11.I5 A Schematic representatio of a heat engine.*

From the figure we see that an amount of heat $Q2$ at temperature $T_2$ is transferred from the cold reservoir to a hot reservoir at temperature. $T_1$ with the help of external work $W$. Thus $Q_2 + W$ is the amount of heat $Q_1$ which is rejected to the HTR. From first law of thermodynamics

$$Q_1 = Q_2 + W$$

In refrigerators instead of efficiency we use the term called coefficient of performance $E$. It is defined as

$$E = \frac{\text{desired out put}}{\text{required out put}} = \frac{Q_2}{W}$$

Considering the refrigerator as Carnot cycle, $Q_2/Q_1 = T_2/T_1$ ; then

$$E = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2} \qquad (11.30)$$

which is the coefficient of performance of an ideal refrigerator. $T_2$ is the temperature of the refrigerator and $T_1$ is the exhaust temperature or the room temperature.

From Eq 11.30 we see that as the room temperature or the temperature of the environment increases the coefficient of performance of the refrigerator or an airconditioner dcereases. It is due to this reason that the performance of airconditioners and the deep freezers etc; is affccted durning the extremely hot days. The air conditioners too often trip

**Example 11.11**

A refrigerator freezes 5 kg of water at 0°C into ice at 0°C in 15 minutes. the room temperature is 30°C. Calculate the cost of making 5 kg of ice, if the cost of electrical energy is Re. 0.50 per kWh, also calculate the minimum amount of power needed to do so. Take latent heat of fusion of ice as 334400J $kg^{-1}$

**Solution**

The amount of heat that must be extracted from the water to convert in into ice is

$$Q_2 = mL_f = 5\text{ kg} \times 334400\text{ Jkg}^{-1} = 1672000\text{ J}.$$

Since $T_2 = 0°C = 273$ K and $T_1 = 30°C = 303$ K, the minimum work that must be done from equation $\frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$ is

$$W = Q_2 \frac{T_1 - T_2}{T_2}$$

$$= 1672 \times 10^3\text{J} \left(\frac{30\text{K}}{273\text{K}}\right)$$

$$= 1672 \times 10^3\text{J} \times 0.109 = 183736\text{ J}$$

Because 1 kWh = $\left(1 \quad 10^3\text{Js}^{-1}\right)$ (3600s) = $3.6 \times 10^6$ J

Thus

$$W = 0.183 \times 10^6\text{ J} \left(\frac{1\text{kWh}}{3.6 \times 10^6\text{ J}}\right) = 0.05\text{ kWh}.$$

The cost of electrical energy per kWh = Re. 0.50, hence the total cost of making 5 kg of ice

$$= (0.50 \times 0.50) = \text{Rs. } 2.5$$

As this work must be accomplished in 15 minutes, the minimum power required to run the refrigerator is

$$P = \frac{W}{t} = \frac{0.183 \times 10^6 \text{ J}}{15 \times 60 \text{s}} = 203 \text{ W}$$

This calculation is for an ideal refrigerator where there are no heat losses.

As we see the second law of thermodynamics tells us that it is impossible to make use of thermal energy unless it can flow to region of lower temperature. For this reason the thermal energy stored in the water of oceans is of no use because we can not find cooler place to which it can flow. Another statement of the second law is given below.

"Natural process tends to proceed towards a state of greater disorder". Exactly what we mean by disorder may not always be clear. To make it clear we study the concept of entropy.

## 11.8 ENTROPY

Consider a jar containing separate layers of salt and pepper arranged neatly on top of one another. This arrangement is more orderly. Now if the jar is shaken, the salt and pepper are all mixed together. No amount of shaking brings back the layer arrangement of the salt and pepper as it was in the beginning. The natureal process is from a state of relative order (layers) to one of relative disorder (a mixture), and not the reverse. That is disorder increases. Tea cups break when they fall, but they do not spontaneously mend themselves, because a solid tea cup is more orderly than the pieces of broken cup. Thus normal course of events is an increase of disorder.

When a hot object is put in contact with a cold body, heat flows from the high temperature to the one at a low temperature until the two bodies reach the same intermediate temperature. Before bringing the two bodies in contact we can distinguish between molecules of high average kinetic energy and molecuels of low kinetic energy. After having brought the two bodies in contact all molecules become molecules of the same average kinetic energy. We no longer can distinguish one class of molecules from the other. The order has given way to disorder. When the two objects were at different temperatures we could get useful work by making them the hot and cold region of heat engine. But once the two objects are in contact with one another and they reach the same temperature, no work can be obtained from them. Disorder has increased.

The implication of order and disorder in a system can be approached in two quite different ways. Both approaches use a quantity called entropy. Entropy was first use around the middle of the nineteenth century by R. Clausius. Clausius described the behaviour of systems in terms of *P*, *V*, *T* and *U*. To describe the consequences of the fact that heat flows from a hot body to a cold one, he found it convenient to define a quantity that he called entropy in the following way.

Let an amount of heat $\Delta Q$ be added to a system in such a way that its temperature remains constant. As a result of it there is a change in the entropy of the system. It is denoted by $\Delta S$ and is defined mathematically as

$$\Delta S = \frac{\Delta Q}{T} \tag{11.31}$$

$\Delta S$ is positive i.e., the entropy of the system increases if heat flows into the system. While using this relation, it must be remembered that $T$ is absolute temperature and that heat must be added to the system in a reversible way i.e., the accompanying change in the system must be reversible. A reversible change is the change that takes place in identical ways in both the forward and backward direction. For example, in changing reversibly from state A to state B, the state variables for the system take on definite values at each step during the cahnge. In the reverse process, going from B to A, state variables of the system repeat these steps in reverse order.

It was shown by Clausius that under indentical conditions, a system has the same entropy. Thus, entropy is also a state variable like $Q$, $V$, $T$ and $U$. As entropy is a state variable, it proves to be an important and useful quantity for the description of thermodynamic process. It was further shown that entropy is a measure of disorder. The more disordered the state of a system, the larger will be its entropy. Thus we see that entropy is a state variable which measure disorder. Second law of thermodynamics can be stated in the following way also.

"If an isolated system undergoes change, it will change in such a way that its entropy either remains constant or it tends to be maximum".

**Example 11.12**

What is the change in entropy of 15 g of water as it is cooled from 20°C to 18°C. (Assume $T$ to be nearly constant).

**Solution**

As $\Delta S = \frac{\Delta Q}{T}$

We calculate $\Delta Q$ using Eq. (11.23)

$mc\,\Delta T. = C_p\, n\Delta T$

where

$C_p$ (of water) $\cong$ 4200 J/kg K

$$\Delta T = (20 - 18)\ \text{K}$$

$$m = \frac{15}{1000}\ \text{kg}$$

On substituting these values in the above equation, we find that

$$\Delta Q = 4200\ \text{J/kgK} \times \frac{15\text{kg}}{1000}\ (20 - 18)\ \text{K} = 126\ \text{J}$$

Average temperature is

$$T = 273 + 19\text{C}^{\circ} = 292\ \text{K}$$

$$\Delta S = \frac{126\ \text{J}}{292\ \text{K}} = 0.43\ \text{J K}^{-1}$$

The examples of order going to disorder illustrate another important aspect of the second law of thermodynamics. It reveals that the thermal energy can be considered as a less useful form of energy than the mechanical energy. Mechanical energy can be used.to do work. But after it is changed into thermal energy, the possibility of doing work is lost. Thus we can consider mechanical and other forms of energy as more ordered than thermal energy. This transformation energy from more orderly form to less ordered form is called degradation of energy.

Entropy has also been called as "time's arrow", because it tells us the direction in which the time is going. To appreciate it, let us consider that we are watching a film in which funny occurrences are taking place. We see that the broken pieces of tea cup lying on the floor assemble and then rise to the top of the table, the tea spilt on the table top enters into the cup and from there the stream of tea enters the spout of the teapot. From these observations we conclude that the film is being run backward, because the normal sequence of events is contrary to the events observed in the film. The normal sequence is that it is the disorder which increases with time as is also warranted by the second law of thermodynamics. Thus we are justified to term entropy as "Time's Arrow".

A prediction of second law of thermodynamics is that as time goes on the universe will approach a state of maximum disorder. Matter will become a uniform mixture, heat will have flowed from high temperature regions to low-temperature region until the whole universe is at one temperature. No work can then be done. All the energy of the universe will have become degraded to thermal energy. All changes will cease. This is the so called heat death of the universe.

# SUMMARY

***Heat as Energy in Transit :*** The sum of all the energy of all the molecules in an object is known as internal energy or thermal energy. Heat is not the energy that a body contains, it refers to the amount of energy transferred from a hot to a cold body.

***Unit of Heat :*** Heat is a form of energy, therefore the, unit of heat is joule.

***Temperature :*** Temperature is a measure of "hotness" or "coldness" of a body.

There are three scales of temeprature measurement; namely, Celsius Scale, Fahrenheit Scale and the Kelvin Scale.

***Kinetic theory and Gas laws :*** Pressure of an ideal gas is given by the following

$$p = \frac{1}{3} m v^2 N$$

and in terms of the average kinetic energy of the molecules of the gas

$$p = \frac{2}{3} N \left( \frac{1}{2} m v^2 \right)$$

***Avogadro's Hypothesis :*** Equal volumes of gases at the same temperature and pressure contain equal number of molecules.

***Expansion of Solids :*** The chane in length per unit length per kelvin rise in temperature is called coefficient of linear expansion $\alpha$ i.e.,

$$\alpha = \frac{\Delta L}{L_o} \times \frac{1}{T}$$

Coefficient of cubical expansion $\beta = 3\,\alpha$.

***Specific Heat Capacity :*** The amount of heat required to rasie the temperature of 1.0 kg of a substance through 1 K is called specific heat of the substance.

***Thermodynamics :*** Thermodynamics is the study of relationship between heat and other forms of energy.

***System :*** A system is any obejct or set of objects that we wish to consider. Everything other than the system is termed as "surroundings"

***Closed System :*** A system in which the mass of the system remains constant is called a closed system with respect to matter. a closed system is said to be an isolated system if no energy in any form leaves or enters its through its walls.

***State variables :*** The variables like $p$, $V$ and $t$ are called state variables.

***Reversible Process :*** The process in which the state variables acquire the same values at all stages of the process independent of the direction in which the process is being carried out is called reversibel process.

***Cyclic Process :*** A cyclic process is that in which during each cycle, it extracts heat from a heat reservoir maintained at one temperature, converts part of this energy to useful work and rejects the remainder to a heat reservoir at a lower temperature.

***Heat Engine :*** A heat engine is a thermodynamic system that is capable of a absorbing energy in the form of heat from a soruce, and converting part of it into mechanical energy.

***Efficiency of an Engine :***

$$\text{Efficiency} = \frac{\text{net out put work per cycle}}{\text{input energy per cycle}}$$

$$\text{Efficiency} = 1 - \frac{Q_2}{Q_1}$$

***Carnot's Cycle :*** A cycle that has highest possibel efficiency is called Carnot's cycle and the engine working on this principle is called Carnot's engine.

$$\text{Its efficiency} = 1 - \frac{T2}{T1}$$

***Second Law of Thermodynamics :*** Heat flows spontaneously from a hotter to a colder body, but not the other way round.

***Clausius Statement :*** It is impossible for a cyclic system to transfer heat from body at a low temperature to a body at a higher temperature unless external work is done on the sytem.

***Kelvin's Statement :*** It is impossible to construct an engine, operating continuously in a cycle, which does nothing other than to take heat from a source and perform an equivalent amount of work without having a sink.

***The Refrigerator :*** A heat engine operating in reverse is called a refrigerator. This device causes heat to flow from a low temperature reservoir to a high temperature reservoir at the expense of suplying external mechanical work to the system. Coefficient of performance of refrigerator

$$E = \frac{\text{desired out put}}{\text{required out put}} = \frac{T_2}{T_1 - T_2}$$

***Entropy :*** Entropy is a measure of disorder. The change in entropy

$$\Delta S = \frac{\Delta Q}{T}$$

Second law of thermodynamics can be restated in terms of entropy as stated below.

If an isolated system undergoes change, it will change in such a way that its entropy will increase or at least remains constant.

# QUESTIONS

**11.1** When two systems are in equilibrium, which of the following statements are true and which are false (a) they have the same amount of kinetic energy (b) they are at the same temperature (e) the molecules have the same amount of heat.

**11.2** Why is the average velocity of the molecules in a gas zero but the average of the square of the velocity is not zero?

**11.3** Why does the pressure of a gas in an automobile tyre increase if the automobile is driven for a while?

**11.4** Why is the molar specific heat of a diatomic gas usually larger than that of monatomic gas?

**11.5** Why must more work be done when a gas is heated at constant pressure than at constant volume?

**11.6** If a pendulum clock has to keep correct time at different temperatures, will it be better to use aluminium or steel?

**11.7** Under what conditions can heat be added to a system without exchanging its temperature?

**11.8** Why does the temperature drop in an adiabatic expansion process?

**11.9** If work is done by friction, will the entropy increase or dcerease?

**11.10** Why is it not possible to convert a given amount of heat completely into work? Is the reverse possible, why?

**11.11** Is it possible, according to the second law of thermodynamics, to construct an engine that will be free of thermal pollution.

**11.12** Is it possible to cool a room by keeping the refrigerator door open? Explain.

**11.13** Using the first law of thermodynamics, explain why the total energy of an isolated system is always conserved.

**11.14** Is it possible to convert internal energy into mechanical energy? Explain with examples.

**11.15** Under what conditions is it possible for the entropy of a system to decrease ? Give an example.

## PROBLEMS

**11.1** Prove that the Celsius and Fahrenheit scales have the same reading at $-40^\circ$.

**11.2** A test tube filled with air is sealed off at *S.T.P*. It is then heated to 300°C. What then is the air pressure in the tube in newton per square metre, in pascles, and in atmosphere? Given 1 atmospheric pressure =76 cm Hg $=1.01 \times 10^5\ P_a = 1.01 \times N\ m^{-2}$

[Ans. $2.12 \times 10^5\ Nm^{-2}$, $2.12\ P_a$, 2.10 atm]

**11.3** Air at 27°C and atmospheric pressure is suddenly compressed to one twentieth of its original volume and a pressure of 30 atmosphere. What is the temperature of the gas?

[Ans 450 K, 177°C]

**11.4** One steel metre bar is correct at 0°C and another at 25°C. What is the difference between their lengths at 20°C? $\alpha$ for steel $= 1.2 \times 10^{-5}\ k^{-1}$.

[Ans. 0.003cm]

**11.5** An electric heater supplies 1800 W of power in the form of heat to a tank of water. How long will it take to heat the 200 kg of water in the tank from 10°C to 70°C? (Assume heat losses to surrounding to be negligible. Specific heat of water is $4200\ J\ kg^{-1}\ K^{-1}$.)

[Ans. 7.77 h]

**11.6** A copper calorimeter has a water equivalent of 5.9 g. That is in heat exchanges, the calorimeter behaves like 5.9 g of water. It contains 40 g of oil at 50°C. When 100g of lead at 30°C is added to this, the final temperature is 48.0°C. What is the specific heat capacity of oil? (Specific heat of lead is $128\ J\ kg^{-1}\ K^{-1}$.)

[Ans. 2260.5 J $kg^{-1}$ $K^{-1}$]

**11.7** What is the change in internal energy of 200 g of nitrogen as it is heated from 10°C to 30°C at constant volume? (For nitrogen $C_V = 20760$ J $kmol^{-1}$ $K^{-1}$.)

[Ans. 2948 J]

**11.8** Five grams of helium gas is heated from –30°C to 20°C. Find the change in its internal energy and the work done by the gas if the heating occurs at (a) constant volume, (b) constant pressure. For He $C_V = 12470$ $Jmol^{-1}$ $k^{-1}$, $Cp = 20780$ $Jkmol^{-1}$ $K^{-1}$.

[Ans. (a) 1558.75 J (b) 1299J, 1038.75J ]

**11.9** An air conditioner absorbs heat from its cooling coils at 10°C and expels heat to the outside at 40°C. Calculate the maximum coefficient of performance of the air conditioner.

[Ans. 9.43]

**11.10** Temperature differences between the surface water and bottom water in Mancher lake might be 5°C. Assuming the surface water to be at 20°C, what is the highest efficiency a heat engine could have if it operates between these two temperatures?

[Ans. 1.7%]

**11.11** A Carnot engine whose low temperature reservoir is at 7°C has an efficiency of 40%. It is desired to increase the efficiency to 50%. By how many degrees must the temperature of the high temperature reservoir increase?

[Ans. 93K]

**11.12** What is the change in entropy of 30g of water at 0°C as it is changed into ice at 0°C. Take latent heat of fusion of ice 33600 J $kg^{-1}$.

[Ans. 36.92 J $K^{-1}$]

# 12

# ELECTROSTATICS

Most of us are familiar with the electrical effects produced by friction in every day situations. We find that on rubbing the shoes on a rug and then reaching for the door knob produces a spark between the fingers and the knob. The combing or brushing hair briskly produces spark crackle. These electrical effects have been known for ages. In previous classes we have studied that the process of rubbing an object produces charge called static electricity.

## 12.1 COULOMB'S LAW

The qualitative investigations made in the previous classes revealed that similar charges repel and opposite charges attract each other with a force, known as force of interaction. Here we make quantitative analysis of the nature of these forces. We would like to determine the magnitude and direction of such forces. The quantitative measurement of the force between two electric charges was first made by a French scientist Charles Augustin de Coulomb (1736-1805). He carried out a series of experiments to measure the force between electric charges using an apparatus known as torsion balance.

The apparatus used by Coulomb is shown in Fig. 12.1. It is similar to the one used by Cavendish to determine the value of gravitational constant *G*. The torsion balance consists of a horizontal insulated rod R carrying two metallic spheres a and a′ at the ends of the rod. The rod is suspended from the middle with a fibre supported from F.A small mirror M is attached to the fibre and a beam of light reflected from this mirror falls on the scale S. Another insulated rod P, carrying a small sphere b, equal in size to a, can be brought near a, as shown in the Fig. 12.1.

The sphere b is charged by rubbing it with some suitable material. When sphere b is touched with sphere a, it shares its charge so that both have equal charges $q_1 = q_2 = q$. The charge on b can be further divided into $q/2$, $q/4$, $q/8$, .... by touching it with the spheres of equal sizes. By keeping the distance between a and b fixed, we find that the twist in the fibre, which is measured by deflection on the scale S, is directly proportional to the magnitudes of the charges on a and b. On varying the distance between them, it is found that the force is inversely proportional to the square of the distance between the charged spheres.

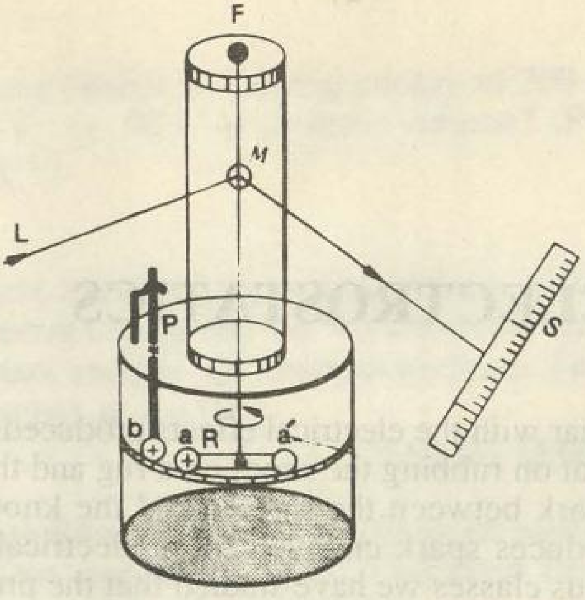


*Fig. 12.1 The schematic diagram of a torsion balance.*

Coulomb enunciated his experimental data in the form of a statement which is known as Coulomb's law. It states "The magnitude of the force between two point charges is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them."

If we use symbols $q_1$ and $q_2$ for two electric charges separated by distance $r$ (Fig. 12.2), the magnitude of the force $\mathbf{F}_e$, according to this law, is expressed as

$$F_e \propto \frac{q_1 \, q_2}{r^2}$$

or

$$F_e = k \frac{q_1 \, q_2}{r^2} \tag{12.1}$$

where $k$ is a constant of proportionality and its value depends upon the system of the units used and the medium between the charges. The electric charges $q_1$ and $q_2$ are assumed to be point or localized charges, provided the size of the bodies carrying the charges is very small as compared to the distance between them.

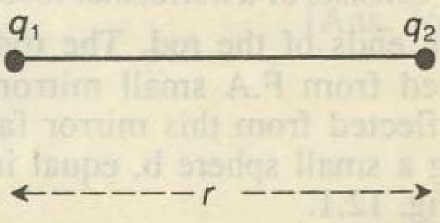


*Fig. 12.2 Two charges separated by a finite distance*

Eq. (12.1) gives the magnitude of the force between two charges but does not say anything about the direction of the force. If $\hat{r}_{12}$ is taken to be a unit vector along the line joining the two charges, pointing from the charge $q_1$ towards the charge $q_2$ as shown in Fig. 12.3, then the force on charge $q_2$ due to charge $q_1$ is given by

$$\mathbf{F}_{21} = k\,\frac{q_1\,q_2}{r^2_{12}}\,\hat{\mathrm{r}}_{12} \qquad (12.2)$$

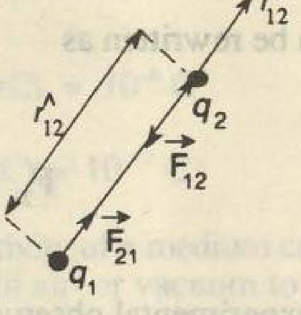


*Fig. 12.3 A unit vector $\hat{r}_{12}$ gives the direction of force.*

If the charges are alike, the right hand side of Eq. (12.2) will be positive and $\mathbf{F}_{21}$ will be a force of repulsion between the two charges. Similarly, if the charges are unlike, the right hand side of the Eq. (12.2) will be negative and $\mathbf{F}_{21}$ will be a force of attraction. It can very easily be seen that if we denote $\mathbf{F}_{21}$ as the force exerted by $q_1$ on $q_2$ and $\mathbf{F}_{12}$ as the one exerted by $q_2$ on $q_1$, then

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

Negative sign here shows that the two forces are in opposite direction.

The SI unit of charge, is the coulomb and is denoted by C. Dictated by the ultimate accuracy of standards, it was agreed to define coulomb in terms of moving charges which constitute an electric current. The SI unit of current is ampere, (denoted by A), and is equal to one coulomb/second. The ampere is standardized and considered to be the fundamental unit for the electric current. In SI unit, the unit of charge is defined as follows:

"The coulomb (C) is the quantity of charge that flows across any cross section of a wire in one second when there is a steady current of one ampere (A) passing through it".

In Eq. (12.1) if we take $F = 1$N, $q_1 = q_2 = 1$C and $r = 1$m, the value of the constant $k$ turns out to be,

$$k = 8.98742 \times 10^9\ \mathrm{N{-}m^2C^{-2}} \approx 9 \times 10^9\ \mathrm{N{-}m^2C^{-2}} \qquad (12.4)$$

The constant $k$ is generally expressed in terms of another quantity known as permittivity of the free space $\varepsilon_o$. The measured value of $\varepsilon_o$ turns out to be

$$\varepsilon_o = 8.85 \times 10^{-12}\ \mathrm{C^2(N^{-1}{-}m^{-2})}$$

therefore, $$k = \frac{1}{4\pi\varepsilon_o} = 9 \times 10^9 \text{ N–m}^2\text{C}^{-2}$$

Eq. (12.2) can be rewritten as

$$\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r_{12}^2} \hat{\mathbf{r}}_{12} \qquad (12.5)$$

It is an experimental observation that the presence of an insulator in place of air or vacuum reduces the force between the electric charges by a factor $\varepsilon_r$. This is known as relative permittivity or dielectric constant for particular insulator. The values of relative permittivity for various dielectrics are given in Table 12.1.

*Table 12.1: The relative permittivity of various dielectrics*

| Dielectric | Relative Permittivity at room temperature (293K) | Dielectric | Relative Permittivity at room temperature (293K) |
|---|---|---|---|
| air | 1.0006 STP | diamond | 6.0 |
| oil | 3.0 | sulphur | 3.4 |
| Paraffin wax | 2.8 | mica | 5.7 – 6.7 |
| benzene | 2.6 | ice 268K | 75.0 |
| water | 80.0 | glass | 5 – 10.0 |
| | | marble | 8.5 |

The force in a medium of relative permittivity $\varepsilon_r$ is given as

$$\mathbf{F}' = \frac{1}{4\pi\varepsilon_o\varepsilon_r} \frac{q_1\ q_2}{r_{12}^2} \hat{\mathbf{r}}_{12} \qquad (12.6)$$

$$\mathbf{F}' = \frac{\mathbf{F}}{\varepsilon_r} \qquad (12.7)$$

**The values of $\varepsilon_r$ for various dielectrics are always larger than unity (except for air in which case $\varepsilon_r = 1$), so**

$$F' < F \tag{12.8}$$

The coulomb is a big unit of charge. Generally, charge is measured in its sub-multiple units which are given as under:

$$\text{One micro–coulomb } (1\ \mu\text{C}) = 10^{-6}\ \text{C}$$

$$\text{One pico coulomb} = (1\text{pC}) = 10^{-12}\ \text{C}$$

Finally from Eq. (12.6) the relative permittivity of a medium can be defined as ratio of the force between two charges placed in air or vacuum to the force when they are placed in the medium.

**Example 12.1**

Two positive charges of magnitude 20 μC and 100 μC are placed at a distance of 150 cm from each other. Calculate the electric force of repulsion between them.

**Solution**

$$q_1 = 20\ \mu\text{C} = 20 \times 10^{-6}\text{C}$$

$$q_2 = 100\ \mu\text{C} = 100 \times 10^{-6}\text{C}$$

$$r = 150\ \text{cm} = 1.5\text{m}$$

$$F = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q_1\ q_2}{r^2}$$

$$F = 9 \times 10^9\ \text{N–m}^2\text{C}^{-2} \times \frac{20 \times 10^{-6}\ \text{C} \times 100 \times 10^{-6}\ \text{C}}{(1.5\ \text{m})^2} = 8\text{N}$$

**Example 12.2**

The electric force between two point charges placed in air is 2N. Calculate the electric force between them when the medium between them is (i) water and (ii) mica.

**Solution**

The electric force $F$, between two charges when placed in air is $F = 2\text{N}$

When placed in a medium of relative permittivity $\varepsilon_r$, the force is reduced to $F'$, such that

$$F' = \frac{F}{\varepsilon_r}$$

For water $\varepsilon_r = 80$

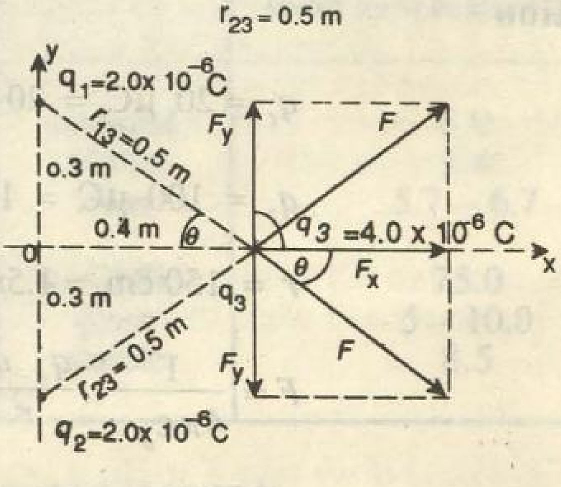
Therefore, $F' = \frac{2\text{N}}{80} = 0.025 \text{ N}$

and for Mica, $\varepsilon_r = 3.8$

Therefore, $F'' = \frac{2\text{N}}{3.8} = 0.526 \text{ N}$

**Example 12.3**

In the figure, two equal positive charges $q_1$ and $q_2$, each of magnitude $2.0 \times 10^{-6}$ C interact with a third charge $q_3 = 4.0 \times 10^{-6}$ C. Find the magnitude and direction of the electric force on charge $q_3$.



r23 = 0.5 m
y
q1=2.0x 10-6 C
r13=0.5 m
0.3 m
0.4 m
θ
Fy
F
q3 =4.0 x 10-6 C
x
o
θ
Fx
0.3 m
q3
r23=0.5 m
Fy
F
q2=2.0x 10-6 C

**Solution**

$$q_1 = 2.0 \times 10^{-6} \text{ C}$$

$$q_2 = 2.0 \times 10^{-6} \text{ C}$$

$$q_3 = 4.0 \times 10^{-6} \text{ C}$$

$$r_{13} = r_{23} = 0.5 \text{ m}$$

**The figure shows that the electric force on $q_3$ due to upper cahrge $q_1$ is,**

$$F = \frac{9 \times 10^9 \ \text{N–m}^2\text{C}^{-2} \times (4 \times 10^{-6}\ \text{C})\ (2 \times 10^{-6}\ \text{C})}{(0.5)^2\ \text{m}^2} = 0.29\ \text{N}$$

In the figure, $\sin\theta = \frac{0.3}{0.5} = \frac{3}{5}$ and $\cos\theta = \frac{0.4}{0.5} = \frac{4}{5}$

The components of this force are given by

$$F_{1x} = F\cos\theta = (0.29\ \text{N}).\left(\frac{4}{5}\right) = 0.23\ \text{N}$$

$$F_{1y} = F\ \sin\theta = (0.29\ \text{N}).\left(\frac{3}{5}\right) = 0.17\ \text{N}$$

The lower cahrge $q_2$ exerts a force of the same magnitude but in a different direction. From symmetry we see that its x-component is the same as that due to the upper charge, but its y-component is in opposite direction. Hence

$$F_x = F_{1x} + F_{2x} = 2 \times 0.23\ \text{N} = 0.46\ \text{N}$$

$$F_y = F_{1y} - F_{2y} = 0$$

The magnitude of total force is, $F = \sqrt{(F_x)^2 + (F_y)^2}$

or $\quad F = F_x = 0.46\ \text{N}$

$$\tan\ \alpha = \frac{F_y}{F_x} = \frac{0}{0.46} = 0$$

Therefore, $\alpha = \tan^{-1}(0) = 0$

## 12.2 THE ELECTRIC FIELD

From Newton's law of universal gravitation and Coulomb's law we can calculate the magnitude and direction of the gravitational and electrical forces, respectively. These laws are limited to an interactin between two point charges or masses. There are some fundamental and basic questions to be answered about these laws. (i) What is the origin of these forces? (ii) How these forces are transmitted from one body to another or from one charge to another?

The answer to (i) is unknown; we acknowledge the existence of force as it is. To answer (ii) two theories are important and are described briefly as under;

(a) action at a distance and (b) the field effect or field theory.

Action-at-a distance means that the force between two charged bodies is conveyed directly and instantaneously (i.e., with no time delay). But this is not in accordance with the experiment. Accelerating electrons in the antena of a radio transmitter influence electrons at a distance receiving antena after a time $lc^{-1}$, where $l$ is the separation of the antenas and $c$ is the velocity of light. Action at a distance idea has never been a satisfying one for scientists.

Michael Faraday (1791 - 1867) introduced field theory concept. According to field theory concept, the interaction between two charges $q$ and $q_o$ separated by some distance is explained in two steps in the following manner.

Step1 : The charge $q$ produces an electric field in the space surrounding it and step 2: the field interacts with a charge $q_o$ which is brought in the field and produces a force **F** on it.

The electric field **E** may be imagined as a sphere around a charge within which other charges are influenced by it. The existence of this electric field can be proved by bringing a test charge $q_o$ into the field. The two step process has been shown in the Fig. 12.4. and the force **F** experienced by the test charge $q_o$ may be given as

$$\mathrm{F} = q_0\mathrm{E}$$

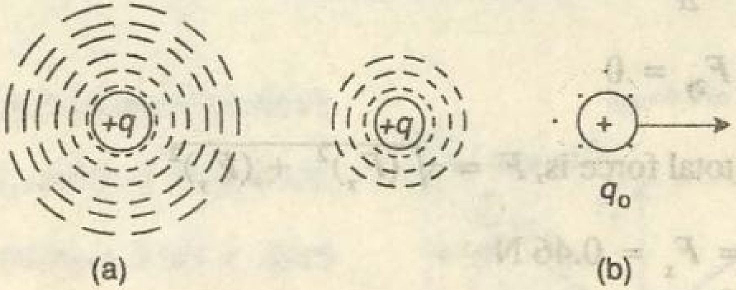


*Fig. 12.4 (a) Broken circles surrounding the positive charge indicate the presence of the electric field (b) Interaction of field with the charge $q_o$*

### 12.2.1 Electric Field Intensity

To measure the strength and direction of an electric field at a point, a unit positive charge is placed at that point. The direction in which this unit positive charge moves or tends to move is the direction of the field. The strength of the field is the magnitude of the force experienced by the unit positive charge when placed at that point. A single vector quantity containing information both about the field strength and its direction at that point is denoted by **E** and is kown as electric field intensity. Quantitatively, it is defined as the force experienced by a unit positive charge placed at that point.

If force experienced by a point positive charge $q'$ in an electric field of $q$ is **F**, then

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

or $$\mathbf{F} = q'\,\mathbf{E} \quad ..... \tag{12.9}$$

According to Coulomb's law, the magnitude of the force experienced by a point charge $q'$ at point p due to the presence of the field of a point charge $q$ at point O, as shown in the Fig.12.5, is given by,

$$F = \frac{k\,q\,q'}{r^2} \tag{12.10}$$

where $r$ is the distance between the point charges $q$ and $q'$. If $\hat{r}$ is the unit vector directed from charge $q$ to $q'$ as shown in the figure, then

$$\mathbf{F} = k\frac{q\,q'}{r^2}\,\hat{r} \tag{12.11}$$

dividing Eq. (12.11) by $q'$ or equivalently replacing $q'$ by a unit positive charge, **F** transfroms to **E** which is the electric field intensity at point p, i.e.,

$$\mathbf{E} = \frac{\mathbf{F}}{q'} = \frac{\dfrac{kqq'}{r^2}\,\hat{r}}{q'} \tag{12.12}$$

$$\mathbf{E} = \frac{kq}{r^2}\,\hat{r};\ \mathbf{E} = \mathrm{k}\frac{q}{r^2}\,\hat{r} \tag{12.13}$$

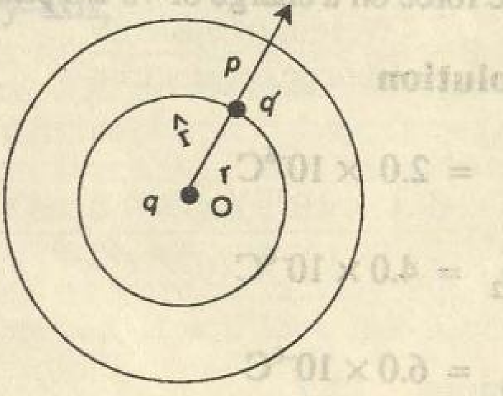


*Fig. 12.5 Coulomb's force experienced by a charge due to the presence of another charge*

**Example 12.4**

Find the magnitude of electric intensity at a point distance 50 cm from a charge of 5 μC.

**Solution**

$$q = 5\,\mu\text{C} = 5.0 \times 10^{-6}\text{C}$$

$$r = 50\,\text{cm} = \frac{50}{100} = 0.5\,\text{m}$$

Utilizing the formula for the magnitude of electric intensity

$$E = \frac{1}{4\pi\varepsilon_\circ}\ \frac{q}{r^2}$$

$$E = \left[9 \times 10^9\ \text{N–m}^2\text{C}^{-2}\right] \left[\frac{5 \times 10^{-6}\ \text{C}}{(0.5\text{m})^2}\right]$$

$$E = \left[\frac{9 \times\ 10^9 \times 0.5 \times 10^{-6}}{0.5 \times 0.5}\right]\left[\ \text{N–m}^2\text{C}^{-2} \times \text{Cm}^{-2}\ \right]$$

$$E = 180{,}000\ \text{NC}^{-1}$$

$$E = 1.8 \times 10^5\ \text{NC}^{-1}$$

### Example 12.5

Consider point charges $q_1 = +2\ \mu\text{C}$, $q_2 = +4\ \mu\ \text{C}$, $q_3 = +6\ \mu\ \text{C}$, $q_4 = -6\ \mu\text{C}$, placed at four corners of a square as shown in the figure. Each diagonal is of length 2 m, (a) Calculate the electric intensity at the centre of the square. (b) What will be the force on a charge of +5 uC placed at the centre?

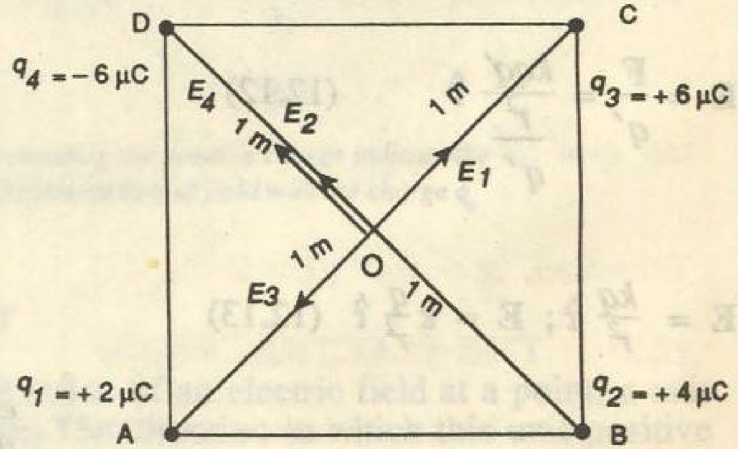
**Solution**

$q_1 = 2.0 \times 10^{-6}\text{C}$

$q_2 = 4.0 \times 10^{-6}\text{C}$

$q_3 = 6.0 \times 10^{-6}\text{C}$

$q_4 = -6.0 \times 10^{-6}\text{C}$



Let the distances of charges $q_1$, $q_2$, $q_3$ and $q_4$ from the centre be $r_1$, $r_2$, $r_3$ and $r_4$, respectively, such that

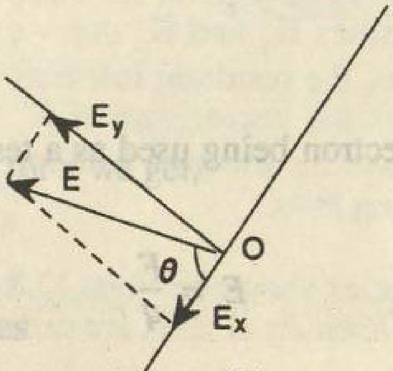
$r_1$ (=OA) = $r_2$ (=OB) = $r_3$ (=OC) = $r_4$ (=OD) = 1 m

Thus $E_1 = \frac{1.}{4\pi\varepsilon_o}\ \frac{q_1}{r_1^2} = 9 \times 10^9\ \text{N-m}^2\text{C}^{-2} \times \frac{2 \times 10^{-6}\ \text{C}}{(1)^2\ \text{m}^2} = 18 \times\ 10^3\ \text{NC}^{-1}$

$$E_2 = \frac{1}{4\pi\varepsilon_o} \quad \frac{q_2}{r_2^2} = 9 \times 10^9 \text{ N–m}^2\text{C}^{-2} \times \frac{4 \times 10^{-6} \text{ C}}{(1)^2 \text{ m}^2} = 36 \times 10^3 \text{ NC}^{-1}$$

$$E_3 = \frac{1}{4\pi\varepsilon_o} \quad \frac{q_3}{r_3^2} = 9 \times 10^9 \text{ N–m}^2\text{C}^{-2} \ \frac{6 \times 10^{-6} \text{ C}}{(1)^2 \text{ m}^2} = 54 \times 10^3 \text{ NC}^{-1}$$

$$E_4 = \frac{1}{4\pi\varepsilon_o} \quad \frac{q_4}{r_4^2} = 9 \times 10^9 \text{ N–m}^2\text{C}^{-2} \ \frac{-6 \times 10^{-6} \text{ C}}{(1)^2 \text{ m}^2} = -54 \times 10^3 \text{ NC}^{-1}$$



(b)

(a) Taking AC along x – axis and BD along y–axis,

$$E_x = E_3 - E_1 = (54 - 18) \times 10^3 \text{ NC}^{-1}$$

$$= 3.6 \times 10^4 \text{ NC}^{-1}$$

$$E_y = E_2 + E_4 = (36 + 54) \times 10^3 \text{ NC}^{-1}$$

$$= 9 \times 10^4 \text{ NC}^{-1}$$

The magnitude of resultant electric intensity at the centre of the square,

$$E = \sqrt{E_x^2 + E_y^2} = \left[\sqrt{(3.6)^2 + (9.0)^2}\right] 10^4 \text{NC}^{-1} = 9.7 \times 10^4 \text{ NC}^{-1}$$

As shown in part (b) the resultant intensity $E$ makes an angle $\theta$ with OA, and is given as

$$\tan\theta = \frac{E_y}{E_x} = \frac{9.0 \text{ NC}^{-1}}{3.6 \text{ NC}^{-1}} = 2.5 \text{ or } \theta = 68^\circ$$

(b) The force on charge + 0.5 μC placed at the centre O

$F = qE = 9.7 \times 10^{4}\ NC^{-1} \times 0.5 \times 10^{6}\ C = 4.85 \times 10^{-2}\ N$

### Example 12.6

Find the intensity of the electric field such that an electron placed in it would experience a force equal to its weight.

### Solution

If $F$ is the force on electron being used as a test charge $q$, then the electric intensity $E$ is given by,

$$E = \frac{F}{q}$$

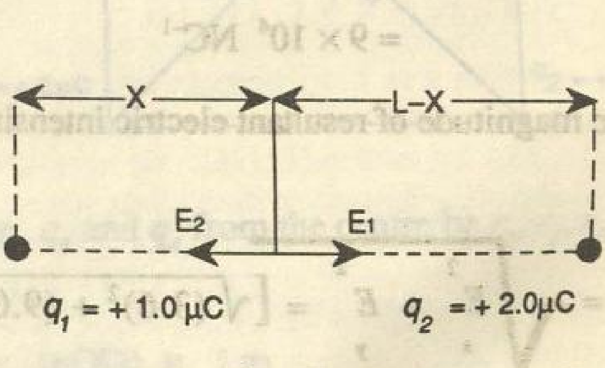
If $F$ is equal to the weight of electron and $q$ is the charge on electron then,

$$E = \frac{\text{Weight of electron}}{\text{Charge on electron}}$$

$$= \frac{(9.1 \times 10^{-31})\text{kg} \times 9.8\ \text{ms}^{-2}}{1.6 \times 10^{-19}\ \text{C}} = 5.5 \times 10^{-11}\ \text{NC}^{-1}$$

### Example 12.7

The accompanying figure shows a charge $q_1 = 1.0\ \mu C$, 10cm from a cahrge $q_2 = +2.0\ \mu C$. At what point on line passing through the two charges is the electric field zero?



**Solution**

The point lies between the charges because only here do the forces exerted by $q_1$ and $q_2$ on a test charge oppose each other. If $E_1$ is the electric field due to $q_1$ and $E_2$ is due to $q_2$, we must have

$$E_1 = E_2$$

$$\frac{1}{4\pi\varepsilon_o}\cdot\frac{q_1}{x^2} = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q_2}{(L-x)^2}$$

Taking $L =$ 10 cm and solving for x we get,

$$x = 4.1 \text{ cm}$$

### 12.2.2 Electric Field Lines

The concept of electric field lines was introduced by Faraday. The most obvious way to construct a mental picture of the electric field is to draw, at many different points, vectors representing the magnitude and direction of **E** at each point. Consider a charge + $q$ as shwon in the Fig. 12.6 (a). The arrows indicate the electric intensity at different points (black dots). The arrow head indicates the direction of the force that would be exerted on a positive charge. If these arrows are joined together, they result in radially outward lines emanating from $+q$ as shown in Fig. 12.6 (b). These lines are called electric field lines or electric lines of force. A great number of lines can be drawn from a single charge. For a given configuration of the lines, the density of the lines decreases when we move out radially. Fig. 12.6 (b) is a two dimensional picture. In three dimension, it will look like needles sticking out of a spherical pin cushion.

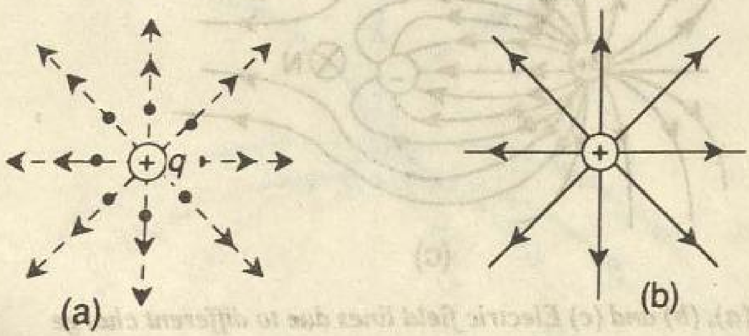


Fig. 12.6 : (a) Electric intensity at different points surrounding a point charge $+q$, (b) Electric field lines emanating from a charge $+q$

The direction of force experienced by a positive charge placed in the vicinity of a negative charge $-q$ is opposite to the force experienced by it in case of a $+q$ charge. Taking this into consideration, electric lines of force due to negative charge $-q$ have been shown in Fig. 12.7.

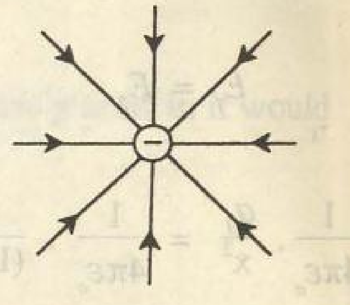


*Fig. 12.7 Electric field lines converging to a point charge $-q$.*

The idea of electric field lines from point charges can be put together to draw the lines for different combinations of charges such as shown in Fig. 12.8. In Fig. 12.8 (a) two charges $+q$ and $-q$ are placed at a certain distance. The field lines start from the positive charge and end on a negative charge. The electric intensity at any point such as p is a combination of intensities $\mathbf{E}_+$ and $\mathbf{E}_-$ due $+q$ and $-q$, respectively. Thus, the resultant intensity at P is $\mathbf{E}$ which is equal to the vector sum of $\mathbf{E}_+$ and $\mathbf{E}_-$ i.e., $\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_-$ and its direction is along the tangent to the field line at P.

In the configurations, as shown in Figs. 12.8 (b) and 12.8 (c) the points like N, where the the resultant intensity is zero, are called "neutral points"

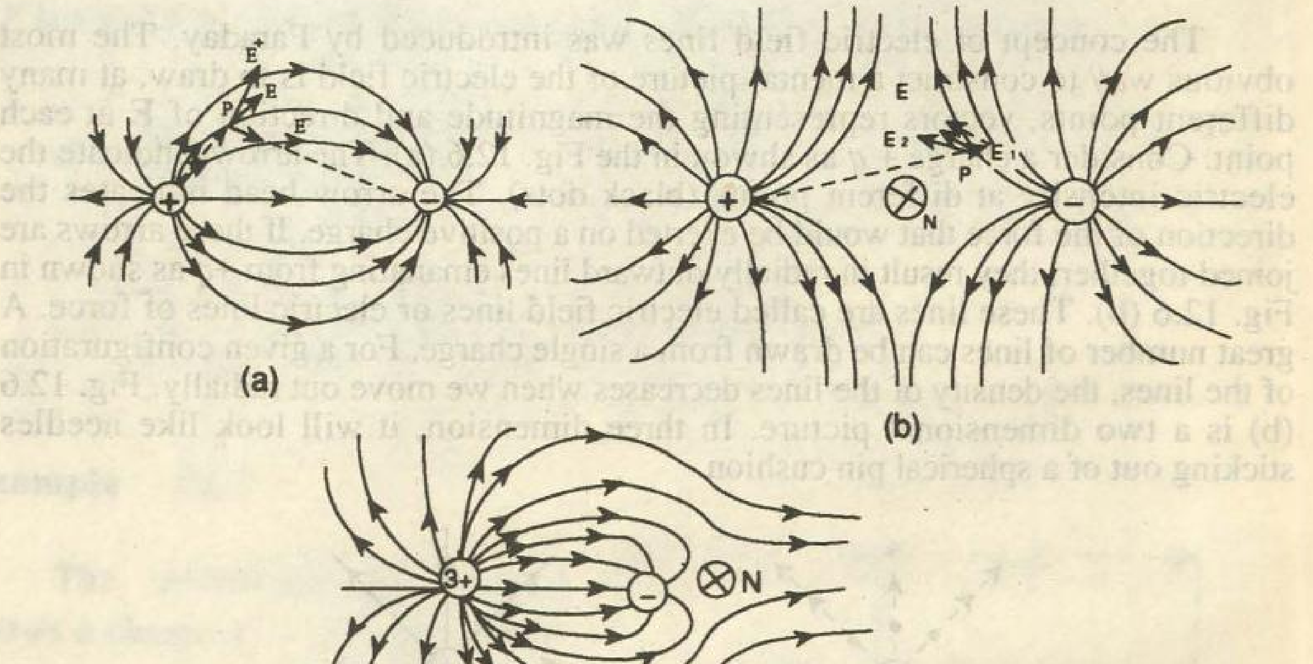


*Fig. 12.8(a), (b) and (c) Electric field lines due to different charge configuration. N indicates the neutral point.*

Fig. 12.9 (a) shows two infinitely large parallel plates separated by a very small distance. The upper plate has a uniform positive charge distribution while the lower one has a negative one. The field lines start from the positive plate and terminate at negative one. The reason is, that the direction of the electric intensity **E** is the direction in which the force acts on a unit positive charge. If the plates are not of infinite length, the field lines at the ends of plates will be a little bulging out, showing that field is not uniform at the ends.

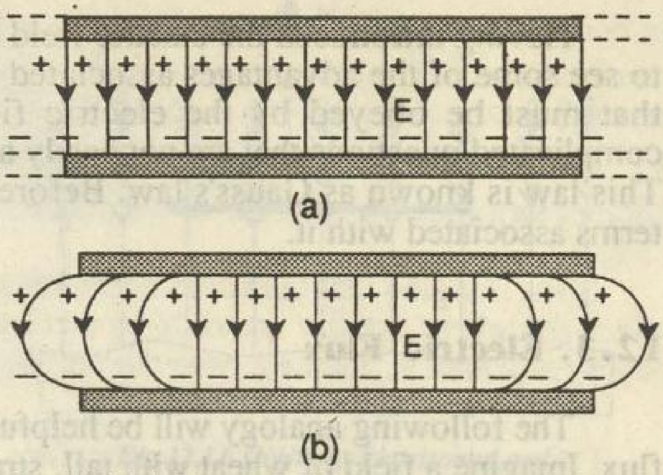


*Fig 12.9 (a) Electric field lines due to oppositely charged plates of infinite size. (b) The fringing field at he ends of two parallel oppositely charged plates due to their finite size.*

The field such as at the ends of plates is called "fringing field" and is pictorially shown in Fig. 12.9 (b).

Another interesting property of field lines can be explained with the help of Fig. 12.10.

Suppose that a charge *+q* is placed near a metal plate. The positive charge will attract the negative charges (electrons) in the metal plate, resulting in the motion of these charges until some of them reach that surface of the metal which is near to *+q* charge where they will be at rest. Thus, the field lines starting from *+q* charge will end on the negative charges of the metal plate. Furthermore, these lines are always perpendicular to the metal surface as shown in Fig. 12.10.

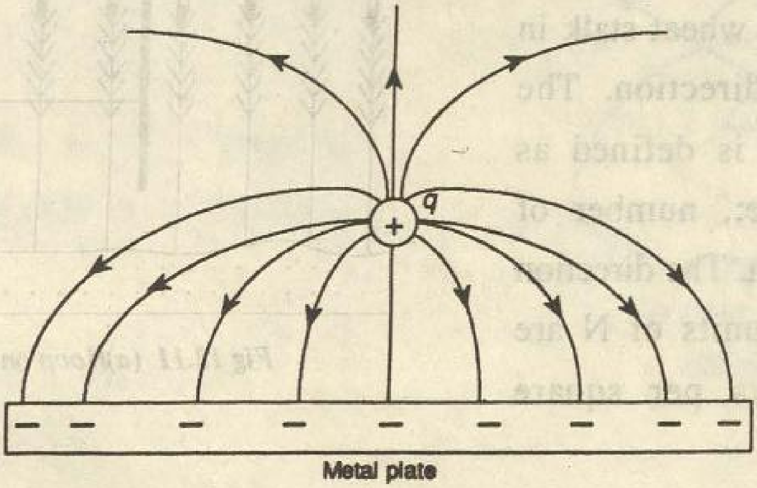


*Fig 12.10 Electric field lines are always perpendicular to the surface of a conductor under electrostatic condition.*

Having introduced the electric field and its pictorial representation, we begin to see some of the advantages associated with it. We can now define a useful law that must be obeyed by the electric field and which can easily solve some complicated questions that are not easily answered by direct use of Coulomb's law. This law is known as Gauss's law. Before we discuss this law let us explain some terms associated with it.

## 12.3. Electric Flux

The following analogy will be helpful in understanding the concept of electric flux. Imagine a field of wheat with tall, straight, uniformly spaced parallel stalks as shwon in Fig 12.11. We have a hula hoop (see Fig..12.11) which is placed suppose down over the wheat stalk. The number of wheat stalks enclosed by the hoop, depend upon the following factor:

i). Number of wheat stalks per unit area

ii). Area of the hoop and also on,

iii). The orientation of the hoop.

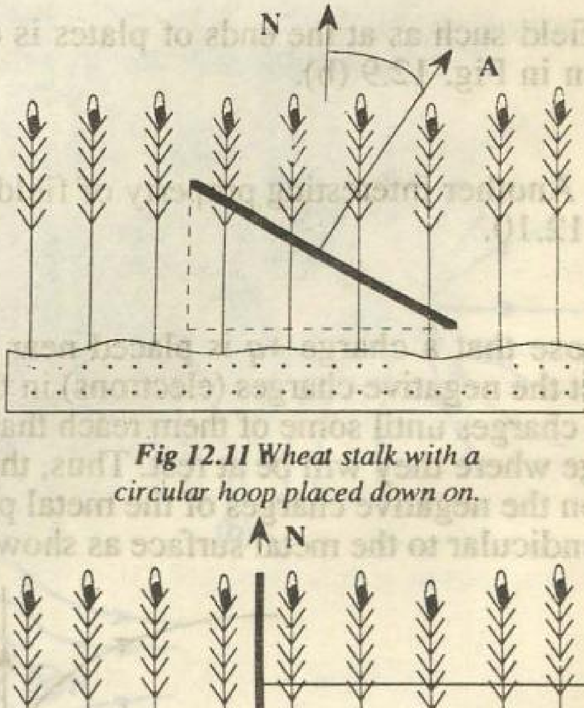


*Fig 12.11 Wheat stalk with a circular hoop placed down on.*

Let us introduce the vector N to describe the wheat stalk in magnitude and direction. The magnitude of N is defined as space density i.e;, number of stalks per unit area. The direction of N is up and units of N are number of stalks per square metre.

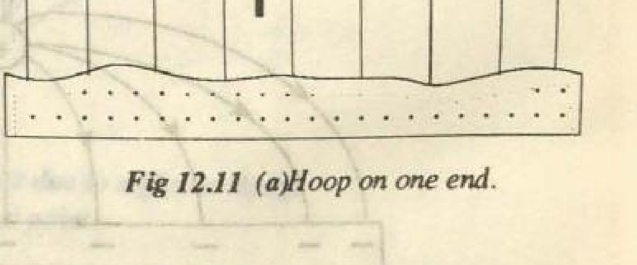


*Fig 12.11 (a)Hoop on one end.*

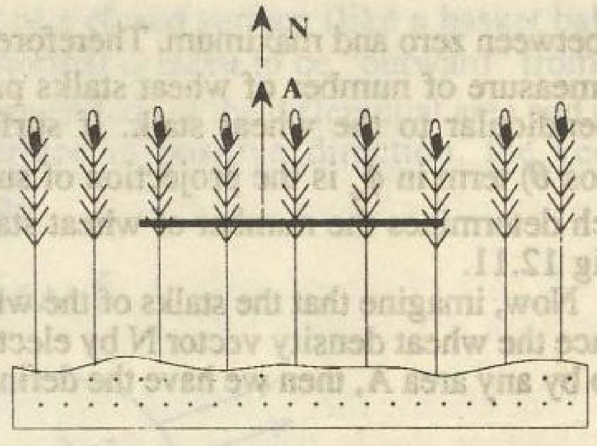


*Fig 12.11 (b)Hoop is horizontal and perpendicular to wheat stalk.*

Now let us introduce another vector A to describe the hoop. **A** is defined as the vector whose magnitude is equal to the area of the hoop and its direction is perpendicular to the circular surface defined by the hoop. If hoop is on one end (hoop plane vertical, **A** horizontal) as shown in Fig 12.11(a), then no wheat stalks passes through the hoop. If the hoop is horizontal and A is vertical (see Fig 12.11 (b), number of stalks enclosed is maximum equal to NA.

For any orientation of the hoop we define the number of wheat stalks enclosed as the "wheat flux" $\phi_w$;

$$\phi_w = \text{Wheat flux} = \text{Number of stalks enclosed}$$

$$= NA\cos\theta = \mathbf{N}\cdot\mathbf{A}$$

This agrees with the above discussion because

(i) when $\theta = 90°$. ( hoop is on one end, **A** is horizontal), then

$$\phi_w = \mathbf{N}\cdot\mathbf{A} = NA\cos 90° = 0$$

No wheat stalks pass through the hoop.

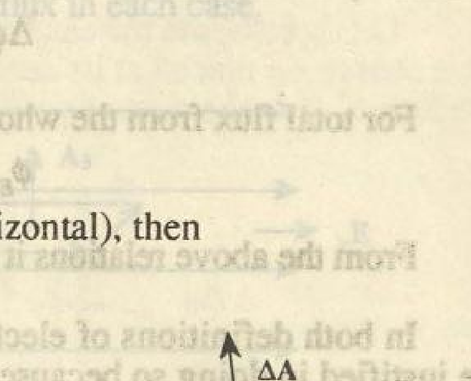


*Fig 12.12 : Curved surface with differential area ΔA.*

(ii) When $\theta = 0$. ( hoop is horizontal and **A** is vertical), then

$$\phi_w = \mathbf{N}\cdot\mathbf{A} = NA\cos 0$$

$$= NA \text{ (maximum)}$$

**For any other orientation of the hoop, cos$\theta$ lies between 0 and 1, and correspondly 'wheat flux', i.e;, number of wheat stalks passing through the area**

lies between zero and maximum. Therefore, it can be concluded that 'wheat flux' $\phi_w$ is a measure of number of wheat stalks passing through certain surface area held perpendicular to the wheat stalk. If surface area is not held perpendicular the (A cos $\theta$) term in $\phi_w$ is the projection of surface area perpendicular to wheat stalks which determines the number of wheat stalks passing through the hoop, as shown in Fig 12.11.

Now, imagine that the stalks of the wheat are actually electric field lines. If we replace the wheat density vector N by electric field vector E and replace the area of hoop by any area A, then we have the definition of "electric flux"

For a uniform field and a plane surface, the electric flux $\phi_E$ is defined to be the scalar product of the electric field vector E and the plane surface area A.

$$\phi_E = \mathbf{E}.\ \mathbf{A} \qquad (12.14)$$

For non-uniform fields and curved surfaces, we divide the surfaces into very small patches of area ΔA called differential area and assume that these are approximately flat and they are so small that electric field **E** is almostly uniform over it. For one of these small patches of area (see Fig 12.12). electric flux is defined by the relation. (differential form),

$$\Delta\phi_E = \mathbf{E}\ .\ \Delta\mathbf{A} \qquad (12.15)$$

For total flux from the whole surface we add flux from each patch such that

$$\phi_E = \sum\ \mathbf{E}\ .\ \Delta\mathbf{A}$$

From the above relations it is clear that electric flux is a scalar quantity.

In both definitions of electric flux we used a vector quantity A for area. We are justified in doing so because, as you may recall in chapter two (Class XI), area is the geometric representation of a vector product. In this context A is a vector whose magnitude is equal to the surface area (of hoop) and whose direction is normal to the surface area as shown in the Fig 12.13 (a). Two conventions specify which normal direction is positive.

1) If the surface is open, that is, if it has a perimeter like a sheet of paper or a flat dish, let the fingers of your right hand point in the direction in which the perimeter is described (Fig 12.13a). Then your thumb gives the direction of positive normal.

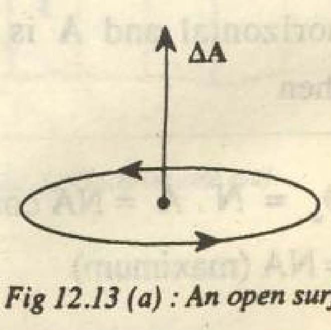


*Fig 12.13 (a) : An open surface*

2) If the differential area **ΔA** is a part of a closed surface (like a basket ball or football, for example) the positive normal is taken to be "outward" from the volume being enclosed. If $dA$ is the magnitude of the differential area and $\hat{n}$ is the unit vector normal to the surface in positive direction, the vector differential area **ΔA** will be written as,

$$\Delta \mathrm{A} = \Delta \mathrm{A}\ \hat{n}$$

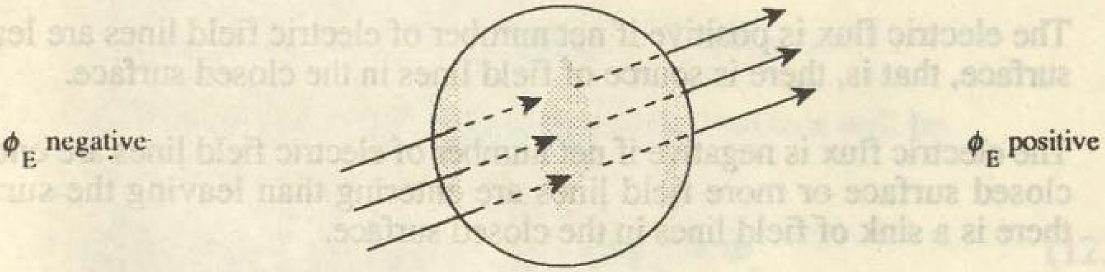
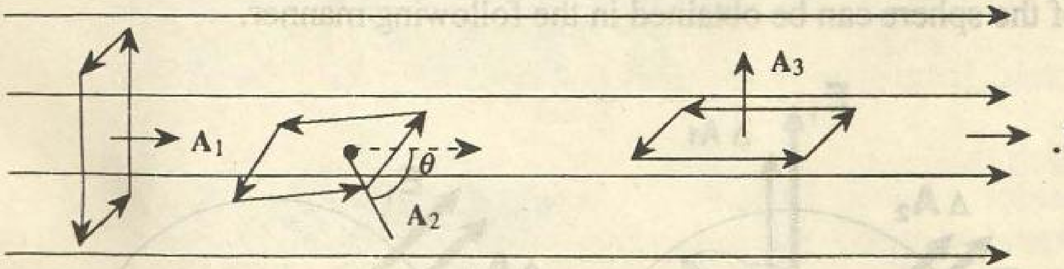


*Fig 12.13 (b) : A closed surface*

**Example : 12.8 :**

In the figure given below calculate the electric flux in each case.



For an area $\mathbf{A}_1$ with positive normal parallel to **E**, the electric flux is

$$\phi_E = \mathbf{E} \cdot \mathbf{A}_1 = EA_1 \cos 0 = EA_1 \text{ (maximum).}$$

In this case surface area is perpendicular to field lines.

For the tilted surface $A_2$, the electric flux is,

$$\phi_E = \mathbf{E} \cdot \mathbf{A}_2 = EA_2 \cos\theta$$

As $\theta = \frac{\pi}{2}$, for $A_2$, dot product of **E** and $\mathbf{A}_2$ vanishes.

The field lines in this case are parallel to surface area and do not intersect the surface.

In terms of field lines, the flux is number of field lines passing through a given surface. The tilted surface intercepts fewer field lines for a given surface area and electric flux is therefore reduced.

If case of closed surfaces (Fig. 12.13b) the electric flux will be positive, negative or zero in cases stated below:

i). The electric flux is positive if net number of electric field lines are leaving the surface, that is, there is source of field lines in the closed surface.

ii). The electric flux is negative if net number of electric field lines are entering the closed surface or more field lines are entering than leaving the surface, i.e; there is a sink of field lines in the closed surface.

iii). The electric flux is zero if number of field lines entering are equal to field lines leaving the surface or no field line intercepting the surface.

### 12.3.1 Electric Flux Through a Closed Surface

Let us compute the electric flux through the surface of a sphere of radius *r* due to a charge *+q* placed at its centre (Fig. 12.14). The electric flux through the whole surface of the sphere can be obtained in the following manner.

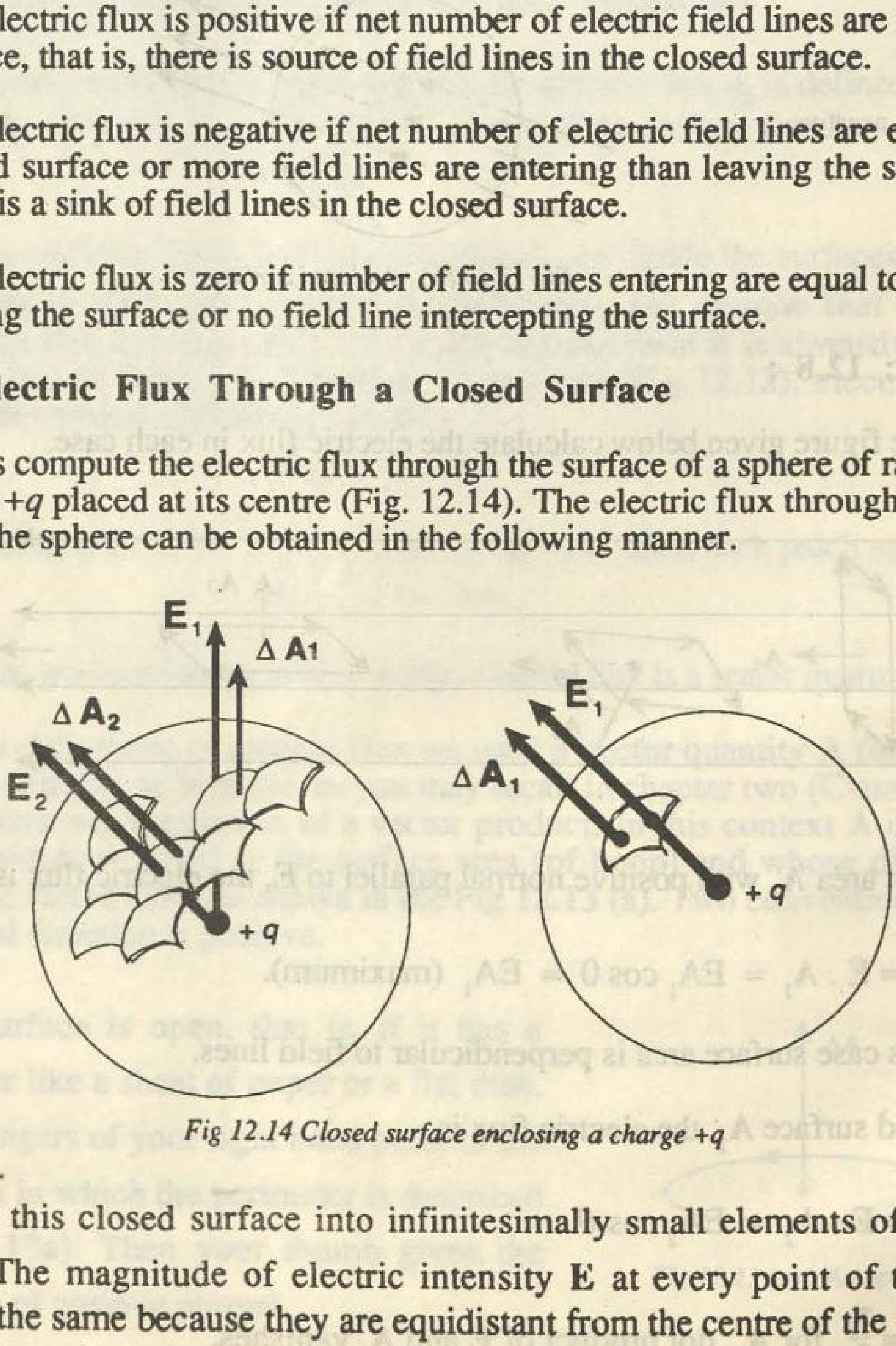


*Fig 12.14 Closed surface enclosing a charge +q*

We divide this closed surface into infinitesimally small elements of area $\Delta A_1$, $\Delta A_2$,....... The magnitude of electric intensity $\mathbf{E}$ at every point of the surface element is the same because they are equidistant from the centre of the sphere. Let the eletric intensity at the centre of a surface element $\Delta \mathbf{A}_1$ is $\mathbf{E}_1$. The direction of $\mathbf{E}_1$ and $\Delta \mathbf{A}_1$ is the same. Therefore, the electric flux through this surface element is

$$(\Delta\phi_e)_1 = \mathbf{E}_1 \,.\, \Delta\mathbf{A}_1 = E_1\Delta A_1 \cos 0 = E_1\Delta A_1$$

where we have used $|\mathbf{E}_1| = E_1 = E$

Similary, the electric flux from the surface elements $\Delta A_2, \Delta A_3, .....$ is

$$(\Delta\phi_e)_2 = E\,\Delta A_2\,,\ (\Delta\phi_e)_3 = E\,\Delta A_3\ ,.......$$

respectively. The total electric flux through the closed surface will be

$$(\Delta\phi_e)_1 + (\Delta\phi_e)_2 + (\Delta\phi_e)_3 + .... + (\Delta\phi_e)_n = \sum_{i=1}^{n} (\Delta\phi_e)_i = \Phi_e \qquad (12.16)$$

**where $n$ is the number of surface elements which could cover the whole closed surface. Thus,**

$$E\,\Delta A_1 + E\Delta A_2 + E\Delta A_3 + .... + E\Delta A_n = E\ (\Delta A_1 + \Delta A_2 + \Delta A_3 + ... + \Delta A_n)$$

$$= E \sum_{i=1}^{n} \Delta A_i$$

$$= E \times (\text{total area of closed surface})$$

$$= \frac{1}{4\pi\varepsilon_o}\ \frac{q}{r^2} \times 4\pi r^2 = \frac{q}{\varepsilon_o}$$

Thus,.

$$\Phi_e = \frac{q}{\varepsilon_o} \qquad (12.17)$$

**Eq. (12.17) shows that the electric flux through a closed surface depends upon the medium and the charge enclosed by the closed surface. It is to be noted that the result of Eq. (12.17) has nothing to do with the shape or the geometry of the closed surface.**

## 12.4 GAUSS'S LAW

**Consider now the flux passing through a closed surface S of some arbitrary shape enclosing point charges $q_1, q_2$ ......., $q_n$, as shown in Fig. 12.15.**

The total electric flux through the closed surface is,

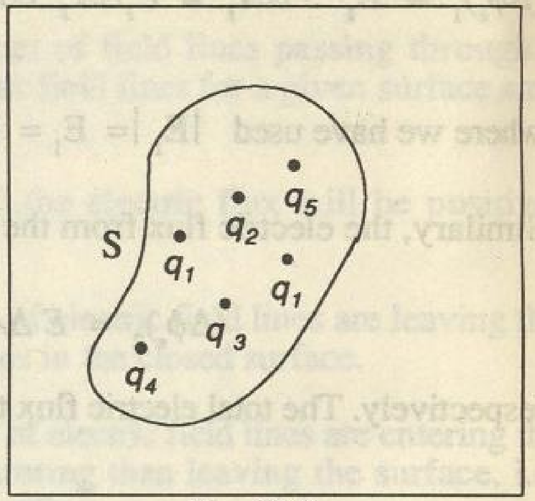


Fig. 12.5
*Electric flux through a closed surface*

$$\Phi_c = \frac{q_1}{\varepsilon_o} + \frac{q_2}{\varepsilon_o} + \ldots\ldots + \frac{q_n}{\varepsilon_o}$$

$$\Phi_e = \frac{1}{\varepsilon_o}\left[q_1 + q_2 + \ldots\ldots + q_n\right]$$

$$\Phi_e = \frac{1}{\varepsilon_o}\sum_{i=1}^{n} q_i$$

$$\Phi_e = \frac{1}{\varepsilon_o} \times Q$$

where Q is the total charge enclosed by the closed surface. Thuse the Gauss's law may be stated as "The flux through any cosed surface is $\frac{1}{\varepsilon_o}$ time the total charge enclosed in it".

### 12.4.1 Applications of Gauss's Law

#### (a) Electric Intensity Due To An infinite Sheet Of Charge

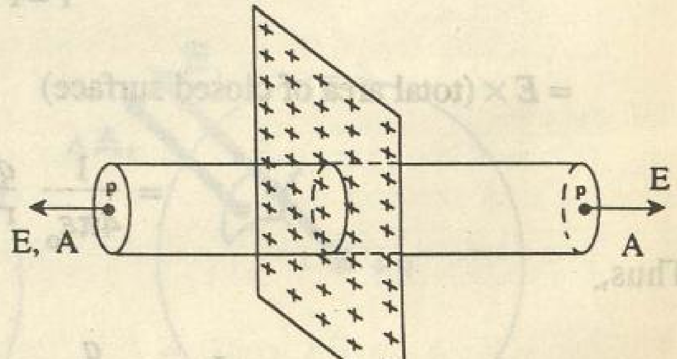


*Fig. 12.16 An infinite sheet of charge pierced a cylindrical gaussian surface.*

Consider a plane sheet of infinite extent on which positive charges are uniformly distributed with surface charge density $\sigma$. A finite part of this sheet is shown in Fig. 12.16. To calculate the electric intensity $E$ at a point P, close to the sheet, imagine a closed surface in the form of cylinder passing through the sheet and whose one face contains P. The sheet is of infinite extent so the direction of $E$ will be along the outward drawn normal passing through P and perpendicular to face containing P. Obviously, direction of $E$ will be parallel to the curved surface. Hence the flux through the curved surface will be zero while it will be 2 $EA$ through the two end faces of the closed surface, where $A$ is the area of cross section, as shown in Fig.12.16. The charge enclosed by the surface will be $\sigma$ A. By virtue of Gauss's law,

$$\text{Flux} = \frac{1}{\varepsilon_o} \times (\text{charge enclosed by the closed surface})$$

$$\Phi_e = \frac{\sigma A}{\varepsilon_o}$$

Hence, $$2EA = \frac{\sigma A}{\varepsilon_o} \tag{12.18}$$

or $$E = \frac{\sigma}{2\varepsilon_o} \tag{12.19}$$

and $$\mathbf{E} = \frac{\sigma}{2\varepsilon_o}\,\hat{\mathbf{r}} \tag{12.20}$$

where $\hat{\mathbf{r}}$ is the unit vector normal to the sheet and directed away from it.

In case of negative charge distribution, a negative sign will be placed the positive sign in Eq. (12.20). Again note that $E$ does not depend upon the size of sheet.

## (b) Electric Intensity Between Two Oppositely Charged Plates

Fig. 12.17 shows two parallel plates of some conducting material. These are oppositely charged so that their charge densities are $+\sigma$ and $-\sigma$. The plates are considered to be of infinite extent to avoid fringing fields at their ends. The separation between them is assumed very small. Under these conditions, the lines of force in Fig. 12.17 indicate that the electric intensity is along the normal to the plates. Due to small separation, the opposite charges attract each other and are in electrostatic equilibrium on the inner surfaces of the plates. Hence they appear to be oppositely charged sheets of infinite extent to a unit positive charge placed at a point P between them. The magnitude of electric intensity at the point P due to each plate will be $\frac{\sigma}{2\varepsilon_o}$. For direction, the presence of unit positive charge at the point P reveals that it will be from positive to negative plate in each case. Hence the net electric intensity at the point P will be

$$\mathbf{E} = 2.\ \frac{\sigma}{2\varepsilon_o}\ \hat{\mathrm{r}}$$

$$\mathbf{E} = \frac{\sigma}{\varepsilon_o}\ \hat{\mathrm{r}} \qquad (12.20a)$$

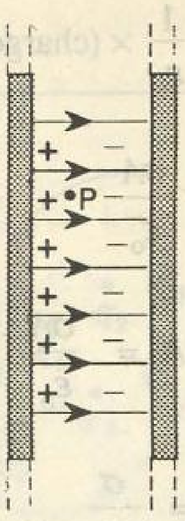


*Fig. 12.17 Oppositely charged parallel plates of infinite extent*

where $\hat{\mathrm{r}}$ is a unit vector directed from the positive to the negative plate.

## 12.5 ELECTRIC POTENTIAL

Let us consider a positive charge $q_0$ which is allowed to move in an electric field produced between two oppositely charged parallel plates, as shown in Fig. 12.18 (a).

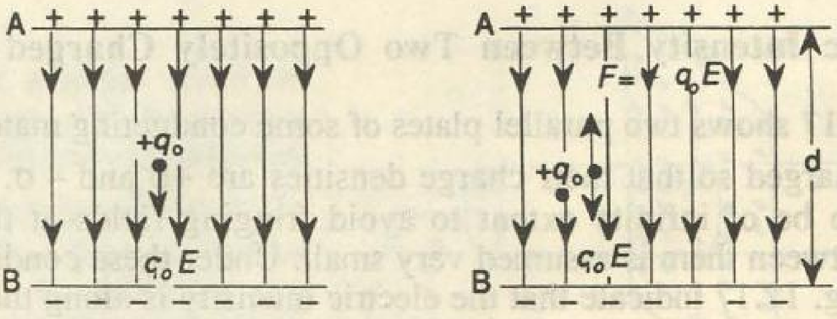


*Fig. 12.18 (a) A positive charge+q moving freely between two oppositely charged plates (b) A+$q_o$ in dynamic equilibrium in an electric field.*

The positive charge will freely move from A to B and acquires kinetic energy. On the other hand, if the positive charge is moved from B to A, an external force must be applied to make the required movement of the charge against the electric field.

We know that a body in equilibrium always moves with a uniform velocity and consequently no unbalanced force acts upon it. Let us impose the condition that equilibrium is maintained in the motion of charge $q_o$ from A to B. Suppose $\boldsymbol{E}$ is the electric intensity, then this condition may be achieved by applying a force $F$ which is equal and opposite to $q_o E$ at every point along its path as shown in Fig.12.18 (b).

This force prevents the charge $q_o$ from accelerating when moving from A to B, and avoids deceleration of the charge as moved from B to A and hence the kinetic energy of the charged particle is zero. The work done by the force in the electric field is therefore stored as electric potential energy similar to the case of potential energy in the gravitational field. The work carried out in moving a charge from one place to another in an electric field is independent of the path and is dependent on its initial and final positions.

Let $W_{AB}$ be the work done by the force in carrying a positive charge $q_o$ from point B to point A without disturbing the equlibrium state of the charge. The change in potential energy $\Delta U$ of charge $q_o$ is defined to be equal to the work done by the force in carrying the charge $q_o$ from one point to the other against the electrical field, i.e.,

$$\Delta U = W_{AB} \text{ or } U_A - U_B = W_{AB}$$

where $U_A$ and $U_B$ represent the potential energies at point A and B, respectively. If this charge is released near A, it will move from A to B and will gain an equivalent amount of K.E. The potential difference between two point is defined as: Potential difference between two points is the work done in moving a unit positive charge from one point to another keeping the charge in electrostatic equilibrium. It implies that

$$\frac{\Delta U}{q_o} = \frac{W_{AB}}{q_o} \text{ or } \frac{\Delta U}{q_o} = \frac{U_A}{q_o} - \frac{U_B}{q_o} = V_A - V_B = \Delta V \qquad (12.21)$$

where $V_A$ and $V_B$ are defined to be the electric potentials at point A and B, respectively. The defining equations are,

$$U_A = q_o V_A, \; U_B = q_o V_B \qquad (12.22a)$$

and therefore,

$$\Delta U = q_o \Delta V = W_{AB} \qquad (12.22b)$$

where $\Delta V$ is potential difference between A and B.

The unit of work is joule (J) and that of charge is coulomb (C). From Eq. 12.22 (b) it is clear that units of PE and work are the same. According to Eqs. (12.21) and (12.22), the unit of potential difference is the joule per coulomb.Since the potential difference is frequently used in physics, it is convenient to define a new unit. This unit has been given the name volt. Hence,

$$1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

A potential difference of 1 volt is said to exist between two points if 1 joule of work is required to move a positive charge of 1 coulomb from one point to the other while keeping the charge in its electrostatic equilibrium. Some multiples and submultiples of volt in use are:

1 milli-volt (mV) = $10^{-3}$ volts

1 micro-volt (μV) = $10^{-6}$ volts

1 kilo volt (kV) = $10^{3}$ volts

1 mega volt ($M_e$ V) = $10^{6}$ volts

1 giga volt ($G_e$ V) = $10^{9}$ volts

To calculate the absolute potenial or simply potential at a point in an electric field, it is necessary to designate some point as reference point. If the potential at such a point is assumed to be zero, then the potential difference between this point and the point at which potential is to be calculated, will be the absoulte potential or simply potential at the point under consideration. The reference point B is arbitrarily taken to be at infinity. Thus from Eq. (12.21),

$$V_A - V_\infty = \frac{W_{A\infty}}{q_o}, V_A = \frac{W_{A\infty}}{q_o}$$

on dropping the subscript, we get

$$V = \frac{W}{q} \qquad (12.23)$$

Thus it can be stated that the electric potential at any point in an electric field is equal to the work done in bringing a unit positive charge from infinity to that point keeping the electrostatic equilibrium.

### 12.5.1 Electric Potential and Electric intensity

To get a relation between potential difference and electric intensity, let us consider situation as depicted in Fig. 12.18 (b). The electric intensity **E** is constant between the plates. The potential difference between points A and B, is given as

$$V_A - V_B = \frac{W_{AB}}{q_o}$$

Now $W_{AB} = \mathbf{F.d}$ or $W_{AB} = Fd\cos 180^\circ$, where the angle between the force and displacement vector is 180°. Therefore,

$$W_{AB} = -Fd.$$

But the force of interaction between the charge $q_o$ and the electric field **E**, as given in Section 12.2.1 is

$$F = q_o E$$

therefore,

$$W_{AB} = -q_o Ed$$

and

$$q_o(V_A - V_B) = -q_o Ed$$

which on simplifying can be rewritten as

$$E = \frac{-(V_A - V_B)}{d} = \frac{-\Delta V}{d} \qquad (12.24)$$

If the points A and B are separated by infinitesimally small distance $\Delta r$, then Eq. (12.24) is modified as

$$E = \frac{-\Delta V}{\Delta r} \qquad (12.25)$$

The negative sign indicates that $E$ will be positive when $\frac{\Delta V}{\Delta r}$ will be negative i.e., if we move from a point of lower potential to a point of higher potential, the direction of the displacement between two points. The quantity $\frac{\Delta V}{\Delta r}$ is the rate of change of potential with respect to displacement and is termed as potential gradient. This is often used to determine the direction of $E$ using Eq. (12.25).

The unit of electric intensity or electric field strength which we have used so far is Newton/Coulomb ($NC^{-1}$). The definition of the volt provides us with another unit of E. From $\mathbf{E} = \frac{V_A - V_B}{d}$ the unit of **E** may be defined as volt/meter. The equivalence of the two definition is shown below:

$$1\,\frac{\text{volt}}{\text{meter}} = 1\,\frac{\text{joule/coulomb}}{\text{metre}} = 1\,\frac{\text{Newton–metre}}{\text{Coulomb–meter}} = 1\,\frac{\text{Newton}}{\text{Coulomb}}$$

### 12.5.1 (b) Absolute potential at a point due to a point charge

Let us now derive an expression for the potential at a certain point in a field of a point charge $q$. As long as $E$ remains constant, this can be accomplished using Eq. (12.25) which yields $\Delta V = -E\Delta r$. $E$ varies as square of the distance from the point charge so it no more remains constant in this case. The field is radial as shown in Fig. 12.19. Let us take two points A and B, infinitesimally close to each other, so that $E$ remains approximately constant between them. The distances of points A and B from $q$ are $r_A$ and $r_B$, respectively, and the distance of midpoint between A and B from $q$ is $r$. Then

According to Fig 12.19,

$$r_A + \Delta r = r_B$$

or

$$\Delta r = r_B - r_A$$

The centre of this displacement from point charge is,

$$r = \frac{r_B + r_A}{2}$$

The magnitude of electric intensity at this point is,

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2}$$

$$\text{where} \quad r^2 = \left(\frac{r_B + r_A}{2}\right)^2$$

Substituting the value of $r_B$

$$r^2 = \left(\frac{r_A + \Delta r + r_A}{2}\right)^2$$

$$= \left(\frac{4r_A^2 + (\Delta r)^2 + 4r_A \Delta r}{4}\right)$$

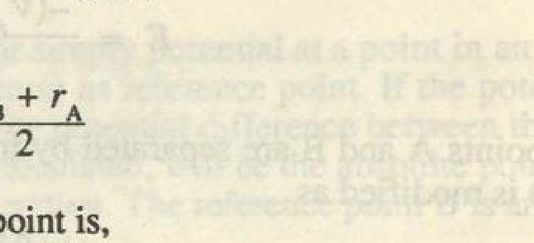


*Fig. 12.19 Electric Field surrounding a point charge for calculating potential at a point charge.*

neglecting $(\Delta r)^2$, $\Delta r$ being very small, we get

$$r^2 = r_A(r_A + \Delta r)$$

But $\Delta r = r_B - r_A$

Hence $r^2 = r_B r_A$

If a unit positive charge is moved from B to A, the work done is equal to potential difference between A and B,

$$V_A - V_B = -E\,(r_A - r_B) = E\,(r_B - r_A)$$

$$= E\,\Delta r$$

$$\text{as}\left(E = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q}{r^2}\right)$$

$$= \frac{1}{4\pi\varepsilon_o}\cdot\frac{q}{r^2}\cdot\Delta r$$

$$= \frac{q}{4\pi\varepsilon_o}\left(\frac{\Delta r}{r^2}\right)$$

$$= \frac{q}{4\pi\varepsilon_o}\left(\frac{r_B - r_A}{r_B\,r_A}\right)$$

$$(\therefore r^2 = r_B\,r_A)$$

$$= \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

To calculate absolute potential or potential at A, let B be the point at infinity. So $V_B = 0$ at $r_B = \infty$, This gives

$$V_A = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q}{r_A}$$

The general expression for electric potential $V(r)$, at a distance $r$ from q is,

$$V(r) = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q}{r} \qquad (12.26)$$

The potential at a point due to a distribution of charges can be evaluated by assuming that potential produced by any charge is not affected by the presentce of other charges. Thus, the resultant potential $V$, at a point due to the distribution of charges consisting of $n$ point charges $q_1, q_2, \ldots q_n$ located at distances $r_1, r_2, \ldots r_n$ from the point, respectively, can be obtained by an algebraic sum of potentials $V_1, V_2, \ldots V_n$ due to the individual charges. Thus, $V = V_1 + V_2 + V_3 + \ldots + V_n$.

or

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_o} \sum_{i=1}^{n} \frac{q_i}{r_i} \tag{12.27}$$

Potential being a dot product of $E$ and $\Delta r$ is a scalar quantity.

**Example 12.9**

Find the potential at a point situated at a distance of 50 cm from a charge of 5 μC.

**Solution**

$q = 5\,\mu\text{C} = 5 \times 10^{-6}\,\text{C}.$

$r = 50\,\text{cm} = 50 \times 10^{-2}\,\text{m} = 0.5\text{m}$

$$V = \frac{1}{4\pi\varepsilon_o} = \frac{q}{r} = 9 \times 10^{9}\,\frac{\text{Nm}^2}{\text{C}^2} \times \frac{5 \times 10^{-6}\,\text{C}}{50 \times 10^{-2}\,\text{m}} = 90{,}000 \text{ volts.}$$

**Example 12.10**

Calculate the magnitude of electric field between two plates if the separation between them is 0.50 cm and a potential difference of 12.0 volts exists between them.

**Solution**

According to Eq. (12.24)

$$\Delta V = V_B - V_A = Ed$$

$$12.0 = E \times 0.5 \times 10^{-2}$$

$$E = 2400\ \mathrm{Vm^{-1}}$$

**Example 12.11**

Find the absolute potential at a point distant 20 cm from a charge of 2 μC.

**Solution**

Absolute potential,

$$V = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r}$$

$$V = \frac{9 \times 10^9\ \mathrm{Nm^2C^{-2}} \times 12 \times 10^{-6}\ \mathrm{C}}{0.2\ \mathrm{m}} = 90 \times 10^3\ \text{volts}$$

### 12.5.2 Equipotential Surfaces

The electric lines of force due to a $+q$ charge are shown in Fig. 12.20. These are radial lines emanating from the charge and are a measure of electric field strength $E$. Consider three concentric circles with their centre at the point where $+q$ is placed. Let us calculate the potential difference between the two points A and B on the circle II. Let the displacement between A and B is $\Delta r$. On each point of the circle, the direction of force is perpendicular to it and, therefore, work done in moving a charge from A to B is

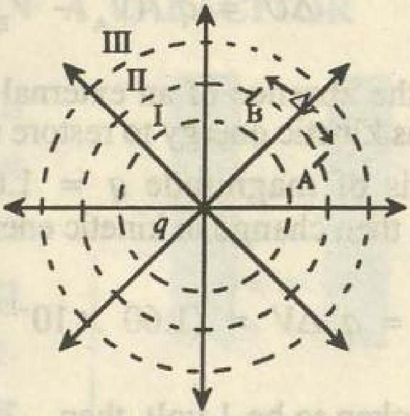


*Fig. 12.20: The dashed-line circles indicate the equipotenial line surrounding a charge +q. Note that the radial electric lines are perpendicular to the equipotenital.*

$$W_{AB} = q\ (V_A - V_B)$$

Also the potential difference between point A and B is

$$V_A - V_B = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) \qquad (12.28)$$

As $r_B = r_{A;}$ So $V_B = V_A$

We infer that the points A and B are at the same potential.

It is clear that no work will be done in moving a test charge from any point on the circle II to any other point on the same circle. Thus, all the points on the circle II are at the same potential. We call this circle an equipotential line. Actually, this circle is a portion of an equipotential surface of a sphere of radius equal to the radius of circle II. Other equipotential lines are shown by circles in Fig. 12.20. One could draw equipotential lines and surfaces at each and every point, although we have drawn only a few representative circles. No two equipotential surfaces (and hence equipotential lines) can intersect each other, otherwise, a single point will be at two potentials at the same time which is physically incorrect. From the discussion, it is clear that equipotential lines and surfaces are always perpendicular to the electric lines of force and therefore do not intersect each other.

### 12.5.3. The Electron Volt (eV)

When a particle of charge $q$ moves from a point at potential $V_B$ to a point at potential $V_{A;}$ keeping the electrostatic equilibrium, the change in potential energy of the particle is related to the potential difference, as calculated in Eq. (12.24)

$$\frac{\Delta U}{q} = (V_A - V_B) = \Delta V$$

or

$$\Delta U = q\ (V_A - V_B) = q\,\Delta V$$

In the absence of an external force the change in dielectric potential energy appears as kinetic energy to restore the electrostatic equilibrium. If the charge on the particle is of magnitude $q = 1.60 \times 10^{-19}$ C, the charge on electron if taken negative, then change in kinetic energy (KE) acquired by the particle is

$$\Delta\,(KE) = q\,\Delta V = (1.60 \times 10^{-19}\ \mathrm{C})\,\Delta V$$

If $\Delta V$ is taken to be 1 volt, then

$$\Delta\,(KE) = (1.60 \times 10^{-19}\ \mathrm{C})\,(1\mathrm{V}) = 1.60 \times 10^{-19}\ \mathrm{C\ J}$$

For practical convenience another unit of energy called elcctron volt (eV) is defined as follows:

one electron volt (eV) is the amount of energy acquired or lost by an electron when it is displaced across two points between which potential difference is one volt".

$$1\mathrm{eV} = 1.60 \times 10^{-19}\ \mathrm{J} \qquad (12.29)$$

However, this is a small unit of energy. The following bigger units are in frequent use:

1 Million electron volt =1Mega electron volt =1MeV=$10^6$ eV

1 Billion electron volt = 1Giga electron volt = 1GeV = $10^9$ eV

**Example 12.12**

An electron has a speed of $10^6\ ms^{-1}$. Find its energy in electron-volt.

**Solution**

The energy of a moving electron will be its kinetic energy, equal to $\frac{1}{2}mv^2$ where *m* is the mass of the electron and *v* is its speed.

Hence, energy of electron = Kinetic energy of electron.

$$= \frac{1}{2} \times 9.11 \times 10^{-31}\ \text{kg} \times 10^{12}\ \frac{\text{m}^2}{\text{s}^{-2}} = 4.55 \times 10^{-19}\ \text{J}$$

$$= \frac{4.55 \times 10^{-19}}{1.6 \times 10^{-19}}\ \text{eV} = 2.84\ \text{eV}$$

## 12.6. CAPACITORS, CAPACITANCE OF A CAPACITOR

A device used for storing electric charge is called a capacitor or a condenser. A simplest capacitor consists of two parallel plates (conductor) that have equal and opposite charges separated by a small distance as shown in Fig. 12.21. When we say that the charge of capacitor is $Q$, it means that one plate has $^+Q$ charge while the other has $^-Q$. The two plates of a capacitor are connected to a battery where the plates acquire opposite charge equal in amount.

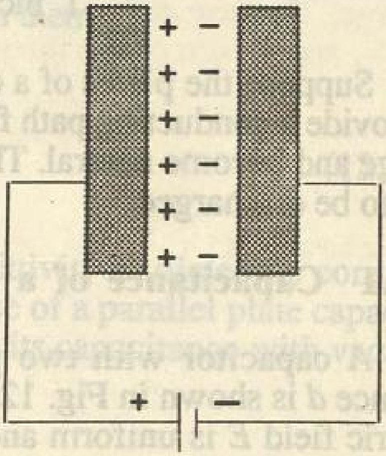


*Fig. 12.21 A parallel plate capacitor connected to a battery*

A typical capacitor is capable of storing a large amount of charge (and hence energy) in a small space. The capability of a capacitor to store charge is called its capacitance. The space in a capacitor may be filled by air or some other insulating material generally called a dielectric. These charges remain on the inner surfaces of the plates as shown in Fig. 12.21. The charge $Q$ which resides on one of these plates will depend upon the potential difference between them. The larger the voltage of the battery, the greater will be the charge on the capacitor. Hence, if $V$ is the potential difference across the plates, then

$$Q \propto V$$

$$Q = CV$$

where $C$ is a constant of proportionality depending upon the geometry of the capacitor and is known as the capacitance of the capacitor as already mentioned. Thus, the capacitance

$$C = \frac{Q}{V} \tag{12.30}$$

SI units of charge and potential difference are coulomb and volts, respectively, therefore units of capacitance will be coulomb/volt according to Eq. (12.30). This unit is called a 'farad', which is defined as: "The capacitance of a capacitor is one farad if a charge of 1 coulomb given to one of the plates produces a potential difference of 1 volt between them".

Farad is a big unit and for practical purposes its sub-multiple units are used. Their relation to farad is given below:—

$$1 \text{ micro farad} = 1\,\mu\text{F} = 10^{-6} \text{ farad}$$

$$1 \text{ pico farad} = 1\,\text{pF} = 10^{-12} \text{ farad}$$

Suppose the plates of a charged capacitors are connected by means of a wire to provide a conducting path for the flow of charge. The plates of the capacitor lose charge and become neutral. The process is called "discharging" and the capacitor is said to be discharged.

### 12.6.1 Capacitance of a Parallel Plate Capacitor:

A capacitor with two parallel plates, each of area $A$ and separated by a distance $d$ is shown in Fig. 12.21. The distance $d$ between plates is small so that the electric field $E$ is uniform and confined almost entirely in the region between the plates and the fringing of the electric lines of force at the boundaries is negligible. The air or any other insulating material can be filled between the plates but for the time being we shall assume air or vacuum to be present as a medium between the plates. From Eq. (12.30)

$$C_{\text{vac}} = \frac{Q}{V}$$

where $Q$ is the charge on the capacitor and $V$ is the potential difference between two plates separated by a distance $d$. Since the field $E$ is uniform, so its relation with the potential difference can be written as:

$$E = \frac{V}{d} \qquad (12.31)$$

The magnitude of the electric intensity between two parallel plates of a capacitor in terms of uniform surface charge density σ, by using Eq. (12.20) is

$$E = \frac{Q}{\varepsilon_o A} \qquad (12.32)$$

As

$$\sigma = \frac{Q}{A} \qquad (12.33)$$

using Eqs. (12.31), (12.32) and (12.33), we get

$$C_{vac} = \frac{Q}{V} = \frac{Q}{Ed} = \frac{\varepsilon_o AQ}{Qd} = \frac{A\varepsilon_o}{d} \qquad (12.34)$$

If an insulating material (or dielectric) of relative permittivity constant $\varepsilon_r$ is present between the plates, the capacitance of the capacitor is enhanced by $\varepsilon_r$. Then Eq. (12.34) reduces to

$$C_{\text{med}} = \frac{A\,\varepsilon_o\,\varepsilon_r}{d} \qquad (12.35)$$

Eq. (12.35) shows the dependence of capacitors upon the area of plates, the separation between the plates and the medium between them.

Using Eqs. (12.34), and (12.35) we have

$$\varepsilon_r = \frac{C_{\text{med}}}{C_{\text{vac}}} \qquad (12.36)$$

This provides us with the definition of relative permittivity or dielectric constant, which is stated below. "It is the ratio of the capacitance of a parallel plate capacitor with a substance as the medium between the plates to its capacitance with vacuum (or air) as medium between them"

### 12.6.2 Combination of Capacitors

There are many situations in electrical circuits where more than one capacitors are used. The capacitors can either be combined in series or in parallel. We wish to find an equivalent capacitor that has the same capacitance as that of combination of capacitors.

#### a) Capacitors in Series

Fig. 12.22 shows three capacitors of capacitances $C_1$, $C_2$ and $C_3$, connected in series, between two points A and B. One of the characteristics of the series combination of the capacitors is, when they are charged, each capacitor acquires the same amount of charge.

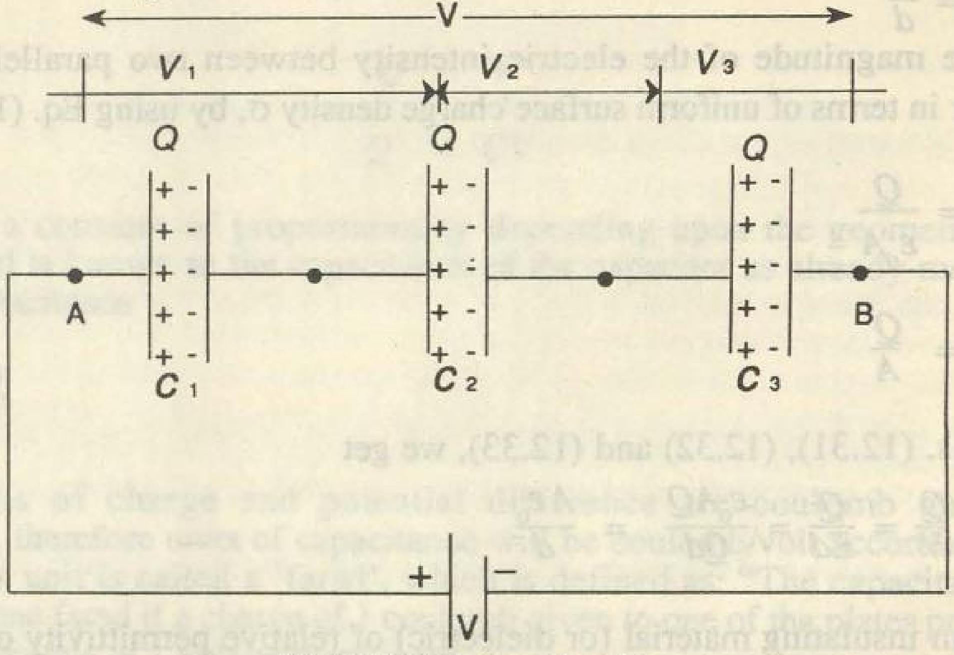


*Fig. 12.22 Capacitors in series*

Suppose we apply a potential difference *V* between points A and B. Assume that the left plate of the capacitor of capacitance *C* acquires a charge +*Q*, so the right plate will have –*Q* charge. The –*Q* charge must come from the left plate of capacitor $C_2$ by transfer of electrons. This makes the left plate capacitor $C_2$ positive with a charge of +*Q*. The other capacitors in series are also charged by the same process shown in the Fig. 12.22. In this way all the capacitors get an equal amount of charge. The potential difference *V* must be equal to the sum of the potential drops $V_1$, $V_2$ and $V_3$ across the three capacitors so.

$$\mathrm{V} = V_1 + V_2 + V_3 \tag{12.37}$$

We know that $V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2}, V_3 = \frac{Q}{C_3}$

Thus $$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \tag{12.38}$$

or $$= Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

or $$\frac{V}{Q} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \tag{12.39}$$

Let $C_{eq}$ by the equivalent of the three capacitors in series. This, too, must have the same charge *Q* as the other capacitors. That is,

$$V = \frac{Q}{C_{eq}}$$

or $$\frac{V}{Q} = \frac{1}{C_{eq}}$$

Thus, comparing it with Eq. (12.39) gives

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \tag{12.40}$$

### (b) Capacitors in Parallel

Fig. 12.23 shows that capacitors $C_1$,$C_2$ and $C_3$ are connected in parallel between two points A and B. As each capacitor is between the same two points (A and B), so when a potential difference $V$ is applied between A nad B, all the capacitors have equal potential difference across them. On giving $Q$ charge to point A, capacitors $C_1$,$C_2$ and $C_3$ will acquire different charges $Q_1$,$Q_2$ and$Q_3$, respectively, depending upon their capacitances.

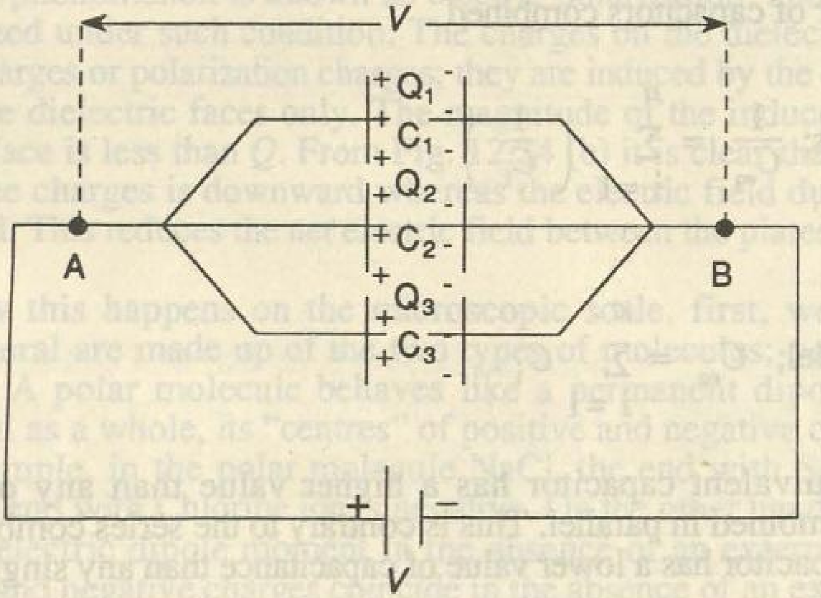


*Fig 12.23 Capacitors in parallel*

Hence, $$Q = Q_1 + Q_2 + Q_3 \qquad 12.41$$

With, $Q_1 = C_1V, Q_2 = C_2V, Q_3 = C_3V.$

Substituting these values of $Q_1, Q_2, Q_3$ in Eq. (12.41) we get

$$Q = C_1V + C_2V + C_3V$$

or $$Q = V(C_1 + C_2 + C_3)$$

or $$\frac{Q}{V} = C_1 + C_2 + C_3 \qquad (12.42)$$

Again if $C_{eq}$ is the capacitance of an equivalent capacitor of this combination, then

$$Q = C_{eq}\ V \text{ or } \frac{Q}{V} = C_{eq}$$

which on comparing with Eq. (12.42) gives.

$$C_{eq} = C_1 + C_2 + C_3 \qquad (12.43)$$

For $n$ number of capacitors combined

(i) in series; $\frac{1}{C_{eq}} = \sum_{i=1}^{n} \left(\frac{1}{C_i}\right)$ (12.,44a)

(ii) in parallel; $C_{eq} = \sum_{i=1}^{n} C_i$ (12.44b)

The equivalent capacitor has a higher value than any one of the single capacitors combined in parallel. This is contrary to the series combination where the equivalent capacitor has a lower value of capacitance than any single capacitor of the combination.

## 12.7 ELECTRIC POLARIZATION; A MICROSCOPIC VIEW

If a medium between two charge configurations is an insulator it is usually referred as a dielectric characterised by dielectric constant $\varepsilon_r$ known as relative permittivity. Above we also noticed that an increase in the capacitance of the capacitor occured when a dielectric material replaced the air and free space between the plates of a parallel plate capacity.

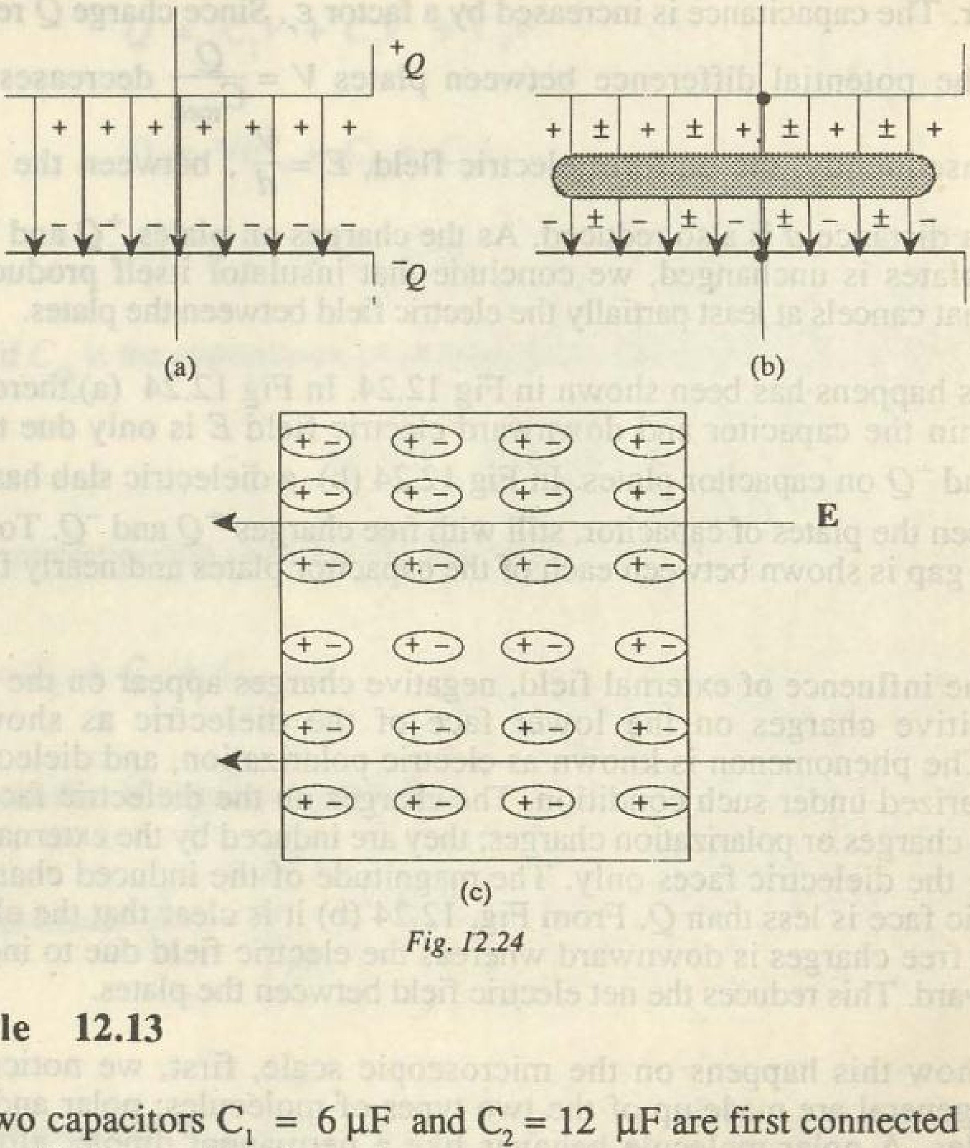
First we recall that what happens when insulating material with relative permittivity (or dielectric constant) $\varepsilon_r$ is inserted into an initially charged parrallel plate capacitor. The capacitance is increased by a factor $\varepsilon_r$. Since charge $Q$ remains unchanged, the potential difference between plates $V = \frac{Q}{C_{\text{med}}}$ decreases by a factor $\varepsilon_r$. Consequently, the uniform electric fleld, $E = \frac{V}{d}$, between the plates separated by a distance $d$ is also reduced. As the charges on plales $^+Q$ and $^-Q$ on each of two plates is unchanged, we conclude that insulator itself produces an electric field that cancels at least partially the electric field between the plates.

How this happens has been shown in Fig 12.24. In Fig 12.24 (a),there is no dielectric within the capacitor and downward electric field $E$ is only due to free charges $^+Q$ and $^-Q$ on capacitor plates. In Fig 12.24 (b) a dielectric slab has been inserted between the plates of capacitor, still with free charges $^+Q$ and $^-Q$. To make things clear, a gap is shown between each of the capacitor plates and nearly face of the dielectric.

Under the influence of external field, negative charges appear on the upper face and positive charges on the lower face of the dielectric as shown in Fig 12.24 b. The phenomenon is known as electric polarization, and dielectric is said to be polarized under such condition. The charges on the dielectric faces are called induced charges or polarization charges; they are induced by the external field and appear on the dielectric faces only. The magnitude of the induced charge on either dielectric face is less than $Q$. From Fig. 12.24 (b) it is clear that the electric field from the free charges is downward whereas the electric field due to induced charges is upward. This reduces the net electric field between the plates.

To see how this happens on the microscopic scale, first, we notice that dielectrics in general are made up of the two types of molecules; polar and non-polar molecules. A polar molecule behaves like a permanent dipole; although electrically neutral as a whole, its "centres" of positive and negative charge do not coincide. For example, in the polar molecule NaCl, the end with Sodium ion is positive while the end with Chlorine ion is negative. On the other hand, a non-polar molecule has no electric dipole moment in the absence of an external field; The centre of positive and negative charges coincide in the absence of an extenal electric field.

When a non-polar dielectric material is placed in an external field, it gets polarized, that is, it displaces the electrons to the opposite of **E** as shown in Fig. 12.24 (b). Under the action of external field, the centres of negative and positive charges shift and form dipoles (Two opposite point charge separated by small finite distanece constitutes a dipole). Within the dielectric a "static charge equilibrium" is always present and induced charges appear on the surfaces of dielectric near to the plates of capacitor as shown in Fig 12.24 (c). As electric field due to induced charges is opposite to the external electric field, so it reduces the intensity of external field due to oppositely charged plates of the capacitor (Fig 12.24b).



(a) (b)

(c)

*Fig. 12.24*

**Example 12.13**

Two capacitors $C_1 = 6\ \mu F$ and $C_2 = 12\ \mu F$ are first connected (a) in series and then (b) in parallel. In each case the external source of voltage is 1000 V. Calculate in each case: the total capacitance, the potential drop accross each capacitor, and the charge on each capacitor.

(a) Let us first consider the capacitors in series. Let $C_{eq} = C$ in this case, such that

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{6\ \mu C} + \frac{1}{12\ \mu C} = \frac{1}{4\ \mu F}$$

$$C = 4\ \mu F = 4 \times 10^{-6}\ F$$

when the capacitors are in series, the charge on each capacitor is the same and is given by,

$$Q = 4 \times 10^{-6}\,\mathrm{F} \times 1000\mathrm{V} = 0.004\,\mathrm{C}$$

The potential difference across each capacitor is different and is given by

$$V_1 = \frac{Q}{C_1} = \frac{4000 \times 10^{-6}\,\mathrm{C}}{6 \times 10^{-6}\,\mathrm{F}} = 666.7\,\mathrm{V}$$

$$V_2 = \frac{Q}{C_2} = \frac{4000 \times 10^{-6}\,\mathrm{C}}{12 \times 10^{-6}\,\mathrm{F}} = 333.3\,\mathrm{V}$$

(b) When the two capacitors are connected in parallel, the total equivalent capacitance, $C_{eq} = C$, is given by,

$$C = C_1 + C_2 = 6\,\mu\mathrm{F} + 12\,\mu\mathrm{F} = 18\,\mu\mathrm{F}$$

The potential difference across each capacitor is the same; 1000 V but the charge on each capacitor is different. Thus,

$$Q_1 = C_1V = 6 \times 10^{-6}\,\mathrm{F} \times 1000\mathrm{V} = 6000\,\mu\mathrm{C}$$

$$Q_2 = C_2V = 12 \times 10^{-6}\,\mathrm{F} \times 1000\mathrm{V} = 12000\,\mu\mathrm{C}$$

### 12.6.3 Different types of Capacitors in use

To understand the function and importance of a capacitor, a parallel plate capacitor provides the simplest example. To meet the needs of modern science and technology, various types of capacitors are in use. Variable capcitors (gang) are commonly used in tuning the radio circuits and television receivers. This is again an example of parallel plate capacitor with variable capacitance. A typical one is shown in Fig. 12.24. In general a capacitor consists of two oppositely charged conductors. This variable capacitor consists of number of fixed metal plates connected together to form one conductor while a second set of movable plates are connected together to form the other conductor. The two sets of plates are interleaved as shown and the effective area can be changed by rotating the shaft connected to the movable set of plates. Hence the effective value of *C* is changed.

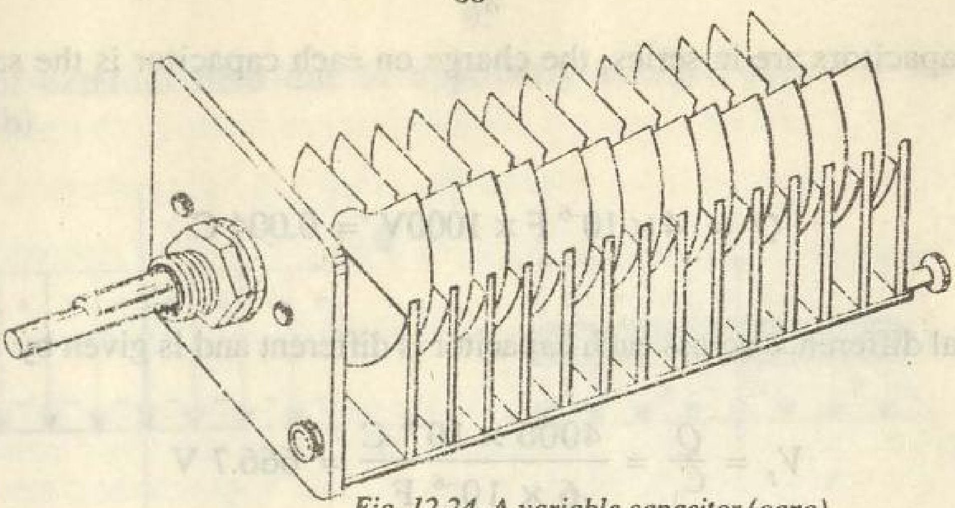


*Fig. 12.24. A variable capacitor (gang)*

Alongwith parallel plate capacitors, spherical and cylindrical capacitors are also in use.

Commercial type capacitors are much more compact and a few of them are shown in Fig. 12.25 (a) shows a tubular capacitor consisting of plates of thin metal foil separated by a sheet of paper or plastic film and rolled up into a small package. The whole unit is then covered in a plastic film.

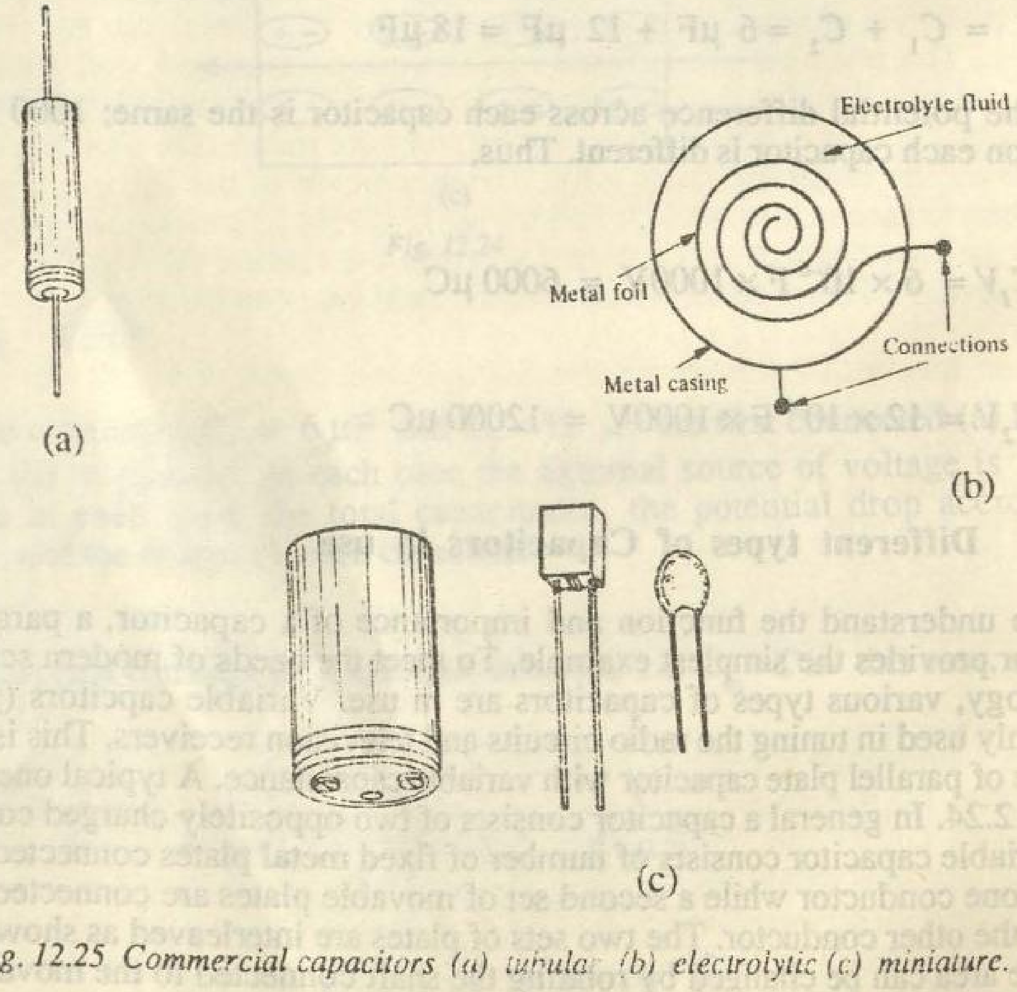

(a)
Electrolyte fluid
Metal foil
Connections
Metal casing
(b)
(c)

*Fig. 12.25 Commercial capacitors (a) tubular (b) electrolytic (c) miniature.*

Fig. 12.25 (b) shows another compact capcitor, called an electrolytic capacitor. The structure is clear from the figure.

Miniature capacitors, shown in Fig. 12.25 (c) form another class of the commercial type compact capacitors. As is evident from their name, they are very small in size and are of great importance not only in reducing the size of the electrical instruments but also help in other modern technologies such as computers, microprocessors etc.

# SUMMARY

**Kinds of Charges :** There are two types of charges, negative and positive. Like charges repel each other and unlike attract each other.

**Coulomb's Law :** The force $F$ between charges $q_1$ and $q_2$ at a distance $r$ apart is given as $F = K\, q_1 q_2 / r^2$, where $K = 1/4\pi\varepsilon_0 \approx 9 \times 10^9$ N–m$^2$C$^{-2}$. The SI. unit of charge is the Coulomb (C), defined as the quantity of charge that flows across any cross-section of a wire in one second when there is a steady current of 1 ampere.

**Electric intensity and field lines :** The concept of the clectric field was introduced by Michael Faraday. The electric field intensity at a point in an electric field is $\mathbf{E} = \mathbf{F}/q_o$ or $\mathbf{E} = \frac{1}{4\mu\varepsilon_o} \cdot \frac{q}{r^2}\, \hat{r}$

The electric field lines, always start from a postitive charge and end on a negative charge. The tangent to the field line at any point gives the direction of the electric field at that point.

The change in the potential energy $\Delta U$ of charge $q$ is equal to work done by the force in carrying the charge from one point to another with a uniform velocity, that is, $\Delta U = \Delta W$. The potential difference $\Delta V$ between two points A and B in an electric field is defined as the work done in carrying a unit positive charge from A to B while keeping the charge in equilibrium. A potential difference of 1 volt is said to exist between two points if 1 Joule of work is required to move a positive charge of 1 coulomb from one point to the other keeping the charge in equilibrium. 1 volt =1 JC$^{-1}$. The electric potential at any point in an electric field is equal to the work done in bringing a unit positive charge from infinity to that point keeping the unit charge in equilibrium.

The quantity $\frac{\Delta V}{\Delta r}$ is known as potential gradient. In an electric field the component of electric intensity in any direction is equal to the negative of potential gradient in the direction, $E = \frac{\Delta V}{\Delta r}$. The unit of $E$, N/C, is equivalent to V/m. The potential at a point a distance $r$ from a charge $q$ is $V(r) = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$.

**Electron Volt :** One electron volt (eV) is the amount of energy acquired or lost by an electron when it is displaced across two points between which the potential difference is one volt; $1\text{eV} = 1.60 \times 10^{-19}$ J.

**Capacitor :** A device to store electric charge is called capacitor or condenser. The capacitance $C$ is equal to the ratio $Q/V$. The unit of capacitance is farad (F) = coulomb/volt.

When a dielectric is placed in an electric field in such a way that the field is directed from up to dowonward direction, then under the influence of this external field, negative charges appear on the upper face and positive charges on the lower side of the dielectric. The phenomenon is known as electric polarization and dielectric is said to be polarized.

# QUESTIONS

**12.1** Write electrical field statements analogous to the following in gravitational field (a) water flows from a higher level to a lower level; (b) water always maintains its level; (c) the total mass is conserved; (d) when a body falls through a height $h$, it loses potential energy and gains kinetic energy.

**12.2** How might a moving conductor like an aeroplane acquire a charge as it flies through the air?

**12.3** Is Coulomb's electrostatic interaction consistent with Newton's third law of motion? Explain.

**12.4** What difference would it make if we called electron positive and proton negative?

**12.5** How could you show that the electrostatic force on a charged object is independent of its mass?

**12.6** Electrostatic force is $10^8$ times stronger than gravitational force. Argue that our galaxy should be almost electrically neutral.

**12.7** Is it possible for electrostatic field lines to intersect? Explan.

**12.8** Gauss's law has been derived by assuming that the charge enclosed by closed surface is positive. If $q$ is negative, what happens to the field lines? What happens to flux?

**12.9** Why does spherical symmetry of a sphere shell demand the *E* due to charge *q* at the centre of sphere, be constant over the surface?

**12.10** An uncharged conducting spherical shell is placed in the field of a positive charge *q*. What will be net flux through the shell?

**12.11** A spherical closed surface of radius R has a uniform volume charge density $\rho$. Deternine the electric flux through the surface?

**12.12** What is the difference between electric potential energy and electric potential difference?

**12.13** Some one claims that a measurement of the potential at a point automatically gives the electric field at that point. Show from scalar nature of *V* and the vector nature of *E* that this claim is ridiculous?

**12.14** Is electron - volt (eV) unit of potential difference or energy? Explain.

**12.15** Describe the charging of a capacitor by battery, assuming the positive charges are transferred?

**12.16** Rubbing a glass rod with a piece of silk produces a net positive charge on the rod. How could you use the positively charged rod to produce a positive or negative charge on metallic conductor?

## PROBLEMS

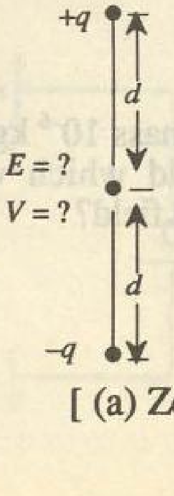
**12.1** What is electric force exerted on each of two identical conductors each with charge of 0.1 μC. and separted by 1.0 cm?

[Ans: 0.90 N ]

**12.2** Charges $q_1$ (=500μC) and $q_2$ (=100μC) are located on the xy-plane at the position $\mathbf{r}_1$ ($=3.0\,\hat{j}$) and $\mathbf{r}_2$ ($=4.0\,\hat{i}$) m. Use equation $F_{12} = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q_1 q_2}{r^2}$ $\hat{r}$ to determine electrostatic force exerted on charge $q_2$.

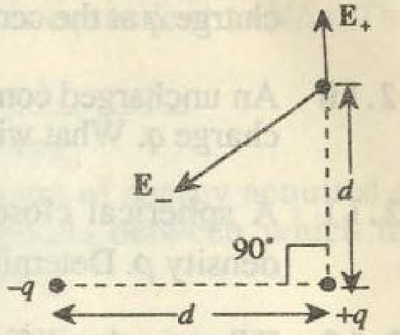
[ Ans: 18N, –37° ]

**12.3** Two opposite point charges, each of magnitude $q_1$are separated by a distance 2*d*. For the point midway between the charges, what is (a) the electric patential (b) electric field (see Fig.)



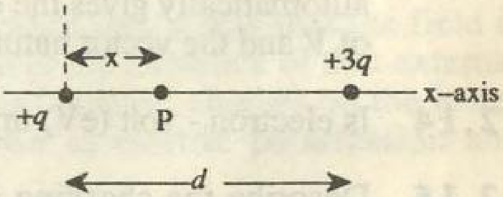
[ (a) Zero (b) $\frac{1}{4\pi\varepsilon_o}\frac{(2q)}{d_2}$ ]

**12.4** Point charges $+q$ and $-q$ are separated by a distance $d$. What is the electric field of the two charges at point P in figure.



[Ans: $0.74\frac{kg}{d^2}$, $\phi = 61°$ ]

**12.5** Charge $+ q$ is at origin. Charge $+3q$ is on the x-axis at $x=d$ as shown in the figure. Where is the electric field of two charges zero?



[ Ans: 0.366, 1.37 ]

[Ans : Hint : For this we obtain $\frac{x}{d}$, which is the answer to the problem ]

**12.6** What repulsive Coulomb force exists between two protons in a nucleus of Iron? Assume a separation of $4.0 \times 10^{-15}$ m between them?

[ Ans: 14 N ]

**12.7** The distance $r$ between the electron and proton in the hydrogen atom is about $5.3 \times 10^{-11}$ m. What are the magnitudes of (a) the electrical force and (b) the gravitational force between these two particles?

[ Ans: $8.2 \times 10^{-8}$ N; $3.62 \times 10^{-47}$ N ]

**12.8** A body of mass $10^{-6}$ kg carries a charge of $10^{6}$. What is the magnitude of electric field which would hold it suspended in equilibrium in the gravitational field?

[ Ans: 9.8 $NC^{-1}$ ]

**12.9** A particle carrying a charge of $10^{-5}$ C starts from rest in a uniform electric field of intensity 50 V/m. Find the force on the particle and the kinetic energy it acquires when it has moved 1m.

[ Ans. $5 \times 10^{-14}$N; $5 \times 10^{-4}$ J ]

**12.10** An electron with an initial speed of $3.0 \times 10^{6}$ ms$^{-1}$ is fired in the same direction as a uniform electric field with a magnitude of 100 N/C. How far does the electron travel before being brought to rest momentarily and turned back?

[ Ans: 0.26 m ]

**12.11** A proton of mass $1.67 \times 10^{-27}$ kg and charge $+ 1.6 \times 10^{-19}$ C is to be held motionless between two parallel and horizontal metal plates 10 cm apart. Find the voltage of the battery which should be used.

[ Ans: $10.23 \times 10^{-9}$ V ]

**12.12** Two small identical conducting spheres first attract one another. Then conductors are touched together and brought back to the same separation as initially. The conductor now repel one another with the same force magnitude as initially. What was the ratio of the original charge magnitudes?

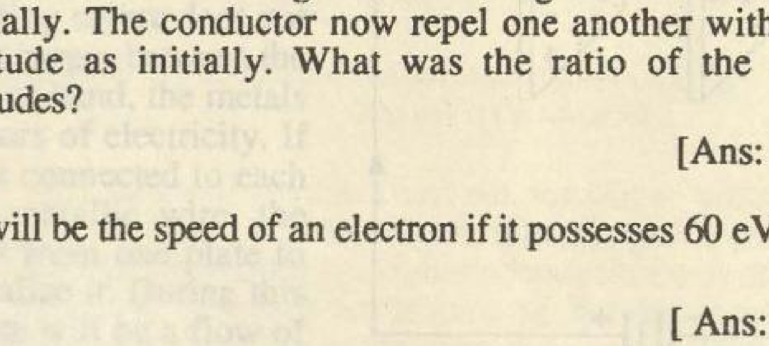
[Ans: $3 + \sqrt{8}$, $3 - \sqrt{8}$ ]

**12.13** What will be the speed of an electron if it possesses 60 eV energy?

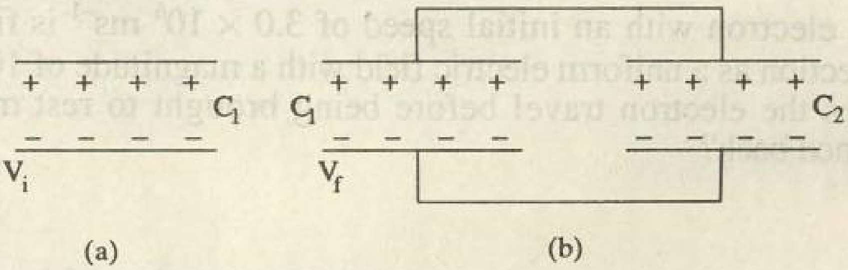
[ Ans: $4.6 \times 10^{6}$ ms$^{-1}$ ]

**12.14** Calculate the equivalent capacitance of the network between points A and B shown in the figure, where $C_1 = 15\mu$F, $C_2 = 5\mu$F and $C_3 = 10\mu$F.

[ Ans: 3.3 μF ]



**12.15** A capacitor $c_1$ is charged imitially with a potential difference $V_i$, as shown in figure (a). Then this capacitor is connected to an initially uncharged capacitor $C_2$ as shown in figure (b). What is the final potential difference across each.



$$\left[\,\text{Ans:} V_f = V_i\left(\frac{C_1}{C_1 + C_2}\right)\right]$$

**12.16** A capacitor of 50μF has a charge of 0.1C on each plate. How much more charge should be added to increase the potential difference by 200 V?

[ Ans: 0.01 C ]

# 13

# CURRENT ELECTRICITY

Some phenomena are associated only with the moving charges andwhich are not found in the situation when they are static. In other words, in making transition from electrostatics to moving charges, we have to study phenomena like magnetic field produced due to the motion of charges, resistance offered by the materials to the moving charges through them and lastly the dissipation of energy when the charges move through the conductors. In this chapter we shall discuss briefly the various features associated with the moving charges.

## 13.1 THE CONVENTIONAL CURRENT

In Fig. 13.1, two oppositely charged metallic plates have been shown. It is a situation where the charges are static because the air due to its non-conducting nature does not allow the flow of charges between the plates. On the other hand, the metals are good conductors of electricity. If the two plates are connected to each other by a long metallic wire, the charges will flow from one plate to the other to neutralize it. During this brief interval, there will be a flow of charge or current in the wire. To get a sense of direction for this current, let us ask ourselves the following question about this situation. Which of the two charges moved along the wire from one plate to the other? If we suppose that positive charge moved, then we note that as soon as the plates were connected by the wire the positive charge would be found to diminish rapidly to zero. Now if instantly, the other plate is examined it is found neutral and it is difficult to decide that whether our assumption was true or not. Perhaps, on account of this lack of conclusive evidence, the old investigators decided, that the positive charges left the plate, crossed along the wire, and neutralized the negative charges on other plate. We know today that it is incorrect. The positive charges are bound tightly at the lattice sites of the crystal structure of a metal and only the valence electrons from each atom are free to move.

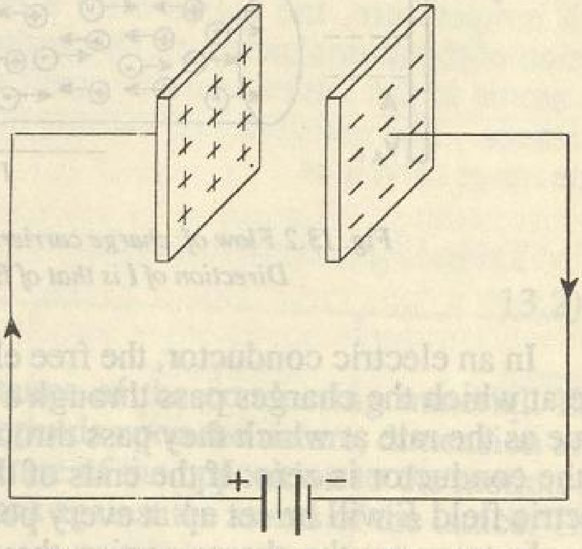


*Fig. 13.1 A static charge configurations*

This makes it certain that current through a metallic conductor is due to the motion of free electrons. However, in other situation like conduction of electricity through

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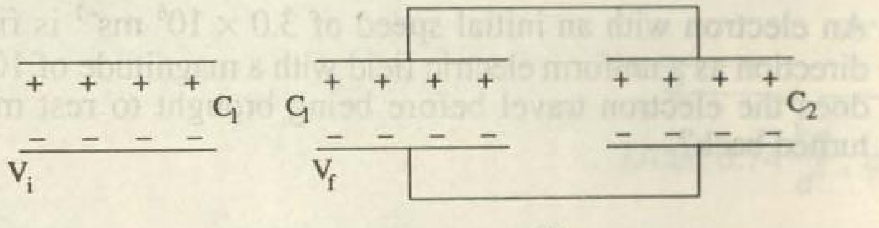

(a) (b)

[ Ans: $V_f = V_i\left(\frac{C_1}{C_1 + C_2}\right)$ ]

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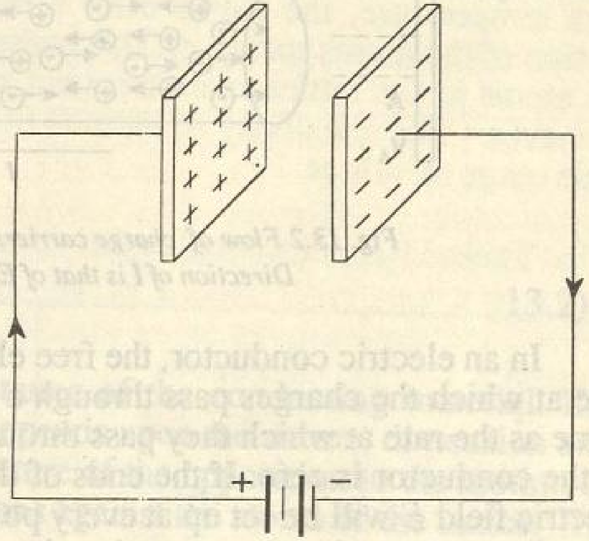

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This makes it certain that current through a metallic conductor is due to the motion of free electrons. However, in other situation like conduction of electricity through

It is experimentally observed that in general the resistance $R$ of a given wire increases with increase in length and decreases with increases in the area of cross section. Mathematically it can be written as $R \propto \frac{L}{A}$

$$R = \rho \frac{L}{A}$$

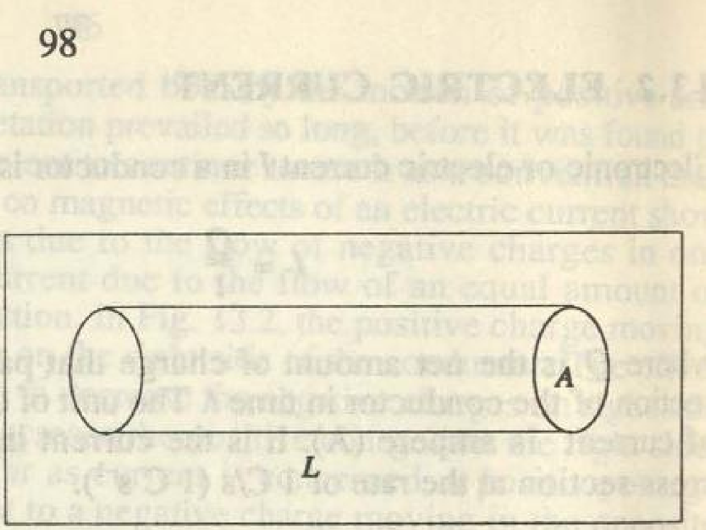


*Fig. 13.3 A conductor of length L and area of cross section A*

where $\rho$ is the resistivity of the meterial of which the conductor is made. As the temperature of the conductor rises, the amplitude of the vibration of the atoms in the lattice increases and hence the probability of their collision with free electrons also increases. It can be said, that at high temperature, the atoms offer a bigger target area, i.e., the collision cross section of the atoms increases. This makes the collision between free electrons and the atoms in the lattice more frequent and hence the resistance of the conductor increases. The resistivity of various materials are given in Table 13.1. Note the wide range of values.

Table 13.1 *Electrical resistivities (at room temp.) of various materials*

| Material | Resistivity (ohm – m) | Material | Resistivity (ohm – m) |
|---|---|---|---|
| Silver | $1.63 \times 10^{-8}$ | Pure Water | $5.0 \times 10^{3}$ |
| Copper | $1.77 \times 10^{-8}$ | Wood (maple) | $3.0 \times 10^{10}$ |
| Gold | $2.44 \times 10^{-8}$ | Glass | $2.0 \times 10^{11}$ |
| Tin | $11.50 \times 10^{-8}$ | Amber | $5.0 \times 10^{14}$ |
| Lead | $22.00 \times 10^{-8}$ | Rubber (hard) | $10^{13} - 10^{16}$ |
| Tungsten | $5.5 \times 10^{-8}$ | Aluminium | $2.83 \times 10^{-8}$ |
| Iron | $10.0 \times 10^{-8}$ | Mahgani | $44.0 \times 10^{-8}$ |
| Nichrome (alloy) | $100.0 \times 10^{-8}$ | Germanium | 0.5 |
| Carbon | $3.50 \times 10^{-8}$ | Siliocon | 20 – 2300 |

*Table 13.2 : Temperature coefficient of resistivity of various materials*

| Substance | Temperature Coefficient $K^{-1}$ | Substance | Temperature Coefficient $K^{-1}$ |
|---|---|---|---|
| Silver | 0.00380 | Platinum | 0.00392 |
| Copper | 0.00393 | Constantan | 0.00001 |
| Gold | 0.00340 | Mercury | 0.00090 |
| Aluminium | 0.00390 | Nichrome (alloy) | 0.00040 |
| Tungsten | 0.00450 | Carbon | 0.0005 |
| Iron | 0.00500 | Germanium | 0.05 |

Experimentally the change in resistivity of a metallic conductor with temperature is found to be nearly linear over a wide range of temperatures below and above 0°C, as shown in the Fig. 13.4 in case of copper. Over such a range, the fractional change in resistivity per kelvin is known as the temperature coefficient of resistivity ($\alpha$), i.e.,

$$\alpha = \boxed{\frac{\rho_T - \rho_o}{\rho_o \times T}} \qquad (13.3)$$

where $\rho_o$ and $\rho_T$ are the resistivities of the conductor at 0°C, and $T$°C, respectively. As $\rho$ is proportional to resistance, therefore, Eq. (13.3) can be expressed in terms of resistance as under

$$\alpha = \frac{R_T - R_o}{R_o \times T} \qquad (13.3a)$$

The different values of temperature coefficient of resistance are given in Table 13.2. The unit of $\alpha$ is $K^{-1}$ on the Kelvin scale. $\alpha$ is also helpful in differentiating between two metals having the same resistivity. For example, iron and platinum have the same resistivities but have different temperature coefficients.

As the resistivity changes with temperature, one may construct resistance thermometers which may register temperature through the change in their resistance.

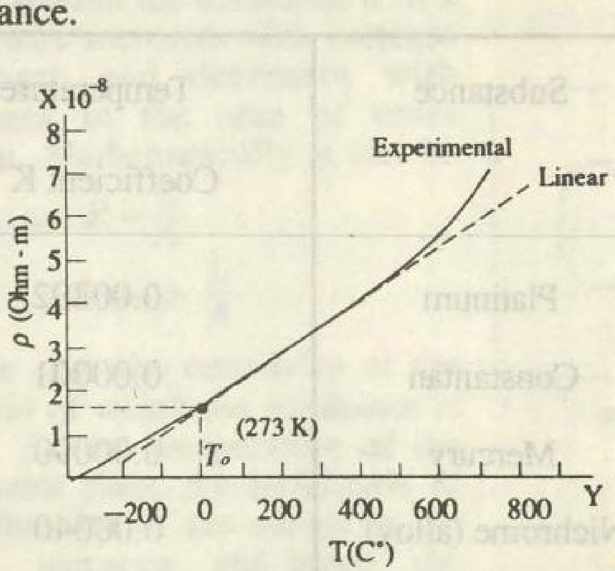
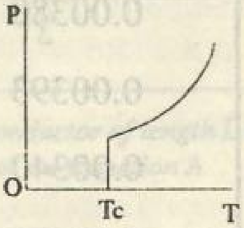


Fig. 13.4



*13.5 Variation of resistivity versus temperature for a super conductor*

As explained above, a fall in the temperature of a material reduces the thermal vibrations of the lattice ion and hence a decrease in the resistance can be observed in a metal. It is, however, difficult to make the resistivity and hence the resistance, exactly equal to zero with temperature. A class of elements (metals, alloys and ceramics) called superconductors show a remarkable variation in electrical resistivity with temperature as shown in Fig. 13.5.

One can observe in this figure that at a certain critical temperature $T_c$, the value of resistivity falls exactly to zero. For example, the value of resistance of some elements falls to zero below 7.175 K. At such low temperatures, the ions are frozen at the lattice sites and therefore do not scatter or hinder the motion of electrons in a superconductor.

In 1986, a remarkable discovery has been made in the field of superconductivity. In a laboratory of Zurich, two scientists Bednorz and Mueller have observed that certain ceramic materials exhibit superconducitvity at much higher temperatures, i.e., at $T = 30$ K, than the previously found highest value of $T_c = 7.175$ K. In the following years (1986 - 1990) some new ceramic materials have been prepared which have been found to be superconductor even at $T = 125$ K. Physcists today are working hard to produce compounds which could show superconductivity even at room temperature. The virtue of superconductivity is its ability to transport its electrical energy without loss. This could lead to substantial energy saving in future,

## Example 13.1

**Find the potential difference across two ends of a copper wire to maintain a steady current of one ampere. The length of the wire is one metre, radius of cross-section is 0.25 cm. ( $\rho$ for copper = $1.54 \times 10^{-8}\ \Omega - \mathrm{m}$ )**

**Solution**

The resistance of the copper wire will be given by the formula,

$$R = \rho \frac{L}{A}$$

Here $L = 1\text{ m}$

$$\rho = 1.54 \times 10^{-8}\ \Omega\text{–m}$$

$$r = 0.25\text{ cm} = 0.0025\text{ m};$$

$$A = \pi r^2 = \frac{22}{7} \times (0.0025)^2\text{ m}^2$$

$$A = 19.6 \times 10^{-6}\text{ m}^2$$

Hence $R = 1.54 \times 10^{-8}\ \Omega\text{–m}\,\dfrac{1\text{m}}{19.6 \times 10^{-6}\text{m}^2} = 0.000785\ \Omega$

and the potential difference required will be

$$V = IR$$

$$= 1.0\ \text{A} \times 0.000785\ \Omega$$

$$V = 0.000785\text{ volts}$$

**Example 13.2**

In the above example, calculate the magnitude of the charge which will pass through the wire in one hour. What will be the value of current passing through it after one hour, when rate of fall in current is $25 \times 10^{-6}\text{ As}^{-1}$.

**Solution**

According to Eq. (13.1) the charge *Q*, current *I* and time *t*, are related by the relation

$$I = \frac{Q}{t}$$

Hence $Q = 1.0\text{ A} \times 3600\text{ s} = 3600\text{ C}$

The decrease in current after one hour will be

$$= 25 \times 10^{-6}\text{ As}^{-1} \times 3600\text{ s} = 0.09\text{ A}$$

The current flowing through the wire after one hour will be

$$I = 1\text{A} - 0.09\text{ A} = 0.91\text{ A}$$

**Example 13.3**

A light bulb whose filament is made of tungsten has a resistance of 240 Ω when white hot (about 1800°C). Find the approximate resistance of the bulb at room temperature 20°C. ($\alpha$ for tungsten = $0.004\,6°\text{C}^{-1}$)

**Solution**

According to the discussion of the last section, such a high temperature makes the nonlinear part of the curve and Eq. (13.3) is not valid. However, as an approximation, one can use

$$\alpha = \frac{R_{T_2} - R_{T_1}}{R_{T_1}\,(T_2 - T_1)}$$

or $\quad \Delta R = R_{T_1}\,\alpha\,\Delta T$

where $\quad \Delta R = R_{T_2} - R_{T_1}$ and $\Delta T = T_2 - T_1$

Substituting the values of $\alpha$ and $\Delta T$, we get

$$\Delta R = R_{T_1}\ (0.0046\text{C}^{-1})\ (1800°\text{C} - 20°\text{C})$$

Where $R_{T_1}$ is the resistance in ohms at 20°C, and may be determined as

$$\Delta R = 240\,\Omega - R_{T_1}$$

Hence, equating the two values of $\Delta R$ we get

$240 - R_{T_1} = R_{T_1}\ (0.00465)\ (1780)$

Thus, we find that $R_{T_1} = 26.7\,\Omega$

## 13.4 COMBINATION OF RESISTORS

We now analyse the electrical circuit containing series and parallel combinations of resistors connected with a source of emf (battery, drycell etc).

### 13.4.1 Resistors in Series

Figure 13.6 a shows two points A and B between which there are three resistrors $R_1$, $R_2$ and $R_3$ connected end to end and are said to be connected in series. One of the characteristics describing series circuit is that the same current $I$ flows through each resistor. Let $V$ be the potential difference between A and B. Let $V_1$, $V_2$ and $V_3$ be the potential drops across the resistors $R_1$, $R_2$ and $R_3$, respectively. Assuming the connecting wires have no resistance, the potential at X is equal to the potential at A and the potential at Y is equal to the potential at B, and therefore, the total potential difference, $V$, is given by

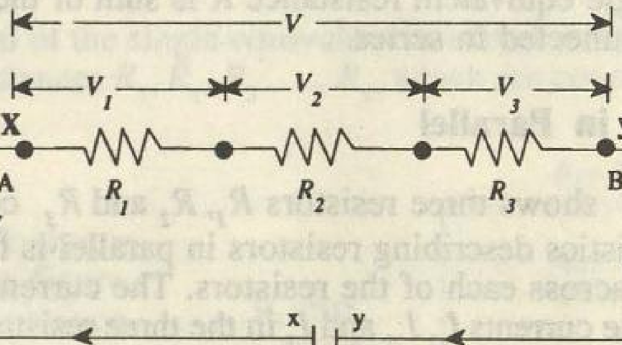


*Fig. 13.6 A schematic representation of resistors in series.*

$$V = V_1 + V_2 + V_3, \tag{13.4}$$

where from the definition $V_1 = IR_1$, $V_2 = IR_2$ and $V_3 = IR_3$. Let $R$ be the equivalent resistance to this series combination. As seen from Fig. 13.6 (b),

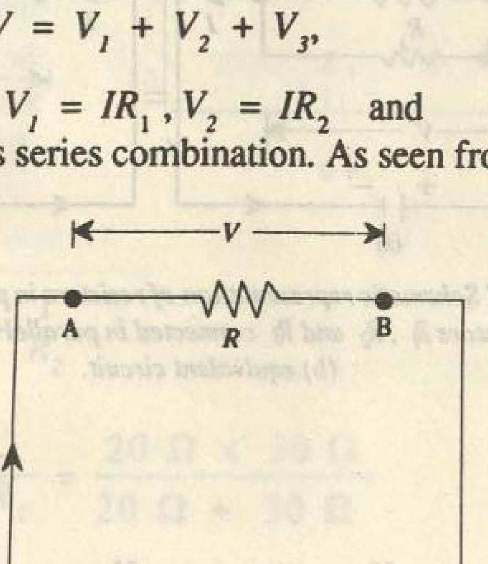


*Fig. 13.6 (b)*

$$R = \frac{V}{I} \quad \text{or} \quad V = IR$$

Therefore, from Eq. (13.4)

$$IR = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$$

i.e.,

$$R = R_1 + R_2 + R_3$$

If there are more than three resistors in series, a single resistance equivalent to this combination will be

$$R = \sum_{i=1}^{n} R_i \tag{13.5}$$

where *n* stands for total number of resistors and $\sum$ stands for summation. That is in a series circuit a single equivalent resistance *R* is sum of the individual resistors, $R_1, R_2, R_3, \ldots R_n$, connected in series.

### 13.4.2 Resistors in Parallel

Figure 13.7 (a) shows three resistors $R_1$, $R_2$ and $R_3$ connected in parallel. One of the characteristics describing resistors in parallel is that there is the same potential difference across each of the resistors. The current *I* in circuit must be equal to the sum of the currents $I_1$, $I_2$, and $I_3$ in the three resistances.

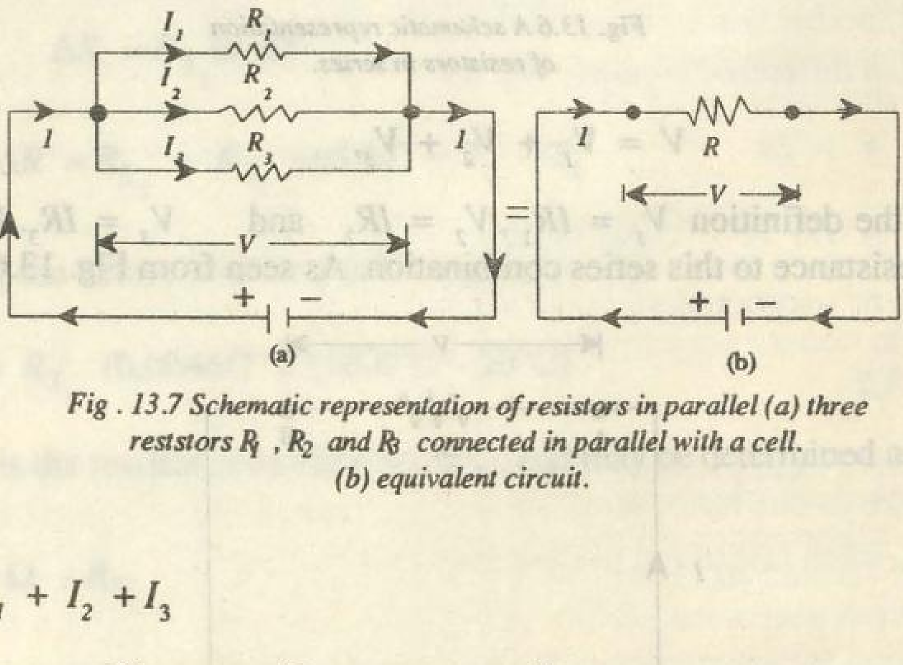


*Fig . 13.7 Schematic representation of resistors in parallel (a) three reststors $R_1$ , $R_2$ and $R_3$ connected in parallel with a cell. (b) equivalent circuit.*

Thus,

$$I = I_1 + I_2 + I_3 \tag{13.6}$$

where $\quad I_1 = \frac{V}{R_1}$ , $I_2 = \frac{V}{R_2}$ and $I_3 = \frac{V}{R}$

Let *R* be the resistance equivalent to the three resistors in parallel as shown in Fig. 13.7. (b). For such an equivalent circuit $I = \frac{V}{R}$. Substituting the values of $I_1, I_2, I_3$ and *I* in Eq. (13.6) we get

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

or

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \tag{13.7}$$

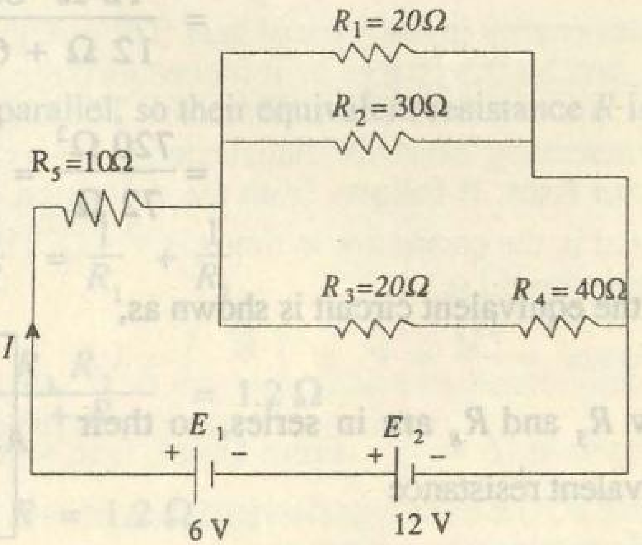
In general, for such a combination of $n$ ($n > 3$) resistors

$$\frac{1}{R} = \sum_{i=1}^{n} \frac{1}{R_i} \tag{13.8}$$

That is , the reciprocal of the single equivalent resistor $R$ is equal to the sum of the reciprocal of the resistances $R_1, R_2, R_3 \ldots\ldots R_n$, which are connected in parrallel.

**Example 13.4**

For the network shown in the accompanying figure, calculate the total resistance, and current $I$.



**Solution**

Resistances $R_1$ and $R_2$ are in parallel while $R_3$ and $R_4$ are in series to each other. let $R_6$ be the equivalent resistance to parallel combination. Then

$$\frac{1}{R_6} = \frac{1}{R_1} + \frac{1}{R_2}$$

or

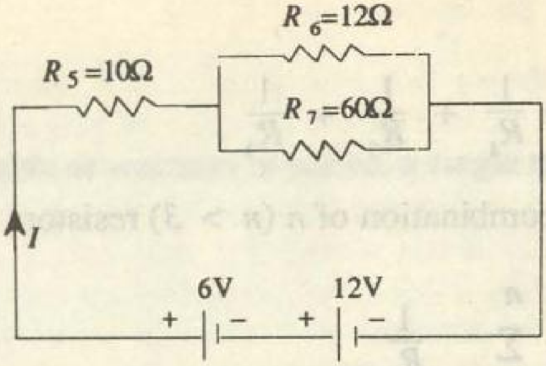
$$R_6 = \frac{R_1 R_2}{R_1 + R_2} = \frac{20\ \Omega \times 30\ \Omega}{20\ \Omega + 30\ \Omega}$$

$$= \frac{600\ \Omega^2}{50\ \Omega} = 12\ \Omega$$

And for series combination, let $R_7$ be the equivalent resistance, which may be calculated as

$R_7 = R_3 + R_4 = 20\ \Omega + 40\ \Omega = 60\ \Omega$

Again $R_6$ and $R_7$ are in parallel and have an equivalent resistance $R_8$, which is determined as

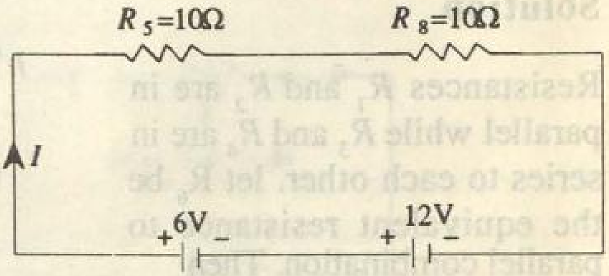

$R_6$=12Ω
$R_5$=10Ω
$R_7$=60Ω
I
6V
12V
+
−
+
−

$$R_{8'} = \frac{R_6 \; R_7}{R_6 + R_7}$$

$$= \frac{12\ \Omega \;\; 60\ \Omega}{12\ \Omega + 60\ \Omega}$$

$$= \frac{720\ \Omega^2}{72\ \Omega} = 10\ \Omega$$

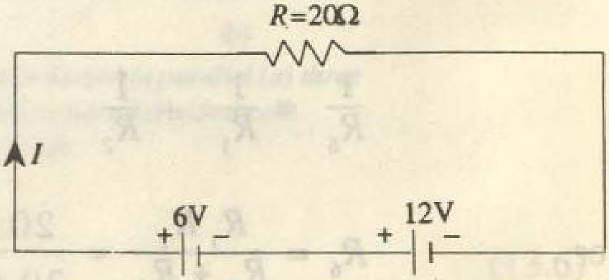
and the equivalent circuit is shown as,


$R_5$=10Ω
$R_8$=10Ω
I
6V
12V
+
−
+
−

Now $R_5$ and $R_8$ are in series, so their equivalent resistance

$$R = R_5 + R_8 = 10\ \Omega + 10\ \Omega = 20\ \Omega$$

and finally the circuit reduces to

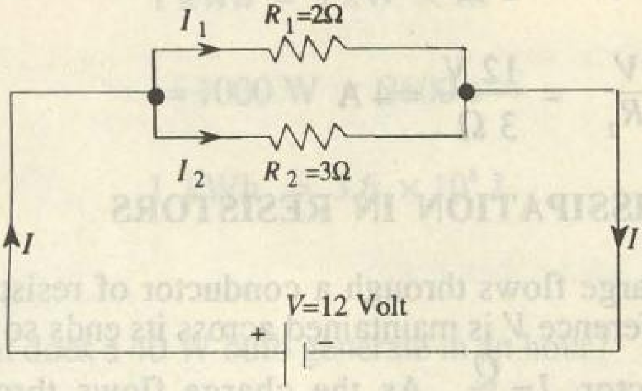

R=20Ω
I
6V
12V
+
−
+
−

Therefore, the current $I$, in the circuit is

$$I = \frac{V}{R} = \frac{18\ \text{V}}{20\ \Omega} = 0.9\ \text{A}$$

**Example 13.5**

Calculate currents $I, I_1$ and $I_2$ in the circuit shown in the accompanying figure.



**Solution**

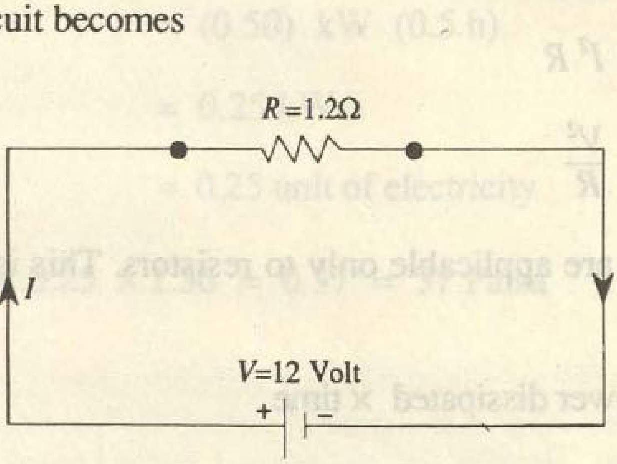
Resistances $R_1$ and $R_2$ are in parallel, so their equivalent resistance $R$ is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R = \frac{R_1 R_2}{R_1 + R_2} = 1.2\ \Omega$$

$$R = 1.2\ \Omega$$

and the equivalent circuit becomes



$$I = \frac{V}{R} = \frac{12\ V}{1.2\ \Omega} = 10\ \text{A}$$ .

The potential difference across both the resistances $R_1$ and $R_2$ is the same, i.e., 12 V, because they are in parallel.

Thus,

$$I_1 = \frac{V}{R_1} = \frac{12\ V}{2\ \Omega} = 6\ \text{A}$$

and

$$I_2 = \frac{V}{R_2} = \frac{12\ \text{V}}{3\ \Omega} = 4\ \text{A}$$

## 13.5 POWER DISSIPATION IN RESISTORS

Suppose $Q$ charge flows through a conductor of resistance $R$ in $t$ seconds when a potential difference $V$ is maintained across its ends so that current flowing through the counductor, $I = \frac{Q}{t}$. As the charge flows through the conductor dissipates energy in the form of heat. The energy dissipated is equal to the potential energy lost by the charge as it moves through the potential difference that exists between the two ends of the conductor. The enegy is delivered to the lattice atoms, hence increasing their vibrational energy. As a consequence, the temperature of the conductor rises. It follows from the definition of potential difference that energy dissipated in the conductor in time $t$ is $V \times Q$. Thus the power dissipated as heat due to electric current is,

$$\text{Power} = \frac{VQ}{t} = V \times \left(\frac{Q}{t}\right) = VI \tag{13.9}$$

where current $I = \frac{Q}{t}$. From Ohm's law, when the conductor is a resistor with resistance $R$, the energy is dissipated as heat in the resistor. The dissipated power, $P$, in this case can be written as

$$P = I^2 R$$

or

$$P = \frac{V^2}{R} \tag{13.10}$$

These two equations are applicable only to resistors. This is also known as Joule heating loss. Thus.

Heat dissipated = Power dissipated × time

$$= VI \times t \tag{13.11}$$

The unit of power, joule/second, is called watt. In case of electrical appliances, if $V$ is expressed in volts and $I$ in amperes, then the electrical power will be described in watts. The practical unit of power in common use is kilowatt, which is equal to 1000 watts.

One kilowatt-hour is commonly termed as 'one commercial unit' of electrical energy. It is the amount of energy delivered by the current in one hour when it supplies energy at the rate of 1000 $Js^{-1}$, i.e.,

Thus $1 \text{ kWh} = 1 \text{ kW} \times 1\text{h}$

$= 1000 \text{ W} \times 3600 \text{ s}$

$1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$

**Example 13.6**

How much heat does a 40 W bulb generate in an hour?

**Solution**

40 W means that the bulb generates 40J of heat per second. Therefore, heat generated in one hour will be

Heat $= (40\text{W})\ (1 \times 60 \times 60) = 144000 \text{ J}$

**Example 13.7**

If the electrical energy costs Rs. 1.50 per unit (kilowatt hour), how much does it cost to operate 500 - watt electrical kettle for 30 minutes?

**Solution** Heat dissipated = Power dissipated × time

$= (0.50) \text{ kW } (0.5 \text{ h})$

$= 0.25 \text{ kW h}$

= 0.25 unit of electricity

Cost $= 0.25 \times 1.50 = 0.37 = 37$ Paisa

**Example 13.8**

Heat energy dissipated by an electrical instrument in 2 hours is 600 J. Calculate the potential difference across the appliance if a steady current of 20 A is to be maintained through the appliance.

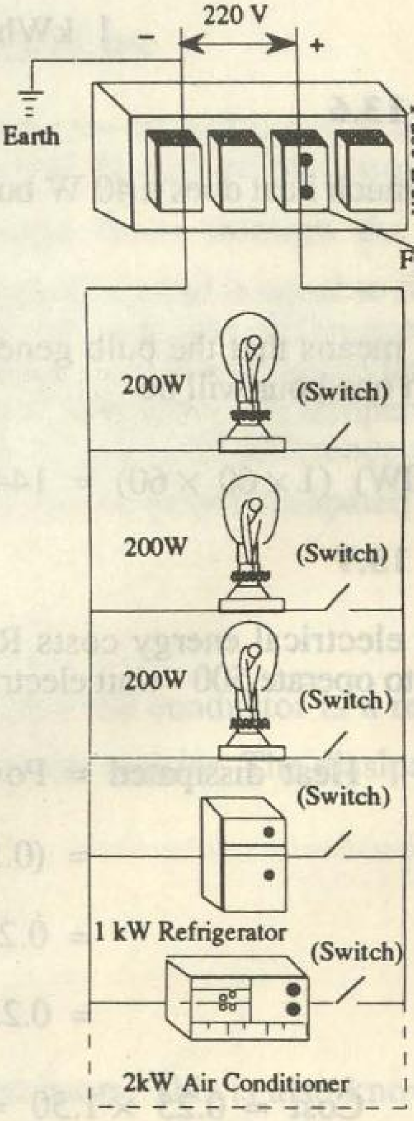
**Solution** According to Eq. (13.11)

$$\text{Potential difference} = \frac{\text{Heat energy dissipated}}{\text{current} \times \text{time}}$$

or $$\text{Potential difference} = \frac{600 \text{ J}}{20\text{A} \times (2 \times 60 \times 60) \text{ s}} = 4.17 \quad 10^{-3} \text{ volts}$$

**Example 13.9**

A newly made house has been provided five connections first for a 2 kW air conditioner, second for a 1kW refrigerator and third for three bulbs each of 200 W. Find (a) the current passing through the main fuse, when all the appliences are on; (b) if bulbs remain on for 8 hourse daily, airconditioner for 4 hours daily and refrigerator for 24 hours daily, find the cost of electrical energy consumed in 30 days of a month when the rate of electricity is Rs. 1.50 per unit?



**Solution**

As shown in the figure a house circuit consists of two parallel wires; one is at a high potential and the other at the ground potential. The potential for the live wire is 220 volts in this case. The electrial appliances are connected parallel to each other between these two wires as shown in the figure.

In most new houses, the wires inside the house are capable of carrying about 20 A without undue heating. However, to protect against too large a current, a fuse or circuit breaker is placed in series with the live wire at the entrance of the house. Its purpose is to disconnect the wire from it. Whenever, any appliance is switched on, it is connected to 220 volts, and the current through it also flows through the main fuse.

(a) The current through any appliance can be calculated by relation

$$\text{Current} = \frac{\text{Power}}{\text{Voltage}}$$

Current flowing through three bulbs $= 3 \times \frac{200\ \text{W}}{220\ \text{V}} = 2.72\ \text{A}$

Current flowing through one air - conditioner $= \frac{2000\ \text{W}}{220\ \text{V}} = 9.09\ \text{A}$

Current flowing through one refrigerator $= \frac{1000\ \text{W}}{220\ \text{V}} = 4.54\ \text{A}$

So the total current flowing through the main fuse

$$= 2.72\text{A} + 9.09\text{A} + 4.54\text{A}$$
$$= 16.35\ \text{A}$$

(b) Energy consumed by each appliance will be calculated as,

Energy consumed = Power × time

Energy consumed by three 200 W bulbs

$$= 200\text{W} \times 8 \times 30 \times 3$$

$$= 144000 \text{ watt hour}$$

Energy consumed by the refrigerator

$$= 1000W \times 24 \text{ hours} \times 30 \text{ days}$$
$$= 720000 \text{ watt hour}$$

Energy consumed by the air conditioner

$$= 2000\text{W} \times 4 \text{ hours} \times 30 \text{ days}$$

$$= 240000 \text{ watt hour}$$

Total energy consumed by the appliances durring one month

$$= 144000 \text{ watt hour} + 720000 \text{ watt hour}$$

$$+ 240000 \text{ watt hour}$$

$$= 1104\ \text{kWh}$$

$$= 1104 \text{ Units}$$

Cost of the electricity at the rate of Rs. 1.50 per unit = 1104 units × Rs. 1.5 = Rs. 1656.00

## 13.6 ELECTROMOTIVE FORCE

The devices which can maintain a potential difference between two points to which they are attached are known as sources of electromotive force. Fig 13.8 shows a source of emf (electromotive force) B (a battery for example) connected to a resistor *R*. In the external circuit to B positive charge will flow in the direction shown by the arrows marked *I*. In other words a colckwise current will be set up.

A source of emf must be able to do work on charge carriers (positive charge) that enter it. In Fig. 13.8, the source acts to move positive charges from a point of lower potential (the negative terminal) to a point of higher potential (the positive terminal) through the course. This act is similar to the act of a water pump which causes the water to move from a point of lower gravitational potential to a point of higher gravitational potential.

In Fig 13.8, *q* positive charge passes through any cross section of circuit in time *t*. This positive cahrge enter the source of emf *E* at negative terminal (lower potential terminal) and leaves at its positive terminal (higher potential terminal). The sourçe must do an amount of work *W* on *q* positive charge to force them to go to the point of higher potential from the point of lower potential. The electromative force of the source is defined by the relation;

$$E = \frac{W}{q}$$

clearly the unit of *E* is joule / coulomb which is volt. By virtue of equation $E = \frac{W}{q}$, we can say that a battery has an emf of 1volt if work done in transporting one coulomb charge through the source from negative to positive terminal is one joule.

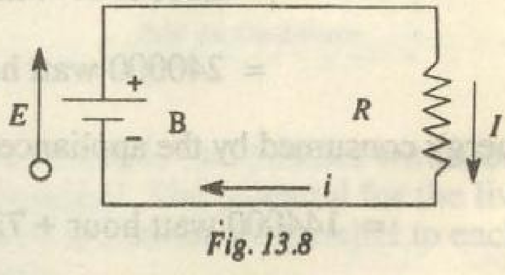


*Fig. 13.8*

***A battery, as a source of emf transports positive charge from point of lower electrical potential (negative terminal) to a point of high electrical potential (pasitive terminal)***

In Fig 13.8, the source, which is a battery in this case, transports positive charge from a point of lower potential (negative terminal) to a point of higher potential (positive terminal) through chemical reaction. Charge flows through the circuit, having electric and magnetic field energy, passes through resistor *R* producing (thermal) energy, reaches the negative terminal of the battery and enters it. The chemical energy is first changed into electricl field energy and then into thermal energy. In all kinds of sources of e.m.f. some kind of energy is transformed into electrical energy. A few examples are given below;

1. Batteries or cells convert chemical energy into electrical energy.
2. Electrical generators convert mechanical energy into electrical energy.
3. Thermocouples convert heat energy into electrical energy.
4. Photo voltaic cell converts light energy into electrical energy.

### 13.6.1 Electromotive Force (emf) And Terminal Potential Difference

Consider a source of emf $E$ connected to a resistance $R$, through which a steady current $I$ flows as shown in Fig. 13.9. According to law of conservation of energy, the electrical power suplied by the source $P_E$ must be equal to the power dissipated $P_{\mathrm{R}}$, as thermal energy in the resistor.

$$P_E = P_{\mathrm{R}} \tag{13.12}$$

$$EI = I^2R$$

$$I = \frac{E}{R} \tag{13.13}$$

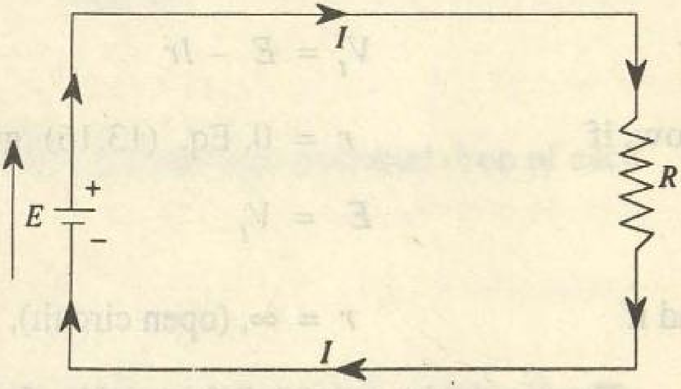


*Fig. 13.9 Simple single loop circuit containing R and E*

In deriving the above equation we have assumed that the resistance of the source or emf is zero and the wires connecting the source and the resistor have negligible resistance. However, in practice, the sources of emf always have some resistance, called the internal resistance of the source, while, the above statement is often true in case of

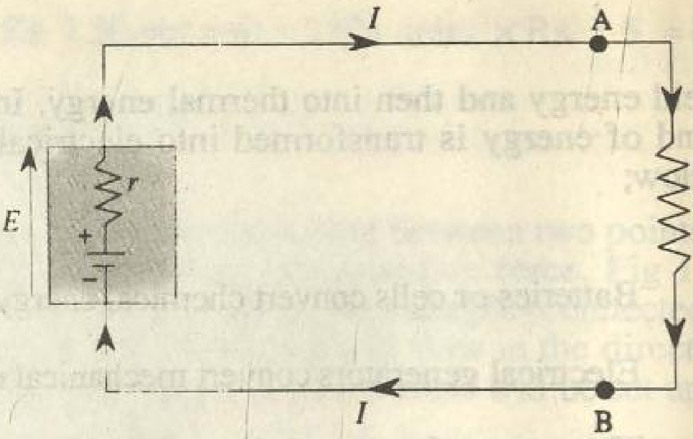


*Fig. 13.10. Simple single loop circuit containing R, E and r (internal resistance).*

connecting wires. Thus a source of emf $E$ and having internal resistance $r$ is equivalent to a source of pure emf $E$ with a resistance $r$ in series. The shaded block in Fig. 13.10 implies that $r$ is not separate from the source. Now in case of Fig. 13.10, some power $P$ must be lost as thermal energy in the resistor of the source as well, and the Eq. (13.12) will be modified accordingly as

$$P_E = P_R + P_r$$

or $$EI = I^2R + I^2r$$

or $$E = IR + Ir \qquad (13.14)$$

Hence $$I = \frac{E}{R + r} \qquad (13.15)$$

Now $IR$ represents the potential difference across the resistance $R$ and it is also the potential difference acorss the terminals of the source as shown in Fig. 13.10. Let us denote it by $V_t$, then Eq. (13.14) becomes

$$E = V_t + Ir$$

or $$V_t = E - Ir \qquad (13.16)$$

Now, if $r = 0$, Eq. (13.16) gives,

$$E = V_t \qquad (13.17)$$

and if $r = \infty$, (open circuit), $I = 0$

**The terminal potential difference$V_t$ of a source of emf (battery in this case) is always less than $E$ unless, either the source has no internal resistance ($r = 0$) or it is an open circuit ($R = \infty$), then $V_t$ is equal to $E$.**

**Example 13.10**

A typical 12 V automobile battery has a resistance of 0.012 Ω. what is the terminal voltage of this battery when the starter draws a current of 100 A? Also calculate $R$, $R_{E,}$ $P_R$ and $P_r$.

**Solution**

According to Eq. (13.16), terminal potential drop

$$V_t = E - Ir$$

$$= 12\text{V} - (100\ \text{A}\times 0.012\ \Omega)$$

$$= 12\ \text{V} - 1.2\ \text{V}.$$

$$E - Ir = 10.8\ \text{V}$$

According to Eq. (13.16)

$$R = \frac{E - Ir}{I} = \frac{10.8}{100\ \text{A}} = 0.108\Omega$$

Now $P_r = I^2 r = (100\ \text{A})^2\ (0.12\ \Omega) = 120\ \text{W}$

$$P_R = I^2R = (100\ \text{A})^2\ (0.108\ \Omega) = 1080\ \text{W}$$

∴ it can be verfied that $P_E = P_R + P_r$,

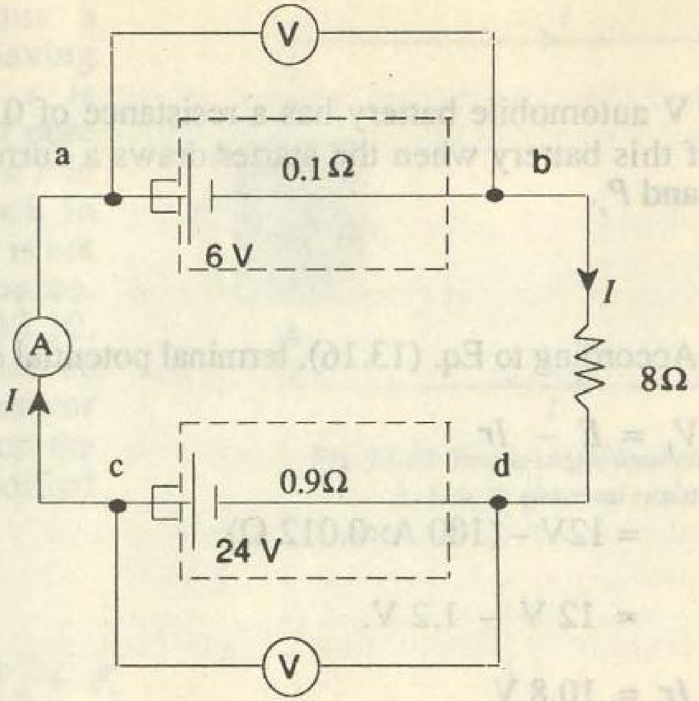
$$P_E = E_I = 12\ \text{V} \times 100\ \text{A} = 1200\ \text{W}$$

**Example 13.11**

In the given figure, calculate the current $I$ and terminal potential drop of each electromotive force.

**Solution**

**As shown in the figure two batteries are opposing each other so the effective driving emf in the circuit is 18 V. The current $I$ due to it will flow in the direction shown.**



The internal resistance of the emf sources are in series with the external resistance of 8 Ω. The equivalent resistance of the circuit is thus 9.0 Ω and the current,

$$I = \frac{18\ \text{V}}{9\ \Omega} = 2\text{A}$$

To calculate the terminal potential drop of the battery of emf 6 V, we note that current is flowing from the point c to the point a and hence enhancing the potential drop between the points a and b. Hence

$$V_t = V_a - V_b = E - IR = 6\ \text{V} + (2\ \text{A})\ (0.1\ \Omega) = 6.2\ \text{V}$$

Similarly, for the other source of emf,

$$V_t = V_c - V_d = E - I_r = 24\ \text{V} - (2\ \text{A})\ (0.9\ \Omega) = 22.2\ \text{V}$$

Notice that the terminal potential of the battery of emf 6V, is larger than its emf. This is the case when a battery is being charged.

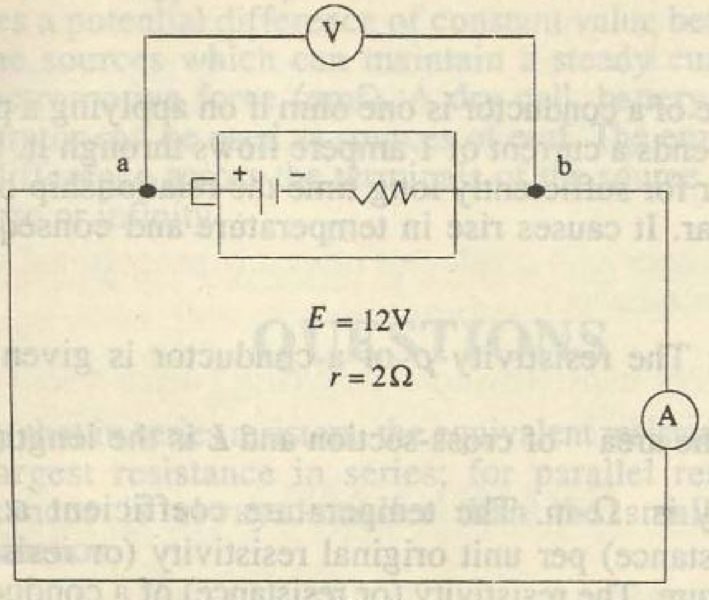
**Example 13.12**

In the given figure the source is short circuited. Calculate $V_t$ and I.

**Solution**

The existance of a negligibly small resistance in the circuit connected to an emf source is called short circuiting.

From Eq. (13.15)



$$IR = E - Ir$$

as $R = 0$

$$I = \frac{E}{r} = \frac{12\ V}{2\ \Omega} = 6A$$

$$V_t = V_a - V_b = 12\ \text{V} - 6 \times 2\text{V} = 0$$

The ammeter reads 6 A and voltmeter reads zero.

# SUMMARY

**Current and Ohm's law:**

Current is the net charge flowing across the cross sectional area per unit time, $I = \frac{Q}{t}$. The unit of current is ampere. The direction of current is that in which positive charge will drift. i.e. in the direction of E. In a electrical circuit, the potential difference across a conductor is directly proportional to the current flowing through it, provided the temperature of the conductor remains the same. Under this condition, the ratio of potential difference $V$ to the current $I$ is constant and is known as resistance $R$; ($R = \frac{V}{I}$) The unit of resistance is ohm ($\Omega$); ohm $= \frac{1 \text{ volt}}{1 \text{ ampere}}$.

The resistance of a conductor is one ohm if on applying a potential difference of 1 volt across its ends a current of 1 ampere flows through it. On passing current through the resistor for sufficiently long time the relationship between $V$ and $I$ no more remains linear. It causes rise in temperature and consequently increase in resistance.

**Resistively;** The resistivity $\rho$ of a conductor is given by the relation $\rho = \frac{R \times A}{L}$; $A$ is the area of cross-section and $L$ is the length of the conductor. The unit resistivity is $\Omega$-m. The temperature coefficient $\alpha$ is the change in resistivity (or resistance) per unit original resistivity (or resistance) per degree change in temperature. The resistivity (or resistance) of a conductor decreases due to fall in temperature but it is very difficult to make it exactly equal to zero. The resistivity (or resistance) of a class of elements at some critical temperature $T_c$, falls to zero. The materials showing such a property are called "superconductors".

## Combination of resistors:

For resistors in series, the equivalent resistor

$R = R_1 = R_2 + R_3 ... = \sum_{i=1}^{n} R_i$, and for resistors in parrallel, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + .............. = \sum_{i=1}^{n} \frac{1}{R_i}$

## Power dissipation in resistance:

On applying potential difference across the two ends of a conductor, a net force acts on the free electrons comprising of the current. During the collisions of the free electrons with the atoms of the conductor, the kinetic energy acquired by them from the external field is lost to the conductor lattice and the power delivered to the conductor is dissipated as thermal energy. Power dissipated $= \frac{VQ}{t} = VI = I^2R = \frac{V^2}{R}$. 1 kilowatt – hour is amount of energy delivered by the current in one–hour when it supplies energy at the rate of 1000 J/hour.

**Source of emf:**

Due to heat energy dissipation, the continuous maintainance of a steady current requires a potential difference of constant value between the two ends of a conductor. The sources which can maintain a steady current are known as the sources of electromotive force (emf). A dry cell, battery, a thermocouple or an electrical generator can be used as sources of emf. The emf of a source is equal to the potential difference across the terminals of the source when either its internal resistance is zero or infinity.

## QUESTIONS

**13.1** Verify that in series resistors, the equivalent resistance is always larger than the largest resistance in series; for parallel resistors, the equivalent resistances is always smaller than the smallest resistance in the combination.

**13.2** Often, you might have noticed crows sitting safely on high tension wires. Why are they not electrocuted, even when sitting on a part of the wire where the insulation has worn off?

**13.3** Why is it dangerous to touch a live wire while standing on earth bare footed?

**13.4** It is extremely dangerous to use a plug in radio near a bath tub when you are taking a bath. Why? Does the some reasoning apply to a battery operated radio?

**13.5** A beame of protons emerging from a particle accelerator such as a cyclotron constitutes an electric current. If this current is used to charge a capacitor, how could a measure of the charge on the capacitor be used to determine the average rate at which proton emerges from the accelerator?

**13.6** If the positive and negative connections to the terminals of an automobile battery were reversed, would you expect the automobile light to function.

**13.7** Is the principle of energy conservation always applicable to electrical circuits? Explain.

**13.8** What is a short circuit and open circuit mean to you?

**13.9** Describe a circuit which will give a continuously varying potential.

**13.10** The current in an electric bulb rises, to a maximum almost at the instant the bulb is turned on. The current then decreases significantly. Why does the current decrease?

**13.11** Is it possible to have a situation in which the terminal voltage will be greater than the emf of the battery?

**13.12** Why a thin region of light bulb filament has more possibility to burn than the thicker one?

**13.13** In the normal operation of a resistor, what limits the temperature rise due to joule heating?

**13.14** Why is the resistance of a conductor inversely propotional to the area of crossection of a conductor?

**13.15** Describe the energy transformations for charges when a source of e.m.f. is connected to a light bulb?`

# PROBLEMS

**13.1** Calculate the resistance of a wire 20 m long that has a radius of 2mm and resistivity $5.5 \times 10^{-8}\ \Omega \times \text{m}$. If the two ends of the wire are connected to a source of 24 V what is the value of current?

[ Ans: $8.75 \times 10^{-2}\ \Omega$, 274 A]

**13.2** A current of 1.0 A is maintained in a copper wire 10m long and having a cross-sectional area 0.25 $\text{mm}^2$. (a) What is potential difference between its end? (b) What is the magnitude of the electric field within the wire driving electrons? (c) At what distance from a proton is the electric field from it if the same magnitude as the driving electric field?

[Ans: 0.69V, 0.069 V/m, 0.14 mm ]

**13.3** The resistance of a tungsten wire at room temperature (20°C) is 1.5 Ω. What will be its resistance at 0°C, 100°C and 1000°C?

[ Ans: 1.37 Ω, 2.0002 Ω, 7.67 Ω ]

**13.4** How much charge passes in 2 minutes through a junction through which a steady current of 5A exists?

[Ans:$t = 2 \times 60 = 120$ s,

$Q = It = 5 \times 120 = 600$C]

**13.5** If there are $10^{18}$ electrons flowing across any cross-section of a wire in 1 min, what is the current in the wire?

[ Ans: $2.67 \times 10^{-3}$ A]

**13.6** A transistor radio operates at 10mW with a 9 volt dry cell. (a) What is current into and out of radio? (b) What is radio's electric resistance?

[Ans: 1.1 mA, 8.2 kΩ ]

**13.7** There is a current of 5 mA in a wire. How much charge flows through cross-section of the wire in 10 s? If the current is due to flow of electrons, how many electrons pass through the wire in 10 second?

[ Ans: $5 \times 10^{-2}$ C, $3.125 \times 10^{17}$ ]

**13.8** A copper wire has a diameter of a 1 mm, and length 1m, If 2 coulomb is the charge which flows through a cross-section of this wire in 30 seconds, how much heat will be dissipated in one hour?
( $\rho$ for copper $= 1.77 \times 10^{-8}\ \Omega$ m )

[ Ans: 0.36 J approx. ]

**13.9** Calculate the resistance of a wire 10 m long that has a diameter of 2mm and resistivity of $2.63 \times 10^{-8}\ \Omega$ m.

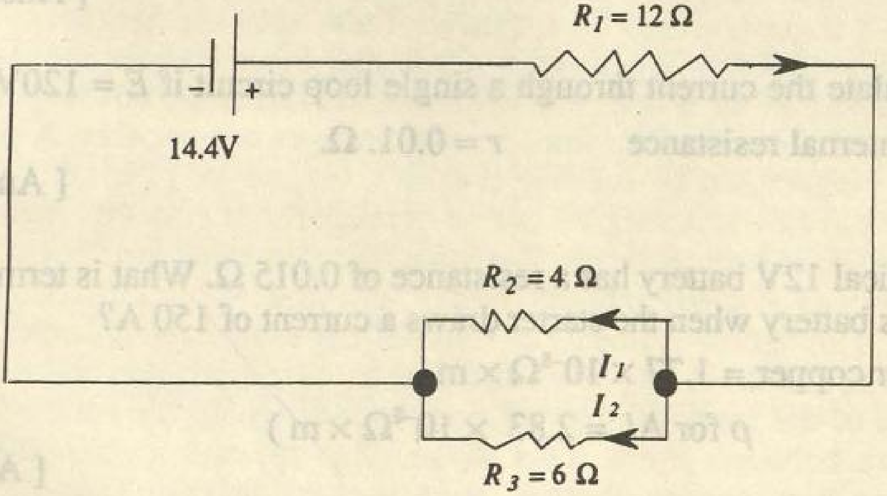
[ Ans: $8.38 \times 10^{-2}\ \Omega$ ]

**13.10** A solenoide is constructed from 500m of copper wire having a diameter of 1.5 mm. Calculate its electrical resistance?

[Ans: 4.88 Ω ]

**13.11** Consider the arrangement shown in figure. Calculate

(a) the current through each resistor?
(b) power lost as heat in each resistor; and
(c) power lost by the source. Does this agree with the sum in part (b)?



[ Ans: $I$ = 1A, $I_1$ = 0.6A, $I_2$ = 0.4A
$P_1$ = watt; $P_2$ = 1.44 watt, $P_3$ = 0.96 watt,
Source = 14.4 watt ; It agrees ]

# 14

# MAGNETISM AND ELECTROMAGNETISM

Magnetism derives its name from Magnesia a region in Asia Minor (modern Turkey) where it was found in the form of lumps of certain iron ore. These lumps of iron ores were found to have the property of attracting small pieces of iron. It was also found that if such a lump was suspended so that it could rotate freely then it always pointed towards north and south direction when at rest. So it was also called lode stone. Later it was found that these properties are exhibited not only by iron, but also by cobalt, nickel and many compounds of these metals. These are called magnetic materials.

Summarized below are the characteristics of magnetic materials. A body that attracts small pieces of iron and points towards north–south direction when suspended freely, is called a magnet. The end of the magnet pointing towards north is called the North pole while the other is called the South pole. Like poles of two magnets repel, while unlike poles attract, one another. The magnetism of the magnet is concentrated in the poles of the magnet. The two poles of a magnet cannot be separated from each other. If a magnet is broken into two pieces, two new magnets are obtained. Each new magnet has both the poles; a N–pole and a S–pole. This process may be repeated as many times as desired but each time a magnet with both the poles is obtained.

Just as electrostatic interactions are represented by means of fields so also are magnetic interactions. The space around a magnet where the influence of the magnet is felt by another magnet or a magnetic substance is called magnetic field. The magnetic field is represented by magnetic field lines (or magnetic lines of force). The path along which an isolated north pole of a magnet moves in the magnetic field is called the field line. The field lines are directed from N–pole of the magnet towards the S-pole, the field lines do not intersect one another. A magnetic substance can be magnetized by rubbing it with a magnet or placing it in a coil through which current flows. A magnet can be demagnetised either by heating it or by dropping (striking) it several times. We have discussed that two point charges located at some distance apart interact with each other with a force called as electrostatic force. The nature of this force remains the same even if one of the two charges moves in space or in any medium. But in case both the charges are in motion, the nature of the force of interaction does not remain the same. Instead a new force appears between them. This force, that appears as a result of the interaction between two moving charges, is known as magnetic force. Since the moving charges give rise to electric currents therefore, the strength of interaction between two currents can be found by the interaction of magnetic effects produced by them as explained below.

## 14.1 MAGNETIC FIELD DUE TO CURRENT

The interaction of one charge with some other charge is generally described by associating fields around the charges. Just as an electric charge brought in the field around a fixed charge interacts with the field and experiences an electrostatic force, similarly, a moving charge interacts with a field around another moving charge and experiences magnetic force. The field around a moving charge is called magnetic field, as opposed to an electric field around stationary charge. The magnetic field is denoted by **B** and is a vector quantity. Its experimental demonstration was carried out by Hans Oersted, a Danish professor of Physics. He showed that an electric current in a wire deflects a near-by compass needle. Thus, a current-carrying wire has a magnetic field around it which is the manifestation of interaction between moving charges in the conductor. The lines of force form concentric circles around the conducting wire and the magnetic field remains effective so long as the current keeps flowing through the wire.

Furthermore, it can be demonstrated that the strength of the magnetic field is greater near the current-carrying wire and decreases as the distance from the conducting wire increases. The direction of the magnetic lines of force is given by right hand rule which is sketched in Fig. 14.1, and stated as follows : "If the wire is grasped in the right hand with the thumb pointing in the direction of the current, the fingers of the hand will circle the wire in the direction of the magnetic field".

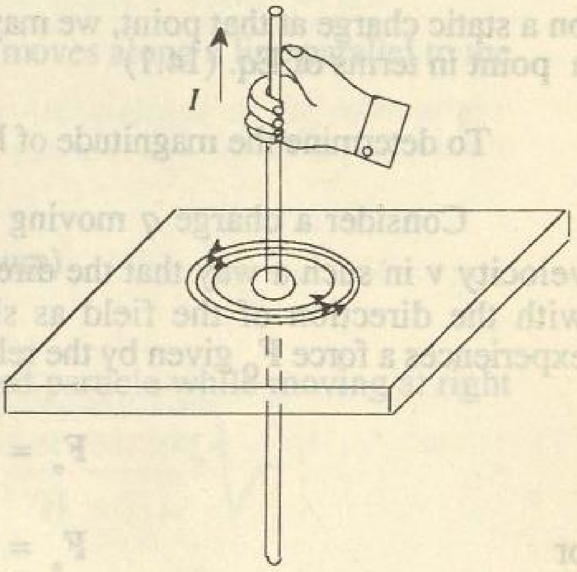


*Fig. 14.1 Schematic representation of right hand rule.*

We can experiment with moving charges in the presence of a magnetic field due to a bar magnet. The magnetic field produced by the moving charges will interact with the magnetic field due to bar magnet in the same manner as the magnetic fields of two magnets interact with each other. It can be experimentally found that:

(i) The charge moving parallel to **B** with a certain velocity **v** experiences no force.

(ii) The charge moving with velocity **v** perpendicular to **B** experiences maximum force.

(iii) The force is always perpendicualr to **v** and **B**.

(iv) The force is proportional to charge *q* and velocity **v**.

The above observations can be expressed in a mathematical form as

$$\mathbf{F}_m = q\,\mathbf{v} \times \mathbf{B} \tag{14.1}$$

where $\boldsymbol{F}_m$ is the force acting on charge $q$ when it moves with a velocity $\mathbf{v}$ through a magnetic field $\mathbf{B}$.

The only undetermined feature of the above experimental observation is the nature of dependence of $\mathbf{F}_m$ on the magnitude of $\mathbf{B}$. This is because we have not yet assigned any magnitude to $\mathbf{B}$. We use Eq. (14.1) to define the magnitude of $\mathbf{B}$. Just as the electric field intensity $\mathbf{E}$ is defined at a point in terms of the force

$$\mathbf{F}_e = q\mathbf{E}$$

on a static charge at that point, we may define the magnitude of magnetic field $\mathbf{B}$ at a point in terms of Eq. (14.1)

To determine the magnitude of $\mathbf{B}$ we proceed as below

Consider a charge $q$ moving through a uniform magnetic field $\mathbf{B}$ with a velocity $\mathbf{v}$ in such a way that the direction of motion of charge makes an angle $\theta$ with the direction of the field as shown in Fig. 14.2 (a). The charge $q$ then experiences a force $\mathbf{F}_o$ given by the relation

$$\mathbf{F}_o = q\,(\mathbf{v} \times \mathbf{B})$$

or

$$\mathbf{F}_o = q\,v B \sin\theta\,\hat{\mathrm{n}}$$

where $\hat{\mathrm{n}}$ is a unit vector in a direction perpendicular to the plane PQRS containing $\mathbf{v}$ and $\mathbf{B}$ (Fig. 14.2 b).

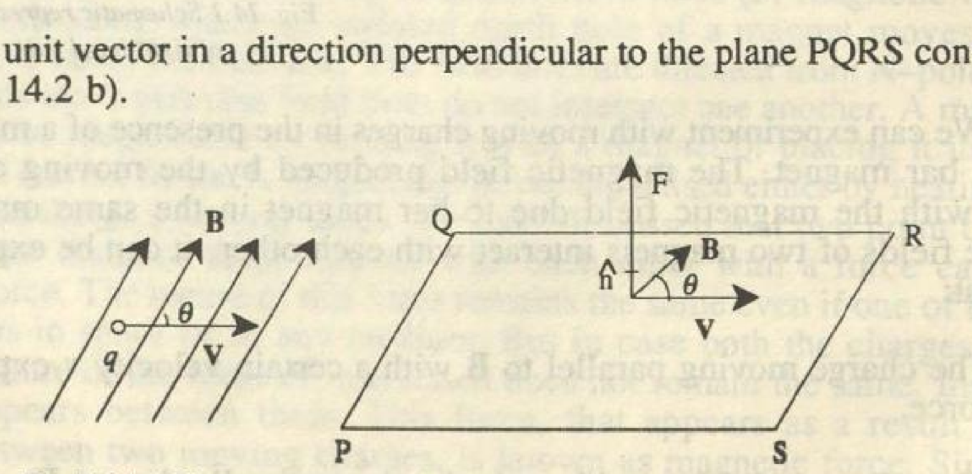


*Fig 14.2 (a) Charge q moves with velocity v through a magnetic field B at an angle θ with the field.*

*(b) Direction of force experienced by the charge q.*

In magnitude

$$|\mathbf{F}| = qvB \sin\theta \qquad (14.2)$$

Let us consider the following cases

Case 1.

$$\text{If } v = 0 \quad |\mathbf{F}| = 0$$

Thus no force is experienced by the charge at rest in the magnetic field.

Case 2.

$$\text{If } \theta = 0; \text{ or } 180^\circ \text{ again } |\mathbf{F}| = 0$$

Thus no force is experienced by the charge which moves along a line parallel to the direction of magnetic lines of force.

Case 3.

$$\text{If } \theta = 90^\circ, \text{ then } |\mathbf{F}| = qvB \text{ (maximum)}$$

Thus a maximum force is experienced by a charged particle while moving at right angle to the magnetic field. From Eq. (14.2)

$$|\mathbf{F}| = qvB \sin\theta$$

or
$$B = \frac{|\mathbf{F}|}{qv \sin\theta}$$

If $q = 1$ coulomb, $v = 1\ \text{ms}^{-1}$, $\theta = 90^\circ$ and $F = 1\ N$, then

$$B = \frac{1.0N}{1.0\text{C} \ \ 1.0\text{ms}^{-1}} = 1.0 \text{ tesla (T)}$$

**Thus we can define tesla, a unit of magnetic induction B in SI units, as given below.**

The magnetic induction is said to be one tesla, if a charge of one coulomb moving at right angle to the magnetic field with a velocity of one metre per second experiences a force of one newton.

### Example 14.1

A uniform magnetic field $B$ of 1.5 T points horizontally from south to north. If a proton with K E 3.0 MeV moves vertically downward through this field, find the force acting on it.

### Solution

The kinetic energy of the proton is

$$KE = 3\text{MeV} = (1.6 \times 10^{-19}\ \text{C}) \times (3 \times 10^{6}\ \text{V})$$

$$= 4.8 \times 10^{-13}\ \text{J}$$

But $KE = \frac{1}{2}\, mv^2$

So $v = \sqrt{\dfrac{2 \times 4.8 \times 10^{-13}\ \text{J}}{1.7 \times 10^{-27}\ \text{kg}}} = 2.37 \times 10^{7}\ \text{ms}^{-1}$

Using Eq. (14.2), we get

$$F = \left[1.6 \times 10^{-19}\ \text{C}\right]\left[2.37 \times 10^{7}\ \text{ms}^{-1}\right][1.5\ \text{T} \sin 90^\circ]$$

$$= 1.6 \times 10^{-19}\ \text{C} \times 2.37 \times 10^{7}\ \text{ms}^{-1} \times \frac{1.5\ \text{N}}{\text{cms}^{-1}}$$

$$= 5.7 \times 10^{-12}\ \text{N}$$

## 14.2 FORCE ON A CURRENT–CARRYING CONDUCTOR IN A UNIFORM FIELD

Current in a conductor is produced when charges move through it. Because magnetic field exerts a side ways force on a moving charge, we expect that it will also exert a sideways force on a current-carrying conductor. Fig. 14.3 shows a wire of length $l$ through which a current $I$ is flowing and which is placed in a magnetic field **B**. For the sake of simplicity the direction of flow of charge through the wire is taken to be at right angle to **B**.

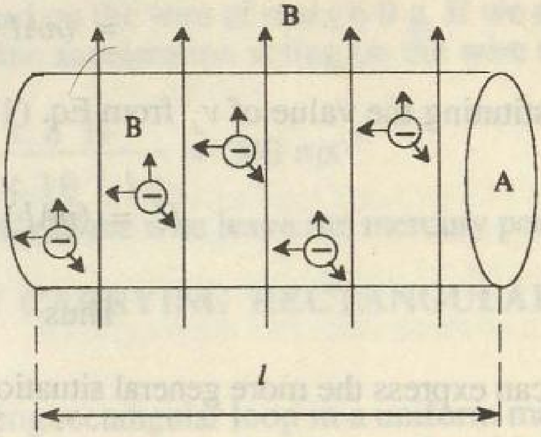


*Fig. 14.3 Force experienced by charges having a drift velocity $v_d$ inside the conductor placed in the magnetic field*

Let $n$ be the number of free electrons per unit volume of the wire when current $I$ is passing through it. The magnitude of the average force on one such electron is given by Eq. (14.2). Because $\theta = 90°$, so

$$F' = qvB \sin\theta = e\,v_d B$$

where $v_d$ is the drift speed. $v_d$ can be obtained by considering that the total number of free electrons in a wire of length $l$ and area of cross-section $A$ is $nAl$ . A charge of magnitude

$$q = (nAl)\,e$$

passes out of the wire through its right cross-section in time $t$ given by

$$t = \frac{l}{v_d}$$

The current $I$ is given by

$$I = \frac{q}{t} = \frac{nAl\,e}{l\,/v_d} = nAev_d$$

or
$$v_d = \frac{I}{nAe} \qquad (14.3)$$

As the force on one electron is $F' = ev_d B$, hence the force experienced by all the electrons in the volume $Al$ is

$$F = (nAl)\ F'$$

$$= (nAl)\ ev_d B \qquad (14.3\ a)$$

Substituting the value of $v_d$ from Eq. (14.3) in Eq (14.3 a) we get

$$F = (nAl)\ e\ \frac{I}{nAe}\ .\ B$$

$$\text{Thus} \qquad F = IlB \qquad (14.4)$$

We can express the more general situation in a vector form as

$$\mathbf{F} = \mathbf{I}\mathbf{L} \times \mathbf{B}$$

where **L** is a vector whose magnitude is the length of the straight wire in the direction of the current. The magnitude of the magnetic force **F** on the wire is given by Eq. (14.4). According to this equation the magnitude of the magnetic force on the wire is directly proportional to the current in the wire the length of the wire, and the magnetic field **B**. This characteristic of the magnetic field is termed as magnetic induction **B**. The magnetic induction **B** has the same direction as the magnetic lines of force at that point. Therefore **B** can be expressed, mathematically, as

$$\mathbf{B} = \frac{\mathbf{F}}{\mathbf{I}\mathbf{L}}$$

Thus the magnetic induction is equal to the force exerted on a wire of length one metre carrying one ampere current placed at right angle to the field. The SI unit of magnetic induction is newton/ampere-meter. It is also called tesla (T) when the magnetic fields are small, a smaller unit called gauss (G) is also used. The two units are related as below

$$1\text{T} = 10^4\,\text{G}$$

**Example 14.2**

Two ends of a copper wire are dipped into two mercury pools to form a complete circuit. The wire is placed between the poles of a magnet so that an upward force is produced when a current passes through the wire. The wire segment is thrown vertically upward by the magnetic force. For a field strength of 0.6 T acting over 3.0 cm length of the wire whose mass is 0.006 kg, find the force **F** on the wire perpendicular to **B** when a current of 100 A flows through it and the acceleration with which the wire leaves the mercury pool.

**Solution**

Since **L** is perpendicular to vector **B**

$$\mathbf{F} = \mathbf{I}\mathbf{L} \times \mathbf{B}$$
$$= (100\ \text{A})\,(0.03\ \text{m})\,(0.6\ \text{T}) = 1.8\ \text{N}$$

This much force is acting vertically upward on the wire of mass 6.0 g. If we neglect the gravitational force on the wire, then the acceleration acting on the wire to start with, will be

$$a = \frac{|\mathbf{F}|}{m} = \frac{1.8\ \text{N}}{6 \times 10^{-3}\ \text{kg}} = 300\ \text{ms}^{-2}$$

This acceleration ceases as soon as the ends of the wire leave the mercury pool.

## 14.3 TORQUE ON A CURRENT CARRYING RECTANGULAR COIL

We now consider a current-carrying rectangular loop in a uniform magnetic field. No net force is exerted on the loop. To have a net force on a closed loop, the magnetic induction must be non-uniform. This can be seen most easily by considering the rectangular loop ABCD as shown in Fig. 14.4.

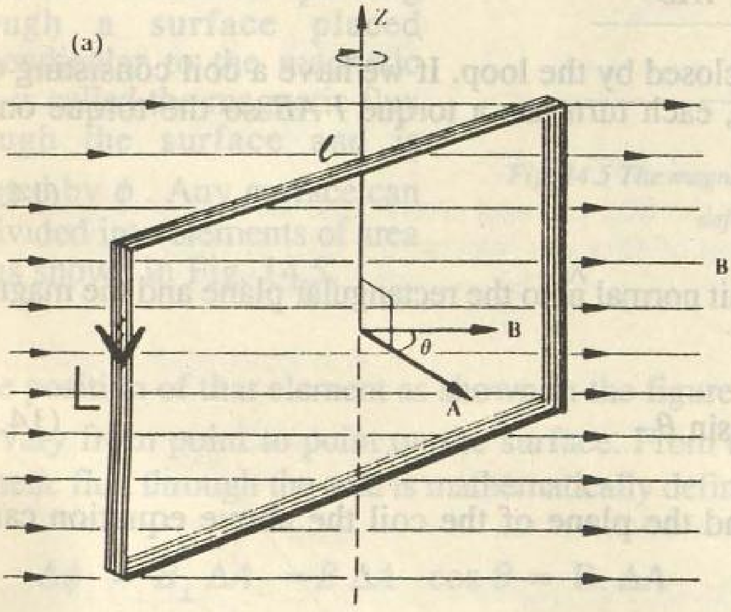


*Fig 14.4 (a) Current carrying rectangular loop in a uniform magnetic field*

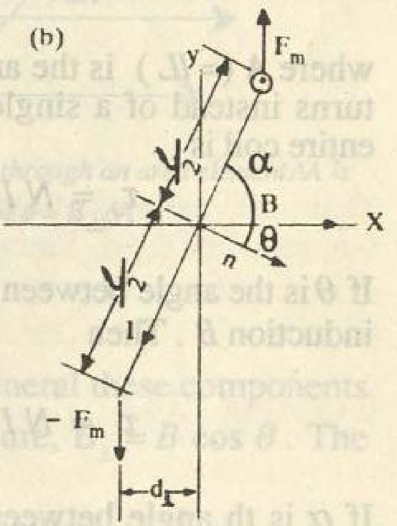


*Fig 14.4 (b) Torque acting upon the rectangular loop when viewed from above*

With uniform magneitc induction **B**, the force on wire segments of equal lengths are equal in magnitude. The right hand rule implies that the forces on opposite sides are oppositely directed because the direction of current is opposite. Thus the net force on the loop is zero.

This is not so with the torque as the forces $\mathbf{F}_m$ and $-\mathbf{F}_m$ on the vertical sides do not act along the same line. There is a net clockwise torque about the z-axis when seen from above. The magnitude of the torque on the loop is given by relation.

Torque = Force × perpendicular distance between the force and the axis of rotation.

$$\text{total torque}\ \tau = \tau_1 + \tau_2$$

$$\text{where}\ \tau_1 = \tau_2 = \tau$$

$$\tau = 2\mathbf{F}_m \times \mathbf{d}_\perp = 2F_m \frac{l}{2} \tag{14.6}$$

As the entire rectangular coil is placed in the magnetic field, so the magnetic forces on each of these sides has the magnitude

$$F_m = ILB \ \sin 90 = ILB \tag{14.7}$$

Substituting the value of $F_m$ from Eq. (14.7) in Eq. (14.6) we get the torque on the loop as

$$\tau = ILl\ B = IAB$$

where $A\ (= lL)$ is the area enclosed by the loop. If we have a coil consisting of $N$ turns instead of a single loop, each turn has a torque $I\ AB$ so the torque on the entire coil is

$$\tau = NIAB \tag{14.8}$$

If $\theta$ is the angle between the unit normal $\hat{n}$ to the rectangular plane and the magnetic induction $B$. Then

$$\tau = NIAB \ \sin\theta \tag{14.9}$$

If $\alpha$ is th angle between $B$ and the plane of the coil the above equation can be wirtten as

$$\tau = NIAB \cos\alpha \tag{14.9 a}$$

This formula is valid even for circular coils.

**Example 14.3**

A circular coil of 50 cm radius has 25 turns. The current in the coil is 20 mA. The magnetic induction $B = 0.20$ T. Determine the magnitude of the torque on the coil when $\theta = 60°$.

**Solution**

$$\tau = NIAB \ \sin\theta = NI\,\pi r^2 B \ \sin\theta$$

$$= 25 \times 0.02\ \text{A} \times \pi\,(0.5)^2 \times 0.2\ \text{T} \times \sin 60^\circ$$

$$= 0.068\ \text{N–m}$$

## 14.4 FLUX AND FLUX DENSITY

In a uniform magnetic field, where $B$ has the same magnitude and direction at every point in the region, the field lines are straight and parallel. If the poles of an electromagnet are large, flat and close together, there is is a region between them where the magnetic field is approximately uniform.

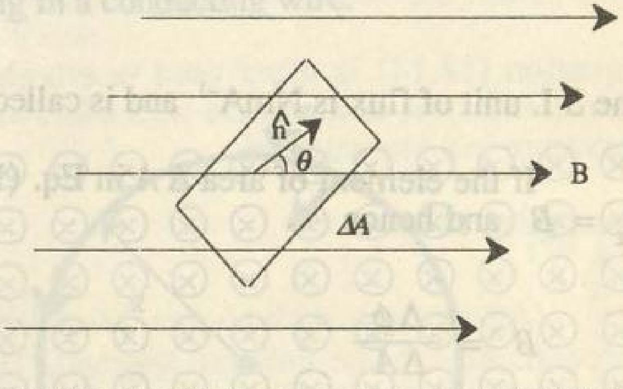


*Fig. 14.5 The magnetic flux through an area element $\Delta A$ is defined to be $\phi = B_\perp \Delta A$*

The magnetic flux across a surface is defined in analogy to the electric flux used in Gauss's law (Sec. 12.4). The total number of lines of magnetic induction passing through a surface placed perpendicular to the magnetic field is called the magnetic flux through the surface and is denoted by $\phi$. Any surface can be divided into elements of area $\Delta A$ as shown in Fig. 14.5.

at the position of that element as shown in the figure. In general these components will vary from point to point on the surface. From the figure, $B_\perp = B\cos\theta$. The magnetic flux through the area is mathematically defined as

$$\Delta\phi = B_\perp \Delta A = B\,\Delta A \cos\theta = \mathbf{B}.\Delta\mathbf{A} \qquad (14.10)$$

The total magnetic flux through the surface is the sum of the contribution from the individual area elements, given by

$$\phi = \sum \Delta\phi = \sum B_\perp \Delta A = \sum B\,\Delta A \cos\theta$$

$$= \sum \mathbf{B}.\Delta\,\mathbf{A} \qquad (14.11)$$

In the special case where **B** is uniform over a plane surface with total area $A$

$$\phi = B_\perp A = BA\cos\theta \qquad (14.12)$$

If **B** happens to be perpendicular to the surface, then $\theta = 0^\circ$ and $\cos\theta = 1$ then Eq. (14.12) reduces to $\phi = BA$. If $\theta = 90^\circ$ then $\cos\theta = 0$ and the Eq.(14.12) reduces $\phi = 0$.

The SI unit of magnetic field $B$ is tesla which is equal to one newton per ampere metre; hence the unit of magnetic flux using equation can be determined as

$$\phi = \mathbf{B.A} = \frac{\text{newton (N)}}{\text{ampere (A) metre (m)}} . m^2 = \text{NmA}^{-1}$$

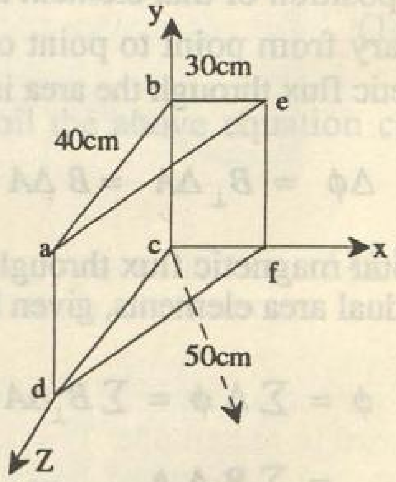
The S I. unit of flux is $\text{NmA}^{-1}$ and is called weber (Wb).

If the element of area $\Delta A$ in Eq. (14.10) is at right angle to the field lines, $B_\perp = B$ and hence

$$B = \frac{\Delta\phi}{\Delta A} \qquad (14.13)$$

That is the magnetic field equals the flux per unit area across an area at right angle to the magnetic field. Since the unit of flux is 1 weber, the unit of field, tesla is equal to one weber per square metre ($1\text{Wb.m}^{-2}$). The magnetic field $B$ is also called flux density.

**Example 14.4**



The flux density **B** in a certain region is 2.0 T and its direction is along +ve x-axis. What is the magnetic flux across the surface (a) abcd (b) befc?

**Solution**

Using equation $\phi = BA \cos\theta$ we have for

(a) Area of abcd = 0.3 m × 0.4 m = 0.12 m$^2$. Angle between vector area abcd and $\mathbf{B} = 0$ because $\cos 0 = 1$

Hence flux$\phi = 2.0\text{T} \times 0.12\ \text{m}^2 \times 1 = 0.24\ \text{Wb}$

(b) Area befc = 0.3 m × 0.3 m = 0.09 $m^2$, $\theta = 90^\circ$

Hence $\phi = 0.2\text{T} \times 0.09\ \text{m}^2 \times \cos 90^\circ = 0$

## 14.5 MOTION OF CHARGED PARTICLE IN A MAGNETIC FIELD

We have discussed that when a current-carrying conductor is placed inside a uniform magnetic field, its own magnetic field intereacts with the magnetic field B of the magnet with a magnetic force $\mathbf{F}_m$ given by Eq. (14.1). It is interesting to examine the influence of the magnetic field B of the magnet when the charges are projected freely across it, instead of moving in a conducting wire.

Consider a charged particle at point O in a uniform magnetic field *B* directed perpenclicularly into the plane of the paper as shown by the symbol ×. The particle is given a velocity **v** in a direction at right angle to the field (Fig. 14.6).

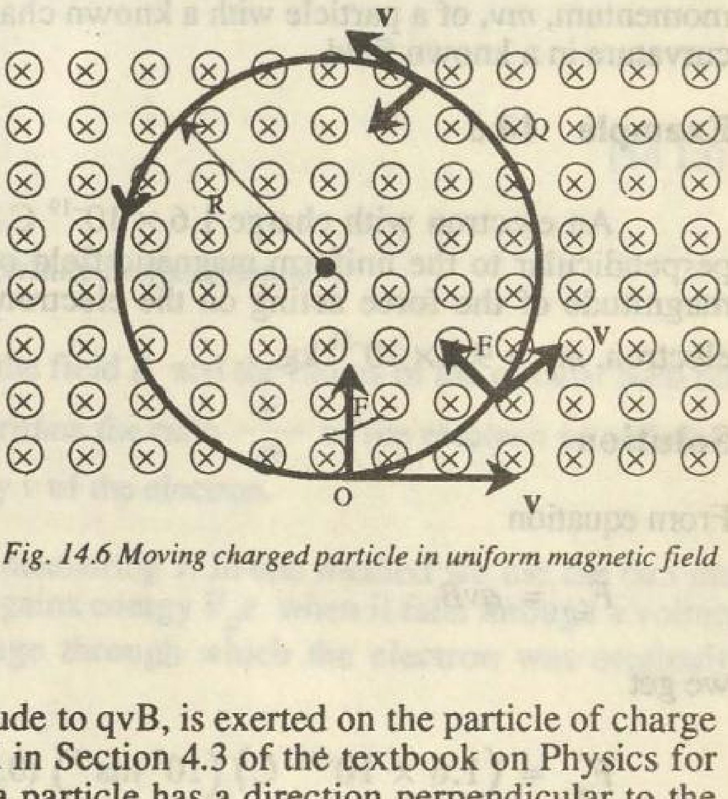


*Fig. 14.6 Moving charged particle in uniform magnetic field*

An upward force **F**, equal in magnitude to qvB, is exerted on the particle of charge *q* at this point. As already explained in Section 4.3 of the textbook on Physics for class XI, when the force acting on a particle has a direction perpendicular to the velocity of particle, the velocity changes only in direction but not in magnitude. Thus the charged particle *q* moves with a constant speed. At points such as P and Q the direction of the force and the velocity will change as shown; the magnitude of the force will be constant, since the magnitudes of **v** and **B** are constant. The particle therefore moves under the influence of a force having constant magnitude but its direction is always at right angle to the velocity of the particle. The orbit of the particle is therefore a circle described with constant tangential speed *v*. Since the centripetal force $F_c$ is given by

$$F_c = m\frac{v^2}{R}$$

where $R$ is the radius of the circular path. This centripetal force $F_c$ is produced due to the action of magnetic field on the charged particle, Thus

$$F_m = qvB = m\frac{v^2}{R}$$

or

$$mv = q\,BR$$

Hence

$$R = \frac{m\,v}{q\;B} \tag{14.14}$$

Equation (14.14) is often used in atomic and nuclear physics to determine the momentum, $mv$, of a particle with a known charge by measurement of its radius of curvature in a known field.

**Example 14.5**

An electron with charge $1.6 \times 10^{-19}$ C moves with a velocity of $10^7$ ms$^{-1}$ perpendicular to the uniform magnetic field of 0.02 T magnitude. Calculate the magnitude of the force acting on the electron and the radius of the path of the electron. $m_e = 9.1 \times 10^{-31}$ kg.

**Solution**

From equation

$$F_m = qvB$$

we get

$$F_m = \left(1.6 \times 10^{-19}\ \text{C}\right)\left(10^7\ \text{ms}^{-1}\right)(0.02\ \text{T})$$

$$= 3.2 \times 10^{-14}\ \text{N}.$$

This force acts perpendicular to the velocity of the electron and hence revolves it on a curved path. Since the force is constant in magnitude and is always perpendicular to the path at any instant, the path must be a circle. The magnetic force acts as a centripetal force. Thus, from Eq. (14.14) we have

$$R = \frac{m\,v}{q\;B} = \frac{9.1 \times 10^{-31}\ \text{kg} \times 10^7\ \text{ms}^{-1}}{1.6 \times 10^{-19}\ \text{C} \times 2 \times 10^{-2}\ \text{T}} = 2.8 \times 10^{-3}\ \text{m}$$

Thus the path of an electron is a circle of radius 2.8 mm.

### 14.5.1 Measurement of $\frac{e}{m}$

In the preceeding article, the motion of a charged particle in the uniform magnetic field $B$ is discussed. By observing the motion of electrons in a uniform magnetic field we can get important information about the nature of electrons. Applying the concepts of the previous article the ratio of the charge $e$ to mass $m$ of an electron can be determined.

As explained earlier a charged particle describes a circular motion when it enters perpendicularly in a uniform magnetic field. If $v$ be the velocity of the electron and $B$ the magnitude of the uniform magnetic field, the radius of the circular path is given by using Eq. (14.14) as

$$R = \frac{mv}{Be}$$

or $$\frac{e}{m} = \frac{v}{BR} \qquad (14.15)$$

from which ratio $\frac{e}{m}$ can be determined experimentally.

Measuring the strength of the field $B$ and the radius of the circular path that the electron describes, we can determine the ratio $\frac{e}{m}$ of the electron provided we know the magnitude of the velocity $v$ of the electron.

There are two methods of measuring $v$. In one method we use the fact that an electron (or a charged particle) gains energy $V_o e$ when it falls through a voltage $V_0$. Hence, if we know the voltage through which the electron was originally accelerated, we can write

$$V_o e = \frac{1}{2} mv^2$$

or $$v = \sqrt{\frac{2V_0 e}{m}}$$

Substituting this value of $v$ in Eq. 14.15 we get

$$\frac{e}{m} = \frac{2V_o}{B^2 R^2}, \qquad (14.16)$$

A more accurate method for measuring a velocity makes use of a particle velocity selector. The device is shown schematically in Fig. 14.7. Two oppositely charged parallel plates produce a field $E$, which exerts a force $Ee$ downward on an electron as shown in the diagram. The indicated magnetic field $B$ exerts an upward force $evB$ on the same electrons. If the voltage across the plates is adjusted in such a way that force exerted by electric field and magnetic field are equal in magnitude i.e.,

$$evB = Ee$$

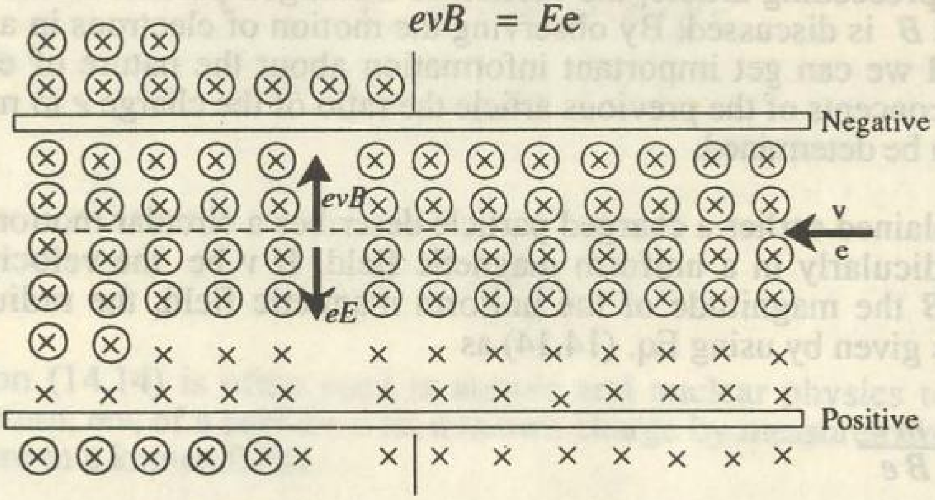


*Fig. 14.7 Force on a charged particle moving in a crossed electric ⊗ magnetic field.*

then the particle will not deflect. Solving for $v$ we have

$$v = \frac{E}{B}$$

To determine the speed $v$ of the electron, we need only to adjust the plate voltage $V$, so that the particle travels undeflected through the crossed electric and magnetic fields. With $E = \frac{V}{d}$ ($d$ is the distance between the plates) known the velocity $v$ of the electron can be calculated. Substituting this value of $v$ in Eq. (14.15) we get

$$\frac{e}{m} = \frac{E}{B^2 R} \qquad (14.17)$$

As the magnetic force on a charged particle is always perpendicular to the velocity of the particle, therefore, the magnetic force does no work on a moving charged particle. It only changes the direction of motion of the particle.

**Exmaple 14.6**

A particle of mass $1.0 \times 10^{-26}$ kg and a charge of $3.2 \times 10^{-19}$ C enters a uniform magnetic field of magnitude $B = 0.4$ T at an angle of 60° with the direction of $B$ with a velocity of $2.25 \times 10^6$ $ms^{-1}$. How far does the particle move along $B$ in one revolution?

**Solution**

The time for one revolution is the time $t$ a particle takes to travel around a circle of radius $R$ with speed $v_\perp$. This time is given by the relation

$$\text{time} = \frac{\text{circumference}}{v_\perp}$$

$$t = \frac{2\pi R}{v_\perp}$$

Using Eq. (14.14) i.e.,

$$R = \frac{mv_\perp}{qB}$$

we find

$$t = \frac{2\pi}{v_\perp} \times \frac{mv_\perp}{qB} = \frac{2\pi m}{qB}$$

Thus, the time required for each complete orbit is independent of velocity $v$. Higher velocity particles complete proportionately large orbits, the net result being a period of revolution that is independent of $v$. The number of complete orbits per time unit is the revolution frequency

$$f = \frac{1}{t} = \frac{1}{2\pi} \frac{q}{m} B$$

This frequency is called the cyclotron frequency named after nuclear particle accelerator cyclotron invented by E. O. Lawrence.

During this period the particle also travels along $B$ a distance

$$d = vt = (v\cos\theta)\,t = v\cos\theta \frac{2\pi m}{qB}$$

where $\theta$ is the angle between vector **v** and **B**. Thus

$$d = \frac{2\pi(1.0\times10^{-26}\,\text{kg})(2.25\times10^{6}\,\text{ms}^{-1})\times\cos60^\circ}{3.2\times10^{-19}\,\text{C}\times0.4\text{T}} = 0.552\,\text{m}$$

The helical motion persists even if **B** is not uniform. In that case the lines of induction act as guide lines and the particle moves helically along them.

## 14.6 AMPERE'S LAW

We have discussed that a current-carrying conductor has a magnetic field around it and its direction is given by the right hand rule. We have not yet dealt with the method for computing the magnitude of $B$. In order to obtain a quantitative value of $B$, we make use of the relation first discovered by Andre Marie Ampere.

Consider a long straight wire carrying a current $I$ away from the reader and a close curve consisting of a circle of radius $r$ and with centre at the wire (Fig. 14.8a).

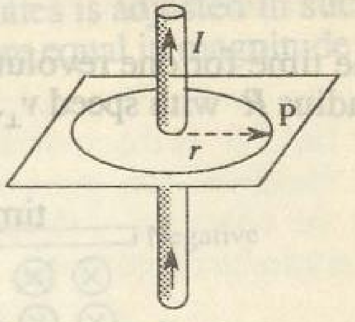


*Fig. 14.8 (a) Magnetic field around a long straight conductor carrying a current*

He showed that the magnitude of the flux density at any point was directly proportional to the strength of the current and inversely proportional to the distance of the point from the conductor. Mathematically it is written as

$$B \propto \frac{I}{r}$$

$$\text{or} \qquad B = \frac{\mu_o}{2\pi} \cdot \frac{I}{r} \qquad (14.18)$$

where $\frac{\mu_o}{2\pi}$ is the constant of proportionality, which can be determined experimentally, $\mu_0$ is called the permeability of free space and its value is $4\pi \times 10^7$ Wb/Am. The direction of $B$ is obtained by right hand rule.

From Fig. (14.8 a) we see that for a given value $I$ of the current in the wire, $B$ has the same value on all the points that lie on a circle whose centre is on the conductor carrying current, thus from the equation (14.18) we have

$$2\pi r B = \mu_o I$$

This shows that the product of $2\pi r$ and $B$ equals $\mu_0 I$. But $2\pi r$ is the length of the path around the wire and on it the value of $B$ is the same at every point. Ampere generalised this result into a law called as Ampere's circuital law. Mathematically it can be written as

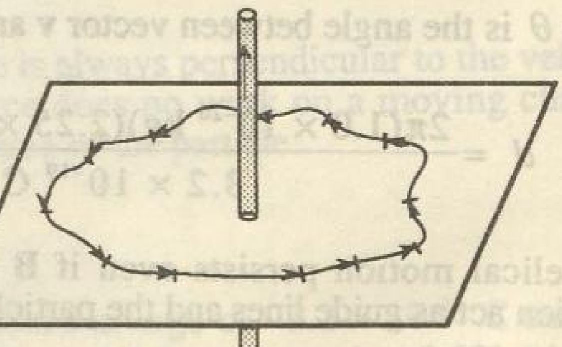


*Fig. 14.8 (b) Magnetic field around conductor carrying a current.*

$$B.l = \mu_o I$$

This law is applicable to closed paths other than circular ones.

Thus Ampere's law states that the dot product of $B$ and l around any closed path equals $\mu_o I$ where $I$ is the total steady current threaded by the path.

Let us apply it to a closed curved path around the current carrying conductor as shown in Fig 14.8 b. This path is divided into a large number of elements each of length $\Delta l$. For each element the value of $\mathbf{B}.\Delta l$ is determined. As the magnitude of $\mathbf{B}.\Delta l$ is $\mathbf{B}\Delta l\ \cos\theta$ where $\theta$ is the angle between the vectors $\Delta l$ and $B$, hence it can be concluded that $\mathbf{B}.\ \Delta l$ is the product of $\Delta l$ and the component of $B$ parallel to $\Delta l$. It was shown by Ampere that the sum of the quantitites $B.\Delta l$ for all path elements in the closed curve equals $\mu_0$ times the current enclosed by the loop. It can be represented as

$$(\mathbf{B}.\Delta l\,)_1 + (\mathbf{B}.\Delta l\,)_2 + \ldots\ldots + (\mathbf{B}.\Delta l\,)_n = \mu_o I$$

or $$\sum_{r=1}^{n} (\mathbf{B}.\Delta l\,)_r = \mu_0 I \qquad (14.19)$$

where $(\mathbf{B}.\Delta l\,)_r$ is the value of $B.\Delta l$ along any element $r$ and $n$ is the total number of elements into which the closed curve is divided.

### 14.6.1 Field Due to Current in Solenoid

A solenoid is constructed by winding wire in a helix around a cylindrical surface. The turns of the winding are ordinarily closely spaced and may consist of one or more layers. The magnetic field produced by it is shown in Fig. 14.9 (a). The field in the middle is stronger and uniform but it is weaker and negligible outside the solenoid. In order to calculate $B$, consider a rectangular path abcda Fig. 14.9 (b). It is divided into four elements of lengths $l_1$, $l_2$, $l_3$ and $l_4$, sides dc, ab, of length $l_1$ and $l_3$ is parallel to the axis of the solenoid. Sides bc and da of lengths $l_2$ and $l_4$, respectively, are taken to be very long so that side dc of length $l_3$ is far from the solenoid and the field at this side is negligibly small. By Ampere's law

$$\sum_{r=1}^{4} (\mathbf{B}.\Delta l_r) = \mu_0 I$$

Consider a long straight wire carrying a current $I$ away from the reader and a close curve consisting of a circle of radius $r$ and with centre at the wire (Fig. 14.8a).

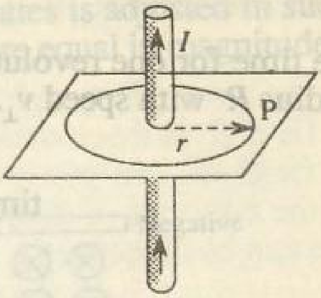


*Fig. 14.8 (a) Magnetic field around a long straight conductor carrying a current*

He showed that the magnitude of the flux density at any point was directly proportional to the strength of the current and inversely proportional to the distance of the point from the conductor. Mathematically it is written as

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where $\frac{\mu_o}{2\pi}$ is the constant of proportionality, which can be determined experimentally, $\mu_0$ is called the permeability of free space and its value is $4\pi \times 10^7$ Wb/Am. The direction of $B$ is obtained by right hand rule.

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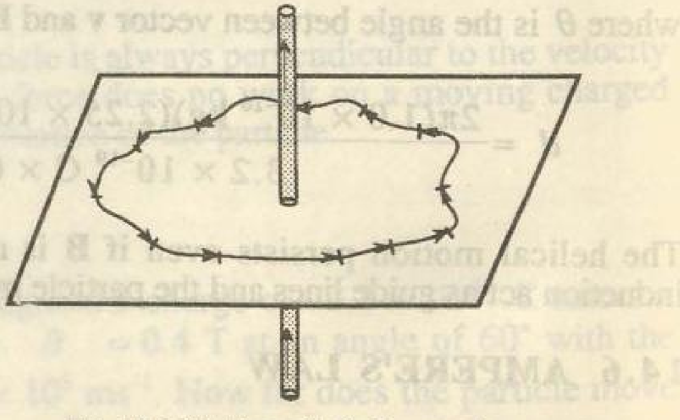


*Fig. 14.8 (b) Magnetic field around conductor carrying a current.*

$$B.l = \mu_o I$$

This law is applicable to closed paths other than circular ones.

Thus Ampere's law states that the dot product of $B$ and l around any closed path equals $\mu_o I$ where $I$ is the total steady current threaded by the path.

Let us apply it to a closed curved path around the current carrying conductor as shown in Fig 14.8 b. This path is divided into a large number of elements each of length **Δ*l*.** For each element the value of **B.Δ*l*** is determined. As the magnitude of **B.Δ*l*** is **BΔ*l*** $\cos\theta$ where $\theta$ is the angle between the vectors $\Delta l$ and $B$, hence it can be concluded that **B.** $\Delta l$ is the product of $\Delta l$ and the component of $B$ parallel to **Δ*l*.** It was shown by Ampere that the sum of the quantitites $B.\Delta l$ for all path elements in the closed curve equals $\mu_0$ times the current enclosed by the loop. It can be represented as

$$(\mathbf{B}.\Delta l\,)_1 + (\mathbf{B}.\Delta l\,)_2 + \ldots\ldots + (\mathbf{B}.\Delta l\,)_n = \mu_o\, I$$

or
$$\sum_{r=1}^{n} (\mathbf{B}.\Delta l\,)_r = \mu_0 I \qquad (14.19)$$

where $(\mathbf{B}.\Delta l\,)_r$ is the value of $B.\Delta l$ along any element $r$ and $n$ is the total number of elements into which the closed curve is divided.

### 14.6.1 Field Due to Current in Solenoid

A solenoid is constructed by winding wire in a helix around a cylindrical surface. The turns of the winding are ordinarily closely spaced and may consist of one or more layers. The magnetic field produced by it is shown in Fig. 14.9 (a). The field in the middle is stronger and uniform but it is weaker and negligible outside the solenoid. In order to calculate $B$, consider a rectangular path abcda Fig. 14.9 (b). It is divided into four elements of lengths $l_1$, $l_2$, $l_3$ and $l_4$, sides dc, ab, of length $l_1$ and $l_3$ is parallel to the axis of the solenoid. Sides bc and da of lengths $l_2$ and $l_4$, respectively, are taken to be very long so that side dc of length $l_3$ is far from the solenoid and the field at this side is negligibly small. By Ampere's law

$$\sum_{r=1}^{4} (\mathbf{B}.\Delta l_{\mathrm{r}}\,) = \mu_0\, I$$

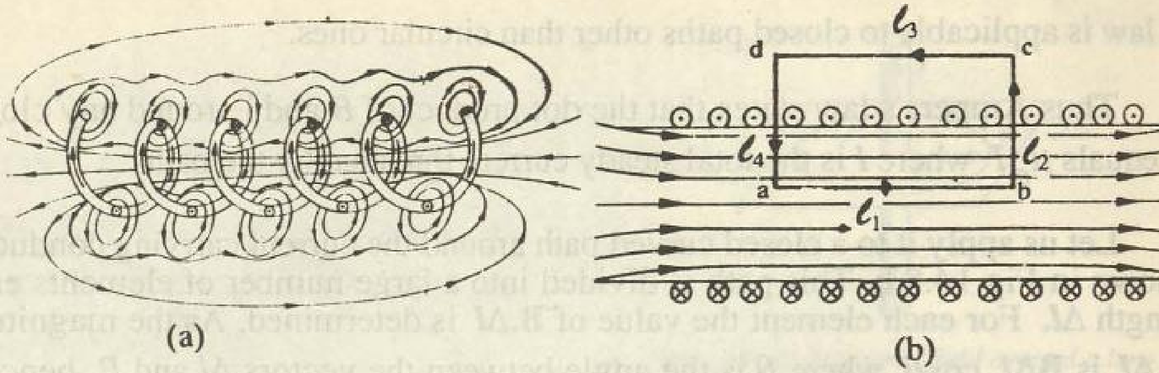


*Fig. 14.9 (a) Field due to a Solenoid (b)* cross *sectional view of the field.*

Inside the solenoid **B** is parallel to $l_1$ and hence the angle between **B** and $l_1$ is zero, so

$$\mathbf{B}.l_1 = \mathbf{B}l_1 \cos 0 = Bl_1$$

The field outside the solenoid is zero, hence $\mathbf{B}.l_3 = 0$

**B** is perpendicular to both $l_2$ and $l_4$, therefore,

$$\mathbf{B}.l_2 = Bl_2 \cos 90^\circ = 0$$

and $$\mathbf{B}.l_4 = Bl_4 \cos 90^\circ = 0$$

Thus $$\sum_{S=1}^{4} (\mathbf{B}.\Delta l)_S = \mathbf{B}.l_1 + \mathbf{B}.l_2 + \mathbf{B}.l_3 + \mathbf{B}l_4 = \mathbf{B}l_1$$

$$\mathrm{B}l_1 = \mu_0 \times \text{current enclosed}$$

If there are $n$ turns per unit length of the solenoid and each turn carries a current $I$, the current enclosed by the loop abcda will be $nl_1 I$. So

$$Bl_1 = \mu_0 \times nl_1 I$$

or $$B = \mu_0 nI \qquad (14.20)$$

The direction of **B** is along the axis of the solenoid.

### 14.6.2 Field Due to Current In A Toroid

A toroid is a solenoid that has been bent into a circle. Fig. 14.10a represents a toroid wound with wire through which current $I$ is passing. The dotted lines in Fig. 14.10b are a number of paths to which we wish to apply Ampere's law.

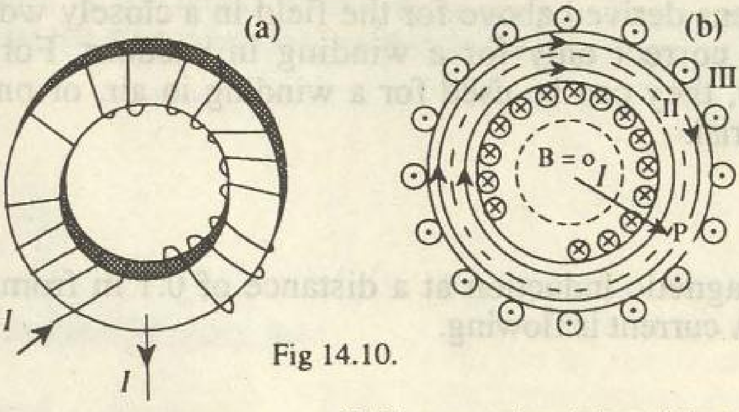


Fig 14.10.

*(a) A Toroid*

*(b) Cross-sectioned view of the toroid to calculate B using Ampere's Law*

Consider path I. By symmetry, if there is any field at all in the region, it will be tangent to the path at all points, and $\mu_o I$ will equal the product of $B$ and the circumference of the path. The current enclosed by this path, however, is zero, and hence from Ampere's law (since the circumference is not zero), the field $B$ must be zero.

Similarly, if there is any field at path number III, it will also be tangent to the path at all points. Each turn of the winding passes twice through the area bound by this path, carrying equal currents in opposite directions. The net current through the area is therefore zero, and hence $\mathbf{B} = 0$ at all points of the path. The field of the toroid is therefore confined wholly to the space enclosed by the windings.

Finally, consider path number II, a circle of radius $r$. Again by symmetry, the field is tangent to the path and $\sum B_{11}\, \Delta l$ equal $2\pi r B$. Each of the winding passes once through the area bounded by path II and the total current through the area is $N I$, where $N$ is the total number of turns in the winding. Thus, from Ampere's law

$$2\pi r \ \mathrm{B} = \mu_o \ N I$$

and

$$\mathrm{B} = \frac{\mu_o}{2\pi} \left( \frac{N I}{r} \right) \qquad (14.21)$$

The magnetic field is not uniform over a cross-section of core, because the path length $l$ is larger at the outerside of the section than at the inner side. However, if the radial thickness of the core is small compared with the toroid radius $r$, the field varies only slightly across a section. In that case, considering that $2\pi r$ is the circumference length of the toroid and that $\frac{N}{2\pi r} = n$ is the number of turns per unit length, the field may be written as

$$B = \mu_o n I$$

just as at the centre of a long straight solenoid.

The equations derived above for the field in a closely wound solenoid or toroid are strictly correct only for a winding in vacuum. For most practical purposes, however, they can be used for a winding in air, or on a core of non-ferromagnetic material.

### Example 14.7

Find the magnetic induction at a distance of 0.1 m from a straight wire through which 50 A current is flowing.

### Solution

Using the relation

$$\mathbf{B} = \frac{\mu_o I}{2\pi r}\text{, we get}$$

$$\mathbf{B} = \frac{(4\pi \times 10^{-7}\ \text{weber/A.m.})\ (50\ \text{A})}{(2\pi)\ (0.1\ \text{m})} = 10^{-4}\, T$$

### Example 14.8

A solenoid is 10 cm long and is wound with two layers of wire. The inner layer consists of 50 turns and the outer of 40 turns. The current is 3A in the same direction in both layers. What is the magnetic induction at point near the centre of the solenoid?

### Solution

Using the equation $B = \mu_o nI$ we can get the value of $B$, As there are two layers of wire and the current in each layer is of the same value and flows in the same direction, therefore, the net magnetic induction is the sum of the magnetic induction due to each layer. Thus

$$B = B_1 + B_2$$

$$= \mu_o n_1 I + \mu_o n_2 I = \mu_o I\ (n_1 + n_2)$$

$$= 4\pi \times 10^{-7}\ \text{weber/A.m} \times 3\ \text{A} \left( \frac{50 + 40}{0.1\ \text{m}} \right)$$

$$= 3.39 \times 10^{-3}\ \text{T}$$

**Example 14.9**

A wooden ring whose mean diameter is 10 cm is wound with a closely spaced toroidal winding of 500 turns. What will be the flux density at a point on the mean circumference of the ring when the current in the winding is 0.3 A.

**Solution**

Here $B = \frac{\mu_o}{2\pi} \frac{NI}{r}$

where $N$ is the total number of turns. So

$$B = \frac{4\pi \times 10^{-7} \text{ weber/A.m}}{2\pi} \times \frac{500 \times 0.3 \text{ A}}{0.05 \text{ m}}$$

$$B = 6.0 \times 10^{-4} \text{ T}$$

## 14.7 INDUCED emf

We have already discussed in this chapter that a current-carrying conductor, when placed in a magnetic field, experiences a force. This led the scientists to consider whether electric current is produced in a conductor when it is dragged in a magnetic field. Consider the following simple experiments to investigate it.

Take a single loop of wire connected with a galvanometer G. A wire of length $l$ shown with a shade, in Fig. 14.11 is palced in the magnetic field.

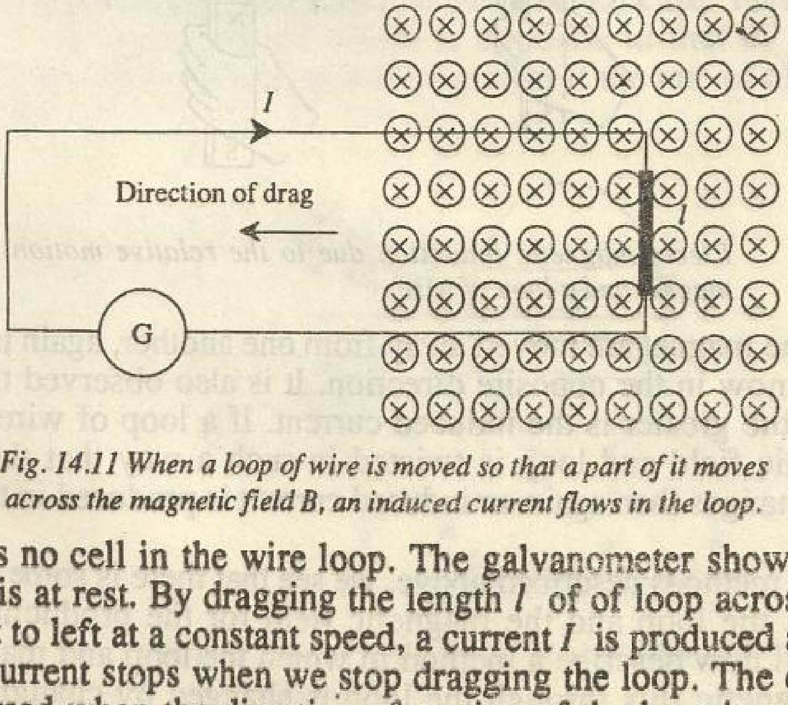


*Fig. 14.11 When a loop of wire is moved so that a part of it moves across the magnetic field B, an induced current flows in the loop.*

There is no cell in the wire loop. The galvanometer shows no deflection when the loop is at rest. By dragging the length $l$ of of loop across the magnetic field from right to left at a constant speed, a current $I$ is produced as shown in the diagram. The current stops when we stop dragging the loop. The direction of the current is reversed when the direction of motion of the loop is reversed. This is indicated by a deflection in the galvanometer in the opposite direction. The current produced will be stronger, if the speed with which the loop is dragged is greater. Moreover, the current is found to be stronger if the magnetic field is stronger.

The current produced by moving the loop of wire across a magnetic field is called induced current, if we change the resistance of the loop by putting different resistances into the loop and drag it with the same length of wire in the same magnetic field with the same speed every time, we find that the product of the induced current and the resistance of the loop ($I \times r$) is constant. The product $I \times r$ gives the induced emf which depends only upon the motion of loop through the magnetic field. This generation of induced emf in a circuit is known as electromagnetic induction. The emf is generated in the circuit due to change of magnetic flux as it moved across the field. Induced emf is produced in a circuit whenever there is a change of flux through it. The change of flux can be affected in many ways.

Described below are some of the methods of changing the magnetic flux through a loop.

If a bar magnet is moved towards a loop of wire or the loop is moved towards the magnet, the flux through the loop changes (Fig. 14.12) and an induced current is produced into the loop as shown by a deflection in the galvanometer.

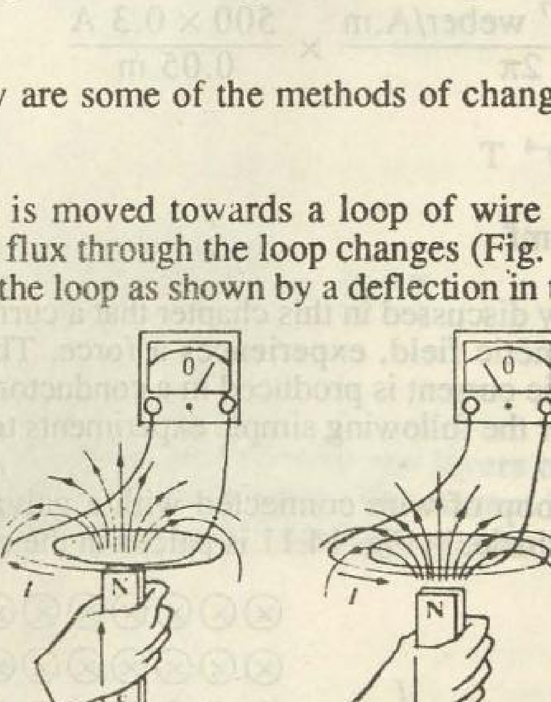


*Fig.14.12 Electromagnetic induction due to the relative motion of a bar magnet and a loop of wire.*

If the loop and the magnet are moved away from one another, again induced current is produced but now in the opposite direction. It is also observed that greater the relative motion the greater is the induced current. If a loop of wire is placed in a uniform magnetic field and loop is twisted in such a way that the flux passing through it also changes then again an induced current is produced in the loop.

In all the methods described above, we see that there is some sort of relative motion between the loop and the magnetic field for the production of induced current. We shall now describe a method in which the loop and magnet remain at rest and the magnetic flux through the loop is changed by changing the current through an electromagnet that produces the magnetic field, thereby changing the strength of the field. In Fig. 14.13 a loop is placed in th magnetic field of an electromagnet. If the current through the electromagnet is changed, the magnetic flux through the loop is changed and an induced current is produced.

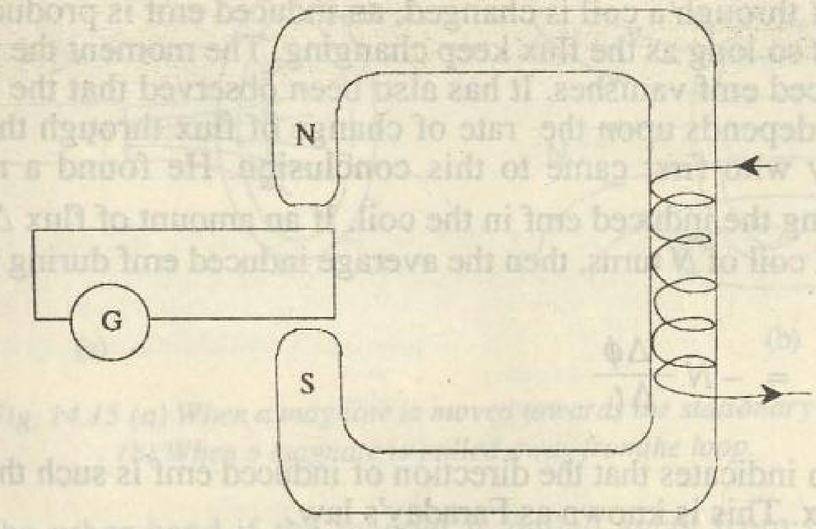


*Fig. 14.13 Current is induced in the loop when magnetic flux passes through, it is changed by varying current through electromegnet.*

Finally we describe an experiment in which the magnetic field produced by the current passing through a loop or a coil is used to change the flux in another loop. Loop L is connected to a battery through a resistance *R*. Another loop P with a galvanometer is placed near the loop L (Fig. 14.14). If we increase the current through loop L by means of a rheostat R, the magnetic field produced by the loop L and passing through the loop P, is changed. Thus an induced current is generated in the loop P. If the current in the loop L is decreased, again an induced current is produced in the loop P but now the direction of the induced current will be opposite to the direction of the induced current in the previous case. This is indicated by the deflection in the galvanometer which is opposite to that of the first case. The magnitude of the induced current will be greater if the rate of change of flux in the loop is greater.

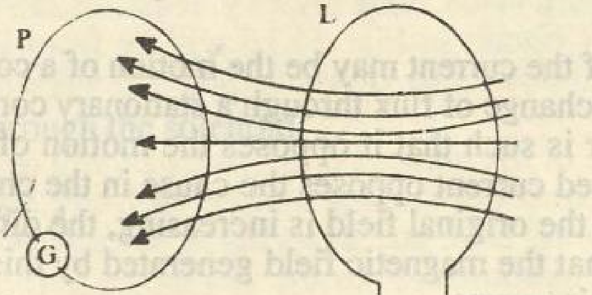


*Fig. 14.14 current is induced in the loop when magnetic flux changes through it*

### 14.7.1 Faraday's Law of Electromagnetic Induction

In each of the experiments given above we have observed that when ever the magnetic flux through a coil is changed, an induced emf is produced in it. This induced emf lasts so long as the flux keep changing. The moment the flux ceases to change, the induced emf vanishes. It has also been observed that the magnitude of the induced emf depends upon the rate of change of flux through the coil. It was Michael Faraday who first came to this conclusion. He found a mathematical relation concerning the induced emf in the coil. If an amount of flux $\Delta\phi$ changes in time $\Delta t$ through a coil of $N$ turns, then the average induced emf during this time is

$$E = -N\frac{\Delta\phi}{\Delta t} \tag{14.22}$$

The negative sign indicates that the direction of induced emf is such that it opposes the cahnge in flux. This is known as Faraday's law.

In order to understand the cause of the induced emf, let us once again consider the experiment shown in Fig. 14.11. The portion of the wire in the magnetic field contains free electrons. While pulling this portion of the wire we move these electrons across the magnetic field. These free electrons experience a force along the wire. Thus as the wire moves across the field, the magnetic force drives the free electrons along the wire. If the loop is closed, a current flows around it. We call this current as an induced current.

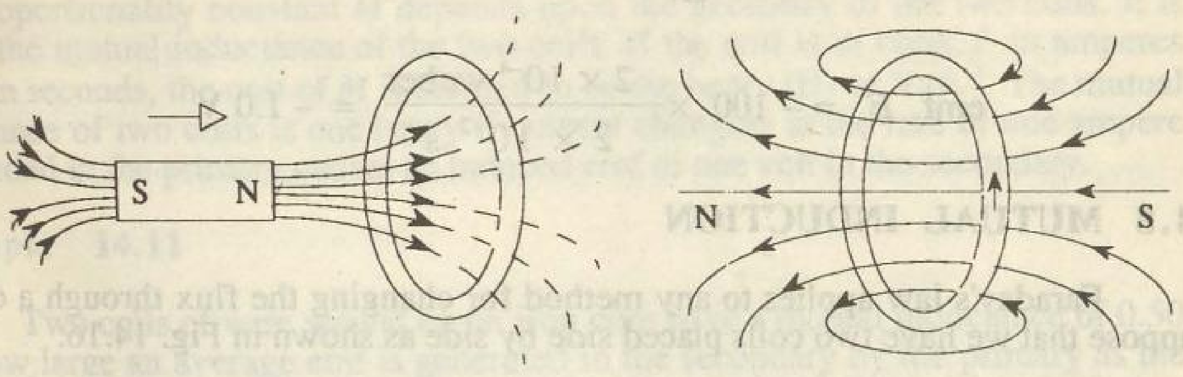
### 14.7.2 Lenz's Law

We now return to the question of determining the sign or direction of the induced emf or current. H.E.F Lenz. a German scientist, enunciated a law which is known after his name. The law states:

" The direction of an induced current is such as to oppose the cause producing it."

The "cause" of the current may be the motion of a conductor in a magnetic field, or it may be the change of flux through a stationary conductor. The current in the moving conductor is such that it opposes the motion of the conductor. In the second case the induced current opposes the cause in the change of magnetic flux through the circuit. If the original field is increasing, the direction of induced emf current will be such that the magnetic field generated by this current will decrease the original field and vice-versa.

To elaborate this, let us consider the situation in which a bar magnet is pushed towards a stationary conducting loop of wire as is shown in Fig 14.15a. As the magnet is pushed to the right towards the loop magnetic flux through the loop increases. To overcome this increase in flux to the right the induced current produces the flux to the left, as in Fig 14.15b. Thus the induced current is in the dirction shown. It may be noted that the magnetic field-lines associated with the induced current oppose the motion of the magnet. So the left face of the current loop will be a north pole and the right face will be a south pole.



(a) (b)

*Fig. 14.15 (a) When a magnate is moved towards the stationary loop.*
*(b) When a magnate is pulled awayfromthe loop.*

On the other hand if the magnet is pulled away from the loop, the flux through the loop will decrease. To compensate this decrease in flux the current through the loop produces a flux to the right. In this case, the left face of current loop would be a south pole and its right face would be a north pole. (Fig. 14.15).

Lenz's law is also directly related to energy conservation. When we drag the wire across the magnetic field we do work against the magnetic force arising from the inter-action of the original magnetic field and that of the induced current and in doing so we impart energy to the loop. This energy is the source of induced current. Thus electromagnetic induction is exactly according to the law of conservation of energy.

### Example 14.10

A solenoid of 100 turns and area of cross section $4.0\ \text{cm}^2$ is transferred from a region where there is no magnetic field to one where the field is 0.50 T in 0.02 second. How large is the average induced emf in the solenoid?

### Solution

Flux through the solenoid,

$$\phi = B_{\parallel} A$$

$$\phi_1 = 0$$

$$\phi_2 = (0.5\ \text{T})\,(4.0 \times 10^{-4}\ \text{m}^2) = 2 \times 10^{-4}\ \text{weber}$$

$$\Delta\phi = \phi_2 - \phi_1 = 2 \times 10^{-4}\ \text{weber}$$

$$\text{emf, } E = -N\,\frac{\Delta\phi}{\Delta t}$$

$$\text{emf, } E = -\,100 \times \frac{2 \times 10^{-4}\ \text{weber}}{2 \times 10^{-2}\ \text{s}} = -\,1.0\ \text{V}$$

## 14.8 MUTUAL INDUCTION

Faraday's law applies to any method for changing the flux through a coil. Suppose that we have two coils placed side by side as shown in Fig. 14.16.

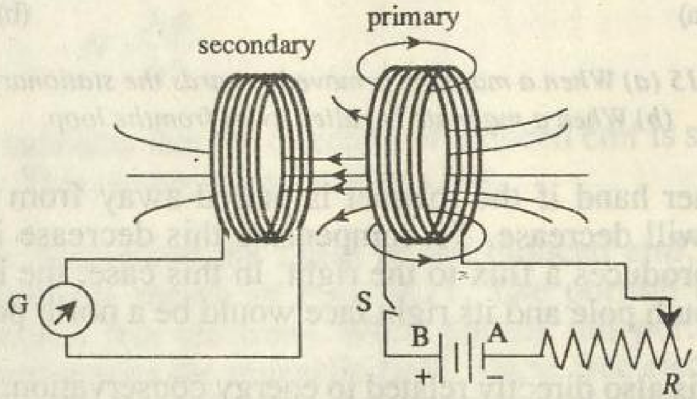


Fig. 14.16 When current in primary changes, induced emf is producced in secondary.

When the switch S is open, both the coils have zero flux through them. We call the coil in the battery circuit the primary coil and other the secondary coil.

If the switch is suddenly closed, a magnetic field is set up due to building up of current in the coil and magnetic flux is produced in its vicinity. Some portion of the flux will pass through the secondary coil. Thus the flux through the secondary coil will change when S is suddenly closed. According to Faraday's law an induced emf will be produced in the secondary for an instant as the current rises from zero to its maximum value in the primary. It can be shown that the direction of the current in primary circuit in Fig. 14.16 will be from B to A when S is just closed. The current will flow in the opposite direction when the switch is opened.

The magnitude of the induced emf generated will depend on many factors. Among these are the number of turns of wire on each coil, the distance between the coils, their cross-sectional area, etc. In addition, since the change of flux through the secondary will be proportional to the change of current in the primary, the induced emf in the secondary will be proportional to the rate of change of current in the primary, $\frac{\Delta I_p}{\Delta t}$. We, therefore, write the following equation for the induced emf in the secondary

$$\text{emf}_{\text{sec}} = -M\frac{\Delta I_p}{\Delta t} \qquad (14.23)$$

The proportionality constant $M$ depends upon the geometry of the two coils. It is called the mutual inductance of the two coils. If the emf is in volts, $I$ in amperes and $t$ in seconds, the unit of $M$ is defined to be the henry (H) or $\text{VsA}^{-1}$. The mutual inductance of two coils is one henry if current changing at the rate of one ampere per second in the primary causes an induced emf of one volt in the secondary.

**Example 14.11**

Two coils of wire wound on an iron core have a mutual inductance of 0.50 H. How large an average emf is generated in the secondary by the primary as the current in the primary is increased from 2.0 to 3.0 A in 0.01 s ?

**Solution**

$$M = 0.50\text{ H}$$

$$\Delta I_p = 3.0\text{ A} - 2.0\text{ A} = 1.0\text{ A}$$

$$\Delta t = 0.01\text{ s}$$

$$\text{emf}_{\text{sec}} = -M\frac{\Delta I_p}{\Delta t}$$

$$\text{emf}_{\text{sec}} = -0.5\text{H}\,\frac{1.0\text{A}}{0.01\text{s}} = 50\text{ volts}$$

### 14.8.1 Self Induction

Faraday's law tells us that any change in flux through a coil will induce an emf in the coil. This means that when a current through a coil changes, the coil induces an emf in itself. Thus if we consider a coil shown in Fig. 14.17, the flux in it will change from zero to some finite value when the switch is first closed.

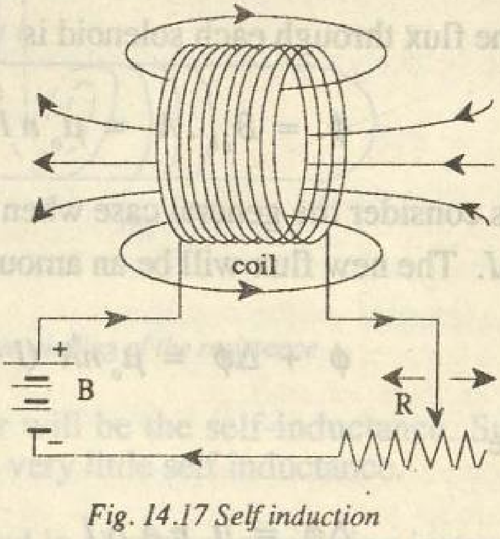


*Fig. 14.17 Self induction*

A flux will be generated by the current and will be directed towards the left through the coil. By Faraday's law, an emf will be induced in the coil and it will try to produce flux to the right through the coil. Hence, an induced emf must be opposite to the emf of the battery. If the switch is suddenly opened, the induced emf will be aiding, rather than opposing the battery.

In this case also, the geometry of the coil, as well as, the material of the core will determine how large the induced emf will be. If $\frac{\Delta I}{\Delta t}$ is the rate of change of current through the coil, we can write for the average induced emf

$$\text{emf} = -L\frac{\Delta I}{\Delta t} \qquad (14.24)$$

The constant of proportionality $L$ is called the self inductance of the coil. It has the same units as mutual inductance i.e., henry.

From this it is apparent, that if the coil is wound on an iron core, the flux through it will be much greater than if no magnetic material was present. Hence, if a large self-inductance is desired, the inductor should be wound on an iron core.

**Example 14.12**

A certain solenoid 0.5 m long and of 4.09 cm$^2$ area of cross-section has 1000 turns per unit length. What is its self inductance?

**Solution**

In this example, we are required to calculate self-inductance of the solenoid, as we know from that the magnitude **B** in a solenoid

$$\mathbf{B} = \mu_o n I$$

and the flux through each solenoid is

$$\phi = B_{11} . A = \mu_o n IA$$

Let us consider the general case when the current in the solenoid changes from $I$ to $I + \Delta I$. The new flux will be an amount $\Delta\phi$ larger than before and so

$$\phi + \Delta\phi = \mu_o nA\ (I + \Delta I)$$

Hence

$$\Delta\phi = \mu_o n A \Delta I$$

But according to Faraday's law, the induced emf in the solenoid is

$$E = -N\frac{\Delta\phi}{\Delta t} = -(n\,d)\,\mu_o\,n\,A\frac{\Delta I}{\Delta t}$$

Comparing it with the equation of self-inductance, namely

$$E = -L\frac{\Delta I}{\Delta t}$$

we see that the self-inductance

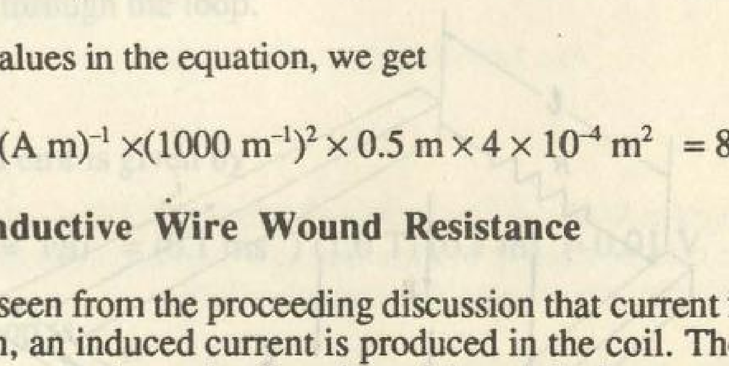
$$L = \mu_o n^2 \mathrm{d}A$$

Substituting the values in the equation, we get

$$\mathrm{L} = 4\pi\times10^{-7}\ \mathrm{Wb\ (A\ m)^{-1}} \times (1000\ \mathrm{m^{-1}})^2 \times 0.5\ \mathrm{m} \times 4\times10^{-4}\ \mathrm{m^2} = 8\pi\ 10^{-5}\ \mathrm{H}$$

### 14.8.2 Non Inductive Wire Wound Resistance

We have seen from the proceeding discussion that current in a coil may vary due to any reason, an induced current is produced in the coil. The direction of this induced current is opposite to the direction of the original current. Thus the current decreases. In order to maintain steady current a possible way that the wire is doubled on itself before being coiled up. The current passing through the wires lying side by side is such that the direction of current in one wire is opposite to the direction of the current in other wire. In this way the magnetic field around one wire cancels the magnetic field around the second wire as is shown in Fig. 14.18 (a).



(a) (b)

*Fig. 14.18 Non inductive winding of the resistance*

The closer the two wires are, the lesser will be the self-inductance. Such a coil, even if it is wound upon an iron core has very little self inductance.

Non inductive resistances are used in electrical measuring instruments like ammeter, voltmeter, etc. The wire wound resistances are used in manufacturing resistance boxes, some telecommunication equipment, etc.

## 14.9 MOTIONAL emf

Consider a situation as shown in Fig. 14.19 where a conductor of length $l$ is sliding with velocity **v** along conducting rails in a region of uniform magnetic field **B**, which is perpendicular to the plane to the rails and the conductor. The conducting path is completed by the resistance $R$, which also represents the total resistance of the closed circuit. An external force $F_{ext}$ pulls the bar along the rails at a constant speed $v$. Let us analyze this situation and calculate the emf generated due to change of magnetic flux.

A positive charge $q$ within the moving conductor of Fig. 14.19 experiences a magnetic force $F_m$ in the j-direction, where

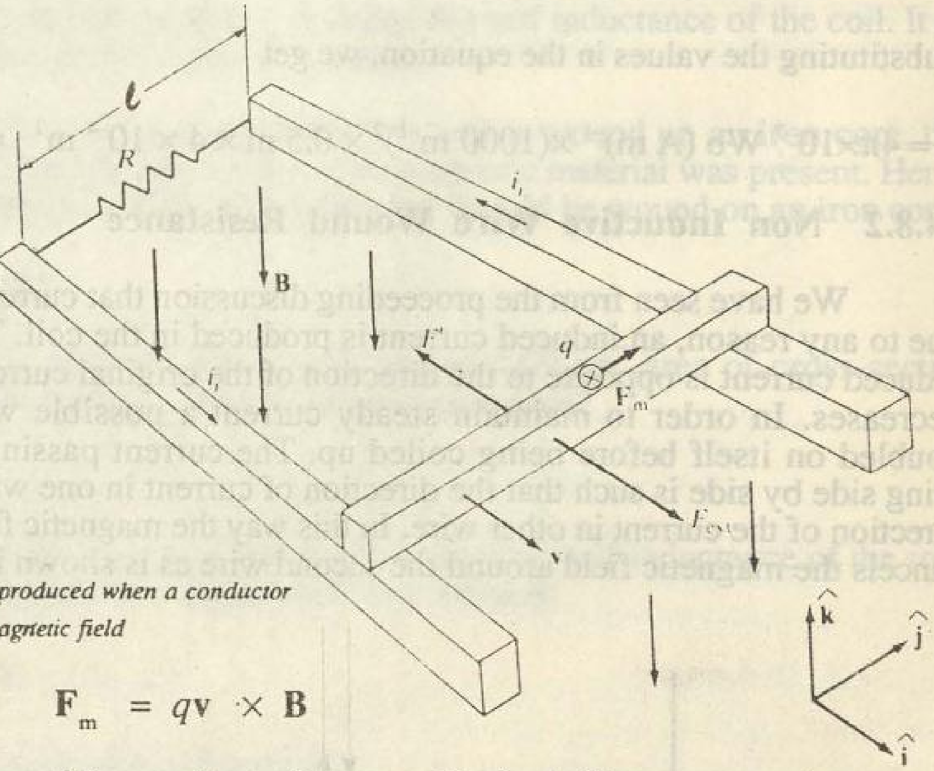


*Fig. 14.19 Current is produced when a conductor moves in a magnetic field*

$$\mathbf{F}_m = q\mathbf{v} \times \mathbf{B}$$

because the velocity $v$ of the conductor is perpendicular to **B**, we may write

$$F_m = qvB \tag{14.25}$$

The external force $F_{ext}$ that pulls the sliding conductor provides the work required to move the charges around the circuit. The work $W$ done by $F_m$ in moving a charge $q$ through the length $l$ of the conductor is

$$W = F_m l = qvBl \tag{14.26}$$

As the emf $E_i$ induced in the conduction loop i.e., along the length of the conductor is equal to $\frac{W}{q}$ we may use Eq. (14.26) to get

$$E_i = \frac{W}{q} = vBl \qquad (14.27)$$

This is known as motional emf. However, if the angle between $v$ and $B$ is $\theta$, then

$$E_i = v\mathrm{B}l \ \sin\theta \qquad (14.28)$$

**Example 14.13**

A conductor of length 0.1 m is sliding along a loop of 0.01 ohm resistance with a velocity of 0.1 $\mathrm{ms^{-1}}$. The loop conductor is placed in a magnetic field $B = 1.0$ T. Find the motional emf $E$ if the conductor moves perpendicular to the field and the current through the loop.

**Solution**

The motional emf is given by

$$E = vBl = (0.1\ \mathrm{ms^{-1}})\ (1.0\ \mathrm{T})\ (0.1\ \mathrm{m}) = 0.01\ \mathrm{V}$$

The current in the loop is

$$I = \frac{E}{R} = \frac{0.01\ \mathrm{V}}{0.01\ \Omega} = 1.0\ \mathrm{A}$$

In the example given above, the force on the loop exerted by the magnetic field is $F = IBl = 0.1$ N. The power $P$ necessary to move the loop against this force is

$$P = Fv = 0.01\ \mathrm{W}$$

The product $E_i$ is also equal to 0.01 W. Thus the rate of energy conversion $E_i$ equals the mechanical power input $Fv$, to the system.

## 14.10 ALTERNATING CURRENT GENERATOR

A generator is a device which converts mechanical energy into electrical energy. The principle of a c generator is illustrated in Fig. 14.20.

A closely wound rectangular coil abcd of N turns rotates about an axis oo′ which is perpendicular to uniform magnetic field of flux density **B**. The terminals of the coil are connected to slip ring S,S′ concentric with the axis of the coil and rotating with it, but insulated from each other. Brushes b and b′ slide along the rings and connect the coil to the external circuit. The magnetic field of a commercial a c generator is provided by an electromagnet and the coil itself is wound on an iron cylinder. (The assembly of coil plus cylinder is also called armature).

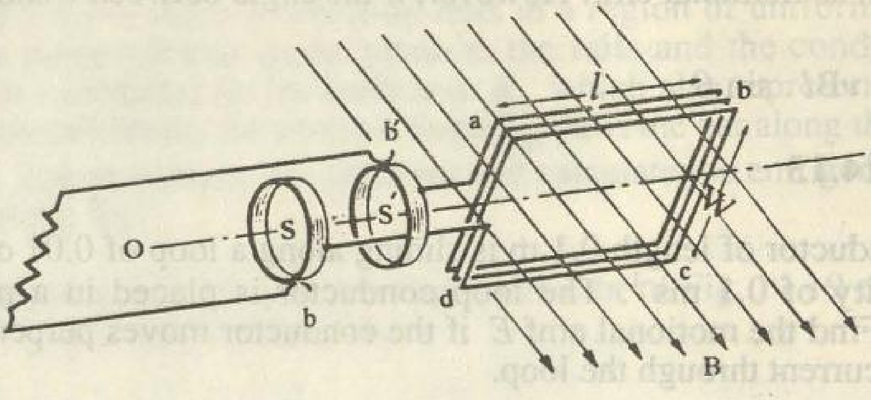


*Fig. 14.20 Principle of an ac generator*

The magnitude of the emf induced in the coil of Fig. 14.20 can be calculated either from the rate of change of flux through the coil, or from the velocities of its sides with which they transverse the magnetic field.

If y is the width and $l$ the length of the coil, the tangential velocity of sides ab and cd is

$$v = \omega \frac{y}{2}$$

where $\omega$ is the angular velocity. If $\alpha$ is the angle between normal to the plane of the coil and the magnetic field **B**, then the component of this velocity perpendicular to the lines of force is

$$v \sin \alpha = \omega \frac{y}{2} \sin \alpha$$

the motional emf in the sides ab and cd is

$$E = vB\, l \sin \alpha = \frac{1}{2} \mathbf{B} l\, \omega y \sin \alpha .$$

(There is no motional emf in sides bc and da) Since these emf's are in series, the net emf in $N$ turns is

$$E = N B\, l\, y \omega \sin \alpha$$

and since $l\,y = A$, So

$$E = N\mathbf{B}A\omega \ \sin\alpha \tag{14.29}$$

From Eq. (14.29) the emf is maximum when the plane of the coil is parallel to the field, and zero when it is perpendicular to the field. This is in agreement with the fact that in the parallel position the sides ab and cd are moving normally to the field, while in the perpendicular position their motion is parallel to the field. The maximum emf is

$$E_{\max} = NBA\omega$$

Hence Eq. (14.29) can be written as

$$E = E_{\max} \ \sin\alpha$$

If $\omega$ is constant, then

$$\alpha = \omega t \quad \text{or} \quad \alpha = 2\pi ft$$

and hence,

$$E = E_{max} \ \sin\omega t = E_{max} \ \sin 2\pi ft \tag{14.30}$$

which shows the dependence of the instantaneous emf on the time *t*. This is shown in the graph of Fig. 14.21. From Eq. (14.30) we find that the induced emf in a rotating coil varies sinusoidally with time. It has its maximum value when $\sin 2\pi ft$ is unity. The maximum emf is therefore, equals to $2\pi f ANB$.

Eq. (14.30) is more often written in the form

$$V = V_o \ \sin 2\pi ft$$

where $V$ is the voltage at any instant and $V_o = 2\pi fAN\mathbf{B}$ is the maximum voltage. The voltage in the rotating coil varies sinusoidally and reverses its direction twice during each cycle of rotation.

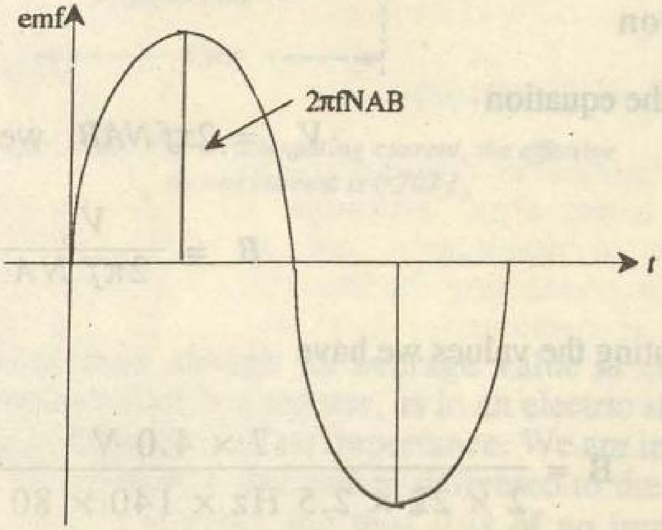


*Fig. 14.21 Time varying emf*

From the above discussion it is clear a coil of wire rotating in a magnetic field has an alternating emf produced between its terminals. If such a generator is used as the power source in a simple circuit, the current through the resistor will reverse its direction $2f$ times per second.

**Example 14.14**

The amplitude of an alternating voltage delivered by an AC generator is 30V and its frequency is 100 Hz. What will be the value of the voltage at $t = 3$ milli second.

**Solution**

From equation

$$V = V_o \sin 2\pi ft$$

we get

$$V = 30\text{V} \times \sin\left[2\pi \times 100\ \text{Hz} \times 3.0 \times 10^{-3}\ \text{s}\right]$$

$$= 30\text{V} \times \sin 1.89^\circ$$

$$= 30 \times 0.033 = 1.0\ \text{V}$$

**Example 14.15**

A 140 loop coil of an area of 80 $\text{cm}^2$ rotates at a frequency of 2.5 Hz in a magnetic field B. If the maximum induced emf in the coil is 4.0 *V*, what is the value of B?

**Solution**

Using the equation

$$V_o = 2\pi f NAB \quad \text{we get}$$

$$B = \frac{V_o}{2\pi f\, NA}$$

substituting the values we have

$$\text{B} = \frac{7 \times 4.0\ \text{V}}{2 \times 22 \times 2.5\ \text{Hz} \times 140 \times 80 \times 10^{-4}\ \text{m}} = 0.227\ \text{T}$$

### 14.10.1 AC Quantities

We know that a coil which rotates in a magnetic field gives rise to a sinusoidal emf (Fig. 14.22). This is the type of voltage and current that is commercially supplied. All sinusoidal currents and voltages have an average value of zero over one or more complete cycles. Even if the ac flowing through a wire makes it white hot (as in the case of incandescent light bulb), the average current in the wire is zero. This fact is explained below.

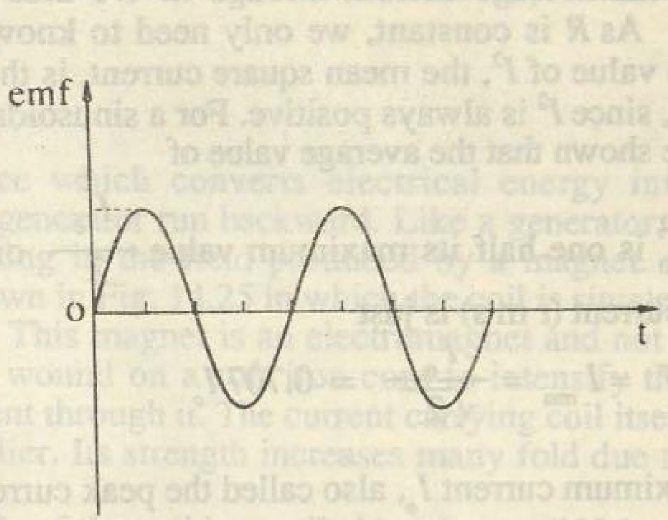


*Fig. 14.22. Time varying graph of emf*

The alternating current in Fig. 14.23 is positive as much as it is negative. To find its average value, we must add its values at various times and divide our sum by the number of values added. Clearly there will be as many positive as negative values for the current. When added, they will give zero, and so the average current is zero. Similarly, the average value of ac voltage is also zero.

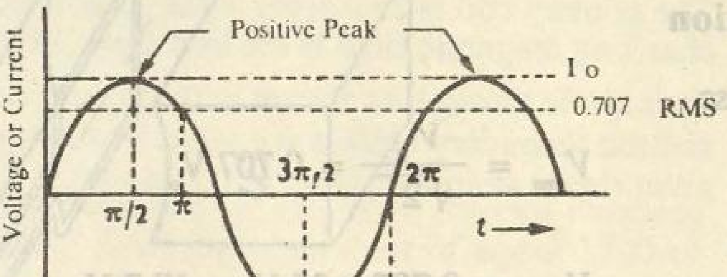


*Fig. 14.23. For an alternating current, the effective or rms current is 0.707 $I_o$*

An ac can do useful work even though its average value is zero. For example, in using electricity to produce heat in a resistor, as in an electric stove, the fact that the current is reversing periodically is of no importance. We are interested only in the heat developed and this is equal to the power delivered to the resistor i.e., $I^2 R$. Here we see that current is squared and thus it is of no importance whether current is positive or negative as $I^2$ will always be positive.

Actually, we are more interested in the average power delivered to a resistor than in the average current through it. We thus wish to find the average value of $I^2R$. As $R$ is constant, we only need to know the average value of $I^2$. The average value of $I^2$, the mean square current, is the average of a group of positive number, since $I^2$ is always positive. For a sinusoidally varying current (or voltage), it can be shown that the average value of

$I_0^2$ or $V_0^2$ is one half its maximum value $\frac{I_o^2}{2}$ or $\frac{V_o^2}{2}$. Thus the root mean square current (r m s) is just

$$I = I_{rms} = \frac{I_o}{\sqrt{2}} = 0.707\, I_o \qquad (14.31)$$

The maximum current $I_o$, also called the peak current, is shown in Fig. 14.23. The rms voltage V is

$$V = V_{rms} = \frac{V_o}{\sqrt{2}} = 0.707\, V_o \qquad (14.32)$$

The rms values are also called effective values. Most of the ac voltmeters and ammeters read the effective voltage or current.

**Example 14.16**

An ac voltage has peak value of 25 V. What is its rms value?

**Solution**

Because

$$V_{rms} = \frac{V_o}{\sqrt{2}} = 0.707\, \mathrm{V_o}$$

hence

$$V_{rms} = 0.707 \times 25\ \mathrm{V} = 17.7\ \mathrm{V}$$

### 14.10.2 D C Generator

A terminal voltage that always has the same sign, although it fluctuates in magnitude, can be obtained by connecting the loop to a split ring or commutator as in Fig. 14.24 instead of slip rings. At the position in which the emf reverses, the connections to the external circuit are interchanged. The graph of the terminal voltage is a straight line. The device is called a dc generatore.

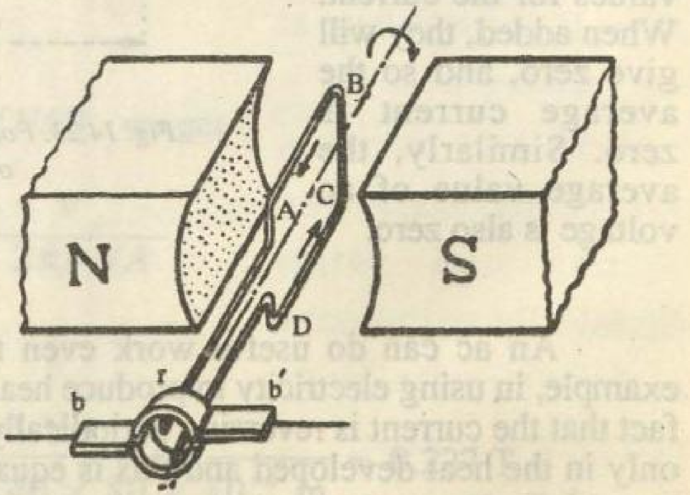


Fig. 14.24 DC generator

Commercial d c generator have a large number of coils and commutator segments, and their terminal voltage is not only uni-directional but also practically constant.

## 14.11 ELECTRIC MOTOR

An electric motor is a device which converts electrical energy into mechanical energy. It is essentially a generator run backward. Like a generator, a simple motor consists of a coil rotating in the field produced by a magnet. A simplified diagram of the motor is shown in Fig. 14.25 in which the coil is situated between the pole pieces of a magnet. This magnet is an electromagnet and not a permanent magnet. The coil itself is wound on a soft iron core to intensify the magnetic field resulting from the current through it. The current carrying coil itself acts as a bar magnet as explained earlier. Its strength increases many fold due to iron core.

In the Fig. 14.25, the north pole of the coil is repelled by the north pole of the permanent magnet, and the coil will be made to rotate in direction by the mutual repulstion of the poles. After rotating through 180°, the north of the coil will be next to the south pole of the permanent magnet and will be retarded from rotating away from that position. However, when it reaches that position, the sliding contacts on the split ring slide over the gap and the current flowing through the coil reverses. This in turn reverses the poles of the coil, and so once again the situation shown in Fig. 14.24 is achieved. Repulsion is maintained, and as a result rotation continues.

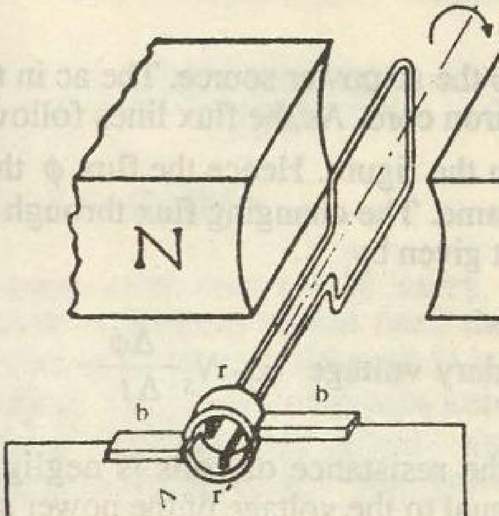
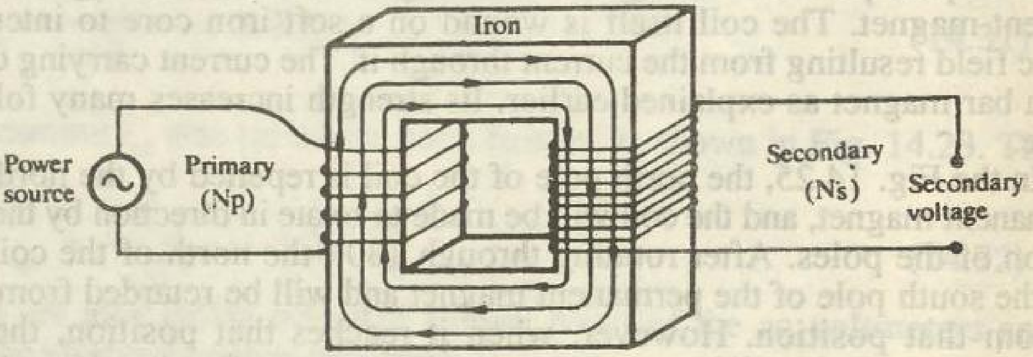


*Fig. 14.25 DC. motor*

There are various modifications of such a motor. Most motors consist of several loops wound with their planes through AA′ but at various angles to one another. Each loop has current flowing through it for only small portion of a cycle during the time when its orientation to the field is right for obtaining maximum torque. Such a motor gives a much more uniform torque than one could obtain from single loop.

## 14.12 TRANSFORMER

One of the most important applications of electromagnetic induction is in the form of a transformer. A transformer is a device which steps up or steps down ac voltage. For example, in power transmission high voltage is stepped down to 220 volts for domestic use with the help of the transformer. The common door bell requires a voltage of about 9 V, and a transformer is used to obtain this voltage from the usual supply of 220 V. Shown in Fig. 14.26 is a typical transformer. A transformer has an iron core on which two coils are wound, the primary with $N_p$ loops and the secondary with $N_s$ loops.



*A transfor mer*

*Fig. 14.26*

The primary coil is connected to the ac power source. The ac in this coil produces a changing magnetic field in the iron core. As the flux lines follow the iron core, the lines circle through as shown in the figure. Hence the flux $\phi$ through the primary and the secondary coils is the same. The changing flux through the secondary coil gives rise to an induced emf in it given by

$$\text{Secondary voltage} = -N_s \frac{\Delta\phi}{\Delta t}$$

In most transformers, the resistance of coils is negligible, therefore, the induced emf in the primary is equal to the voltage of the power source. Therefore, it can be written as

$$\text{Primary voltage} = -N_P \frac{\Delta\phi}{\Delta t}$$

where $\phi$ is the same flux that flows through the secondary coil. Taking the ratio of these two induced voltages, we get

$$\frac{\text{Secondary voltage}}{\text{Primary voltage}} = \frac{N_S}{N_P} \qquad (14.33)$$

This is the transformer equation and it tells how the secondary voltage is related to the primary voltage. The two voltages are in the same ratio as the number of loops in the coil. A transformer that raises the input voltage ($N_s > N_p$) is called a step up transformer. The reverse case in which $N_p > N_s$ is called a step down transformer.

If the secondary circuit is not closed, current cannot flow in it. Hence, there is not power loss in the secondary coil when it is not in use. Due to this fact it is possible for the power company to keep their transformer running even when no one is using the electricity that they are providing. The transformers themselves consume very little energy.

One of the most important uses of transformer is concerned with power transmission and the distribution. Most of the power companies supplying power to cities that are perhaps hundreds of kilometer away from the power generating stations. This poses many problems. Let us assume that in a certain city of 100,000 peoples, each person is using 110 W of power. The power consumed is $(110 \times 100{,}000)$ W and at a voltage of 220 V we have

$$\text{Total power} = VI$$

$$(110 \times 100{,}000\text{W}) = 220\ VI$$

or
$$I = \frac{110 \times 100000\text{W}}{220\text{V}}$$

$$I = 50{,}000\text{ A}$$

Ordinary house wire can safely carry about 30A current without overheating, the power companies would need the equivalent of about 1700 of these wires to carry power to the city although it is impossible, the cost of copper alone will be tremendous. The power companies know that the important quantity is $VI$ and not $I$ alone, so they transmit power over long distances at very high voltages. In the example given above, if $V$ had been 100,000 V, then

$$(110 \times 100{,}000) = 100{,}000\ I$$

or
$$I = 110\text{ A}$$

For this reason high-tension or high voltage transmission lines are used to carry power over large distances. But such high voltages cannot be supplied directly to a house. The danger from electrocution and fire would be too much. At this stage step down transformers are used to convert these high voltages to the normal voltage of 220 V used in house.

**Example 14.17**

A certain transformer is designed to change 220 V line voltage to 9.0 V. (a) What is the turns ratio $\frac{N_P}{N_S}$ for this transformer ? (b) By mistake, it is connected into the circuit backward. About what voltage does it deliver before everything burns out ?

**Solution**

(a) $$\frac{\text{Primary voltage}}{\text{Secondary voltage}} = \frac{N_P}{N_S}$$

or $$\frac{220}{9} = \frac{N_P}{N_S}$$

so $$\frac{N_P}{N_S} = 24.4$$

(b) If by mistake the connections are reversed then the voltage delivered will be

$$220 \times 24.4 = 5368 \text{ V}$$

# SUMMARY

***Magnetic field due to electric current :*** When an electric current passes through a conductor a magnetic field is set up around the conductor in the form of concentric circles. The direction of the field is given by right hand rule.

***Force on a current carrying conductor in a uniform magnetic field :***

When a conductor carrying current is placed in a unifrom magnetic field, it experiences a force which is given as

$$F = Il \times B$$

where $I$ is the current through the conductor, $l$ is the length of the conductor and $B$ is the magnetic field. Torque on a current carrying rectangular loop of area $A$ and having $N$ turns and placed in a magnetic field is given by

$$\tau = IBAN \cos \theta$$

where $\theta$ is the angle between the plane of the coil and the direction of the field.

***Force on a charged particle moving in a magnetic field :*** The charged particle moving in magnetic field experiences a force. The force on a charged particles is perpendicular to both the direction of the field and the direction of motion of the charged particle.

$$\mathbf{F}_m = q\,\mathbf{v} \times \mathbf{B}$$

***Determination of*** $\frac{e}{m}$ ***:*** An electron travelling with a velocity **v** describes a circular path when it enters a uniform magnetic field such that

$$\frac{e}{m} = \frac{v}{B\,R}$$

***Ampere's law :*** Ampere's law is analogous to Guass's law. Gauss's law is a relation between the normal component of electric field at points on a closed surfacae and the net charge enclosed by the surface. Ampere's law is a relation between the tangential component of the magnetic field to points on a closed curve and the net current through the area bounded by the curve. Mathematically, Ampere's law can be written as

$$\sum B_{\perp}\,\Delta l = \mu_o\,I, \quad \text{where } \mu_o = 4\pi k'$$

***Lenz's law :*** An induced emf always drives a current whose magnetic field opposes the original change in flux.

***Mutual induction :*** Mutual induction is numerically equal to emf induced in the secondary coil when the rate of change of current in the primary is one Ampere

$$\text{emf}_{\text{secondary}} = -M\frac{\Delta I_p}{\Delta t}$$

The unit of mutual inductance $M$ is henry (H) or $\text{VsA}^{-1}$

***Self inductance :*** From the equation emf $= -L\frac{\Delta I}{\Delta t}$ we define self inductance as the emf linked with the same coil when the rate of change of current in the coil is unity. The unit of self inductance is also henry.

***Non-inductive wire wound resistances :*** When a current changes in the circuit an opposing current is set up which indicates an increase in the resistance of the circuit. To overcome this we wind the resistance coils by folding the wires into two such folds that the current in one fold is opposite to that in the second. The magnetic field around one wire cancels the magnetic field around the second wire and the original resistance remains unaffected.

***Motional emf. :*** The induced emf generated as a result of the motion of the wire through the magnetic field is called motional emf. (emf.) induced = $v\mathbf{B}l$ , where v is the velocity with which the wire is dragged, $l$ is the distance through which wire moves, and $\mathbf{B}$ is the magnetic field.

***Alternating current generator :*** A generatore is a device which produces a voltage difference between two terminals by changing the flux through the coil. It changes many times during one second, hence it is called generator. The voltage delivered by such a generator at any instant is given by the equation

$$V = V_o \sin 2\pi ft$$

The maximum voltage

$$V_o = 2\pi f NAB$$

where $f$ is the frequency $N$ is the number of turns, $A$ is the area of the coil and $B$ is the magnetic field.

***D C generator :*** A terminal voltage that always has the same sign, although it may fluctuate in magnitude, can be obtained by connecting the loop to a split ring or a commutator instead of slip rings. The construction of a dc generator is similar to that of an ac generator except for the slip rings which have been replaced by commutator. A dc generator gives not only unidirectional but also a practically constant current.

***Electric motor :*** An electric motor is a device which converts electrical energy into mechanical energy. It is essentially a generator run backward.

***Transformer :*** A transformer is a device which transforms ac voltage of one magnitude into another ac voltage of required magnitude. If $N_s > N_p$, the transformer is called step-up transformer and if $N_p > N_s$ the transformer is said to be step-down transforemer.

# QUESTIONS

**14.1** In what direction are the magnetic field lines surrounding a straight wire carrying current that is moving directly towards you ?

**14.2** What kind of field or fields does/do surround a moving electric charge ?

**14.3** Can an electron at rest be set in motion with a magnet ?

**14.4** If a charged particle moves in a straight line through some region of space, can you say that the magnetic field in that region is zero ?

**14.5** A beam of electrons is directed towards a horizontal wire in which the current flows from left to right. In what direction is the beam deflected?

**14.6** A charged particle is moving in a circle under the influence of a uniform magnetic field. If an electric field is turned on that point along the same direction as the magnetic field, what path will charged particle take?

**14.7** A loop of wire is suspended between the poles of a magnet with its plane parallel to the pole faces. What happens if direct current is passed through the coil? What will happen if an alternating current is passed instead?

**14.8** A curret-carrying wire is placed in a magnetic field. How must it be oriented so that the force on it is zero, is maximum?

**14.9** Two wires carrying equal and opposite currents are twisted together in the construction of a circuit. Why does this technique reduce stray magnetic fields?

**14.10** Why is the magnetic field strength greater inside a current-carrying loop of wire?

**14.11** A hollow copper tube carries a current. Why is $B = 0$ inside the tube? Is $B$ non zero outside the tube?

**14.12** Does the inductance of a coil depend on the current in the core? What factors affect the inductance of a coil?

**14.13** Can an efficient transformer step up energy? Explain.

**14.14** In what three ways can a voltage be induced in a wire?

**14.15** Can a current carrying coil be used as a compass? Explain.

**14.16** Does the voltage out-put of a generator change if its speed of rotation is increased? Explain.

**14.17** What are similarities between a motor and a generator? What are the differences?

**14.18** When a beam of electrons is shot into a certain region of space, the electrons travel in a straight line through the region. Can we conclude that there is no electric field in the region? No magnetic field?

**14.19** A very long copper pipe is held vertically. Describe the motion of a bar magnet dropped length wise down the pipe.

**14.20** A solenoid is viewed in such a way that the solenoid current appears clockwise to the viewer. What is the direction of the field within the solenoid.

# PROBLEMS

**14.1** What is the force per metre of length on a wire carrying a 0.5 A current due to a 0.5 tesla magnetic field.

[ Ans : 0.25 N/m ]

**14.2** A power line parallel to the earth's surface carries a current of 20.0 A straight west. At that point the earth's magnetic field is 0.80 gauss parallel to the earth's surface and directed straight north. (a) Find the force due to the field on a 15 m length of wire. (b) What is its direction?

[ Ans : (a) 0.024 N; (b) towards earth ]

**14.3** A 2.0 cm length of wire carries a current of 5.0 A perpendicular to the magnetic field between the pole pieces of a magent. The force on it is 0.080 N. What is the magnitude of the magnetic field?

[ Ans : 0.80 T ]

**14.4** How fast must a proton of mass $1.67 \times 10^{-27}$ kg be moving if it is to follow a circular path of radius 2.0 cm in a magnetic field of 0.70 G?

[ Ans : $1.34 \times 10^{2}$ m $s^{-1}$ ]

**14.5** A proton falls from rest through a potential difference of $3.34 \times 10^{5}$ V. It then enters a region perpendicular to a magnetic field $B = 0.50$ T. How large is the radius of the circle in which the proton travels?

[ Ans : 0.167 m ]

**14.6** A long solenoid has 5000 loops of wire on its 80 cm length. The diameter of the solenoid is 2.0 cm. Find $B$ inside the solenoid when a current of 300 m A flows in it and the flux through it.

[ Ans : (i) $2.36 \times 10^{-3}$ T (ii) $7.38 \times 10^{7}$ Wb ]

**14.7** The radius of a narrow toroid is 10 cm and it has 200 turns of wire on it. How large a current must flow in it if $B$ is to be 0.50 T inside it?

[ Ans : 1250 A ]

**14.8** A 10 cm diameter circular loop of wire is in a 0.50 T magnetic field. It is removed from the field in 10 s. What is the average induced voltage?

[ Ans : $3.9 \times 10^{-4}$ V ]

**14.9** A 20 - loop coil lies on a table top in a region where the magnetic field is vertically upward and has a value of 30 G. The field is reduced to zero in 0.50 s. If the radius of each loop is 7.0 cm, what is the average induced emf in the coil (a) while the field is changing (b) before the field begins to change? (c) As viewed from above, is the induced emf in the coil clockwise or anticlockwise?

[ Ans : (a) 1.85 mV; (b) Zero (c) anticlock wise ]

**14.10** A wire loop is being pulled out of the magnetic field at a constant rate. If the magnetic field is uniform (0.30 T) in this region and zero else where, what is the induced emf in the loop? (The speed of the loop is 2.0 $ms^{-1}$ and its breadth is 4.0 cm).

[ Ans : 0.024 V ]

**14.11** Two coils are wound tightly on the same iron core. The cross-sectional area of both is about 4.0 $cm^2$. When a current of 3.0 A flows in the primary coil, $B = 0.20$ T. There are 100 turns on the secondary. (a) How large an emf is induced in the secondary if the current in the primary drops uniformly to zero in 0.050 s? (b) What is the mutual inductance of the coils?

[ Ans : 0.16 V (b) 2.7 mHz ]

**14.12** Find the self inductance of a 30 cm long solenoid of radius 0.80 cm and 500 loops if it is (a) air filled (b) wood filled, and (c) filled with iron that intensifies the field by a factor of 140.

[ Ans : (a) 0.21 mHz (b) 0.21 mHz (c) 29.4 mHz ]

**14.13** A generator produces 100 V when rotated at a certain speed. If its speed of rotation is doubled, what will be the output voltage?

[ Ans : 200 V ]

**14.14** A transformer changes 12 V to 18000 V, and there are 6000 turns in the secodnary. How many turns are there in the primary?

[ Ans : 4 turns ]

**14.15** Show that the ratio of the current in the secondary and primary of a transformer equals the inverse ratio of the number of turns.

**14.16** An average of 120 kW of electric power is sent to a small town from a power plant 10 km away. The transmission lines have a total resistance of 0.40 ohm. Calculate the power loss if the power is transmitted at (a) 240 V and (b) 24,000 V.

[ Ans : (a) 100 kW (most of the power is lost
(b) 10 W.]

# 15

# ELECTRICAL MEASURING INSTRUMENTS

Electrical measuring instruments are those electrical devices which convert electrical energy into mechanical energy. Galvanometer is a one such example. The general principle, upon which the working of galvanometer depends is that whenever a conductor of length, say $l$, carrying current $I$ is placed in a magnetic field of strength $B$, it experiences a force given by Eq. (14.4) i.e.,

$$F = I\,l\,B$$

If instead of a wire, a current carrying rectangular loop of area $A$ and having $N$ turns is placed in the same field $B$ then the loop will experience a torque as given by Eq. (14.9a), i.e.,

$$\tau = NI\,AB\,\cos\alpha$$

where $\alpha$ is the angle between the plane of the coil and the direction of the magnetic field.

This principle is used in the construction of a galvanometer which is the basic instrument for voltmeter and ammeter. In this chapter we shall discuss the construction and working of galvanometer and its conversion into ammeter and voltmeter. The principle underlying a wheat-stone bridge and the devices based upon it are also discussed in this chapter. The instrument called avometer is also discussed here.

## 15.1 GALVANOMETER

A device used for the detection and measurement of current is called a galvanometer. The working of a galvanometer depends upon the torque exerted on a coil suspended in a magnetic field when a current is passed through the coil. Most of the galvanometers used now-a-days are of the moving coil type.

The construction of a moving coil galvanometer is sketched in Fig. 15.1.

A coil C is suspended between poles N and S of a U shaped magnet by means of a fine suspension wire or thin flat strip. The coil is made up of enameled copper wire on a rectangular frame of some non-magnetic material. The suspension wire F is also used as one current lead to the coil. The other terminal of the coil is connected to a loosely wound spiral which serves as the second current lead. The pole pieces of the magnet are made concave and a soft iron cylinder B is fixed between the poles. This makes the field radial and stronger in the neighbourhood of the coil as shown in Fig. 15.1 (b).

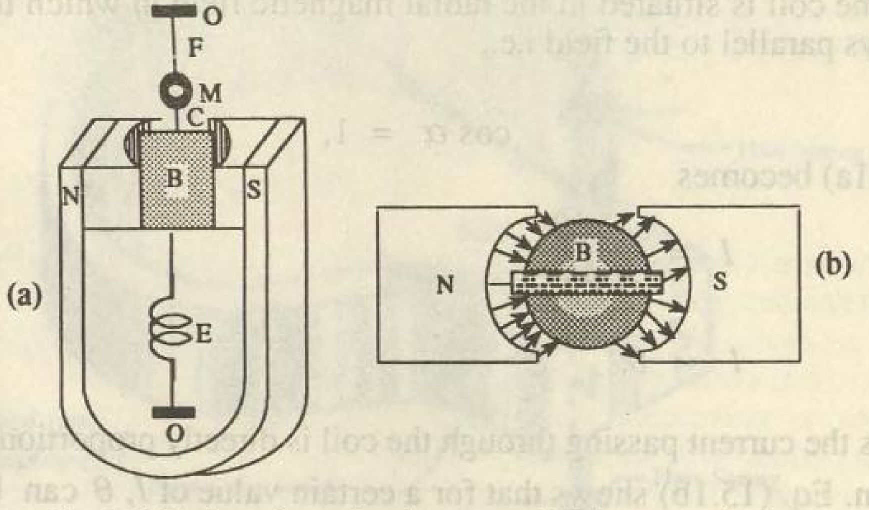


*Fig. 15.1 (a) A moving coil galvanometer (b) Concave pole pieces and soft iron cylinder make, the field radial and stronger.*

When current is passed through the coil, it is acted upon by a torque which tends to rotate it to a position perpendicular to that shown in Fig. 15.1. The coil, while turning, twists the suspension strip and comes to rest where the torque exerted by the magnet field on the coil (called the deflecting couple) is balanced by an opposing torque due to the reaction of the twisted suspension strip (called the restoring couple).

It has already been discussed that the torque of the deflecting couple on a current carrying coil in a magnetic field is given by

$$\text{Deflecting torque} = IBAN \cos\alpha \qquad (15.1)$$

where $\alpha$ is the angle between the plane of the coil and the direction of the field.

We know that the restoring couple due to twisting of a suspension wire is proportional to the angle of deflection provided the suspension wire obeys Hooke's law. Thus

$$\text{Restoring torque} = c\,\theta$$

where $c$ is the torque for unit twist of the coil and $\theta$ is the angle of twist.

When the coil is in the equilibrium position then the deflecting torque is equal to the, restoring couple, i.e.,

$$IBAN \cos\alpha = c\,\theta$$

$$I = \frac{c\,\theta}{BAN\cos\alpha} \tag{15.1a}$$

As the coil is situated in the radial magnetic field in which the plane of the coil is always parallel to the field i.e.,

$$\cos\alpha = 1,$$

the Eq. (15.1a) becomes

$$I = \frac{c\,\theta}{BAN} \tag{15.1b}$$

or $\quad I \propto \theta$

Thus the current passing through the coil is directly proportional to the angle of deflection. Eq. (15.1b) shows that for a certain value of $I$, $\theta$ can be made large in the galvanometer and the galvanometer can be made sensitive if the value of factor $\frac{c}{BA\ N}$ is made small. A galvanometer is said to be sensitive if for a small current through the coil produces a very large deflection in it.

There are two methods of observing the angle of deflection of the coil. In a sensitive galvanometer the angle of deflection is observed by means of a small mirror attached to the coil along with a lamp and scale arrangement (Fig. 15.2). A beam of light from the lamp is directed towards the mirror of the galvanometer. After reflection from the mirror it produces a spot on a translucent scale placed at a distance of one metre from the galvanometer. As the coil along with the mirror rotates, the spot of light moves along the scale. the displacement of the spot of light on the scale is proportional to the angle of deflection (provided $\theta$ is small).

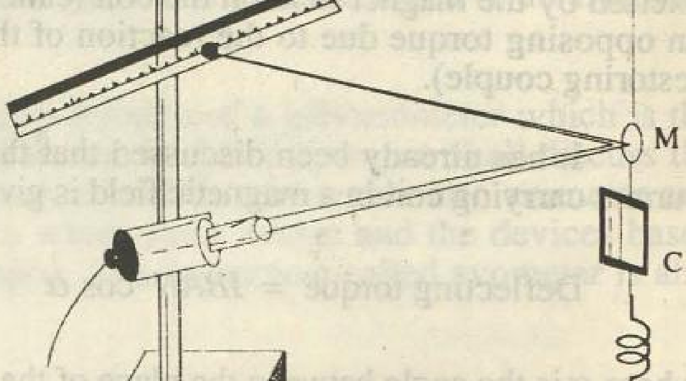


*Fig. 15.2 Lamp and scale arrangement for measuring the angle of deflection.*

The second method of observing $\theta$ is used in the pivoted coil galvanometer. In this type of galvanometer the coil is pivoted between two jewelled bearings. The restoring torque is provided by two hair springs which also serve as current leads. A light aluminium pointer is attached to the coil which moves over the scale (Fig. 15.3), thus giving the angle of deflection of the coil.

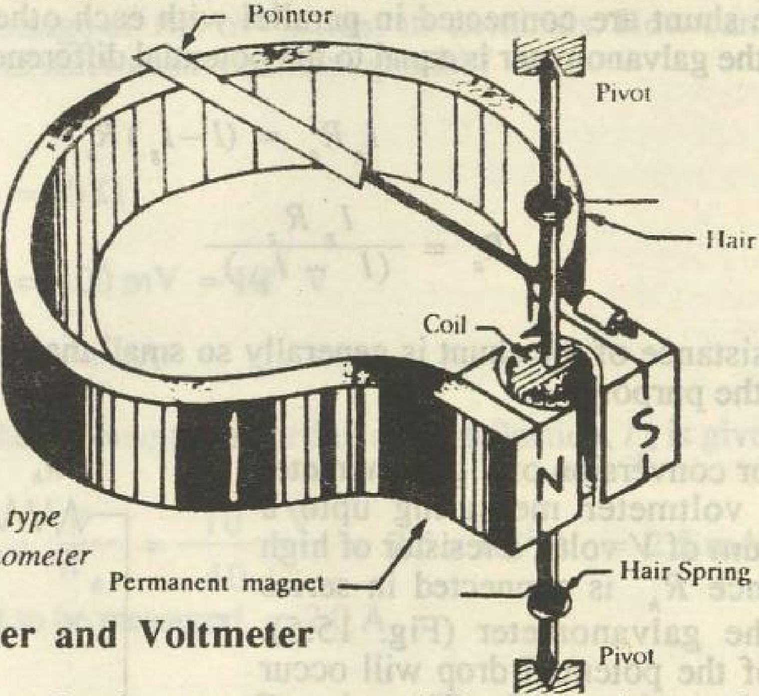


*Fig. 15.3 Pivoted type moving coil galvanometer*

### 15.1.1 Ammeter and Voltmeter

Ammeter and voltmeters are essentially galvanometers in which suitable modifications have been made. Most of the galvanometers give full scale deflection even when a few milliampere current is passed through them, so they cannot be used for measuring large current without some sort of modification.

Suppose we have a galvanometer that gives full scale deflection when a current $I_g$ is passed through it. Let the resistance of this galvanometer be $R_g$. Using Ohm's law we find that the voltage across the galvanometer will be $V_g$ when current $I_g$ is passed and is given by

$$V_g = I_g R_g$$

If we want to convert this galvanometer into an ammeter which can measure a maximum current $I$, it is necessary to connect a low resistance, called a shunt, in parallel with the galvanometer so that excess current passes through the shunt. This is sketched in Fig. 15.4. The resistance of the shunt must be so adjusted that when a current $I$ flows through the galvanometer, the current $I_g$ should pass through the meter movement so that it gives full scale deflection and the remaining current $(I - I_g)$ passes through the shunt.

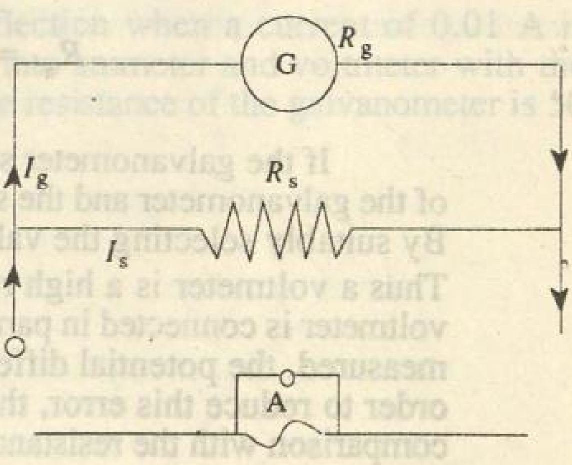


*Fig. 15.4 A galvanometer used as an ammeter*

The shunt resistance $R_s$ can be calculated from the fact that the galvanometer and the shunt are connected in parallel with each other, the potential difference across the galvanometer is equal to the potential difference across the shunt. Thus

$$I_g R_g = (I - I_g) R_s$$

or
$$R_s = \frac{I_g R_g}{(I - I_g)} \tag{15.2}$$

The resistance of the shunt is generally so small that just a piece of copper wire serves the purpose.

For conversion of a galvanometer into a voltmeter measuring upto a maximum of $V$ volts, a resistor of high resistance $R_h$ is connected in series with the galvanometer (Fig. 15.5). Most of the potential drop will occur across the resistance $R_h$. The value of the resistance $R_h$ should be such that the voltage $V$ appearing across the galvanometer and the series resistor $R_h$, Causes a current $I_g$ which is sufficient to produce full scale deflection in the galvanometer. Thus by Ohm's law

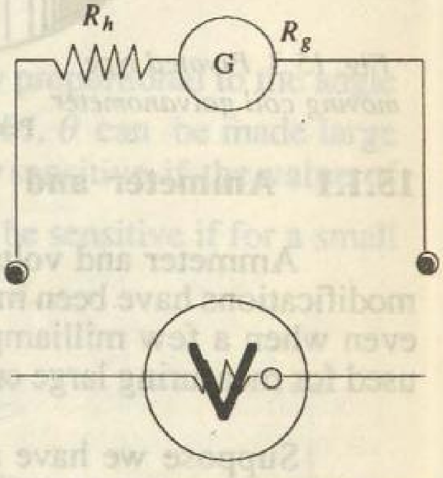


*Fig. 15.5 A galvanometer used as a voltmeter*

$$V = I_g (R_g + R_h)$$

$$R_h = \frac{V}{I_g} - R_g \tag{15.3}$$

If the galvanometer scale is caliberated from 0 to $V$ volts, the combination of the galvanometer and the series resistor acts as a voltmeter with range 0–$V$ volts. By suitably selecting the value of the resistance $R_h$, any voltage can be measure. Thus a voltmeter is a high resistance galvanometer. It may be noted that when a voltmeter is connected in parallel with two points whose potential difference is to be measured, the potential difference is changed by the presence of the voltmeter. In order to reduce this error, the resistance of the volt-meter should be very large in comparison with the resistance of the circuit across which it is connected.

Similarly, when an ammeter is connected in series with the circuit, th current to be measured is changed by the introduction of ammeter. This error ma be reduced if the resistance of ammeter is very small as compared to the tota resistance of the circuit.

**Example 15.1**

A certain galvanometer has a resistance of 40 Ω and gives full scale deflection for a voltage of 100 mV across its terminals. How can it be converted into an ammeter that can measure current upto 2A?

**Solution**

$$R_g = 40\ \Omega$$

$$V = 100\ \text{mV} = 10^{-1}\ \text{V}$$

$$\text{Range} = 2\text{A}$$

Current through the galvanometer for full scale deflection, $I_g$, is given as

$$I_g = \frac{V}{R_g} = \frac{10^{-1}\ \text{V}}{40\ \Omega} = 2.5 \times 10^{-3}\ \text{A} = 2.5\ \text{mA}$$

Maximum current to be measured = 2.0 A

Shunt resistance

$$R_S = \frac{I_g}{I - I_g}\, R_g$$

$$= \frac{2.5 \times 10^{-3}\ \text{A}}{(2\ \text{A} - 0.0025\ \text{A})} \times 40\ \Omega = 0.05\ \Omega$$

A resistance of 0.05 ohm when connected in parallel to this galvanometer will make an ammeter that can measure a current of 2.0 A.

**Exmaple 15.2**

A galvanometer gives full scale deflection when a current of 0.01 A is passed through it. How can it be converted into ammeter and voltmeter with the range of 5 A, 12 V, 120 V, respectively, if the resistance of the galvanometer is 50 Ω?

**Solution**

$I_g = 0.01$ A

$I = 5$ A

$R_g = 50\ \Omega$

$$R_s = \frac{I_g}{I - I_g}\, R_g$$

$$R_s = \frac{0.01\ \text{A}}{5\text{A} - 0.01\ \text{A}}\ 50\ \Omega = 0.100\ \Omega$$

Hence a resistance of 0.1 ohm must be connected across the galvanometer terminals to convert it into an ammeter of range 0 – 5A. To convert this galvanometer into a voltmeter of range 0 – 12V and 0 – 120 V we use the formula

$$R_h = \frac{V}{I_g} - R_g$$

(i) $$= \frac{12\ \text{V}}{0.01\ \text{A}} - 50\ \Omega = 1200\ \Omega - 50\ \Omega = 1150\ \Omega$$

(ii) $$= \frac{120\ \text{V}}{0.01\ \text{A}} - 50\ \Omega = 1200\ \Omega - 50\ \Omega = 11950\ \Omega$$

As a resistance of 1150 ohms has already been connected for 12 V measurement hcnce an additional 10800 ohms will be connected for measurement upto 120 V as shown in the diagram.

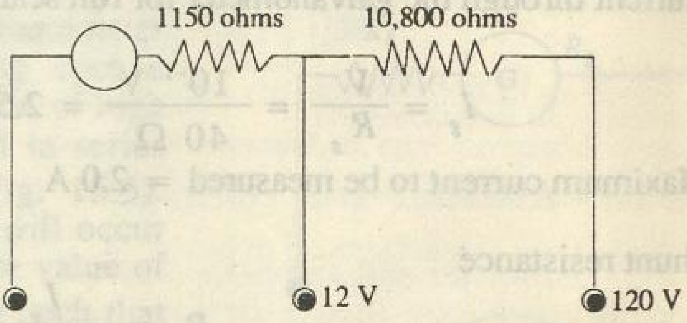
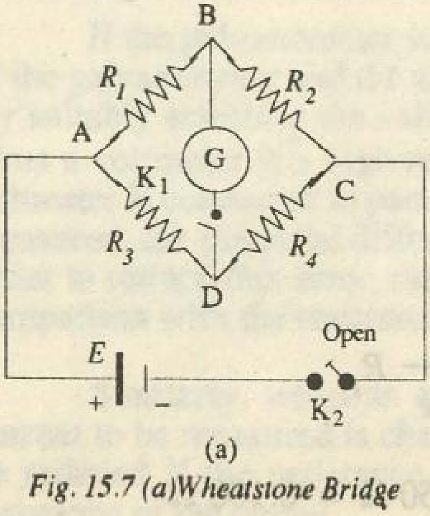

1150 ohms
10,800 ohms
+
12 V
120 V

Fig. 15.6

## 15.2 WHEATSTONE BRIDGE

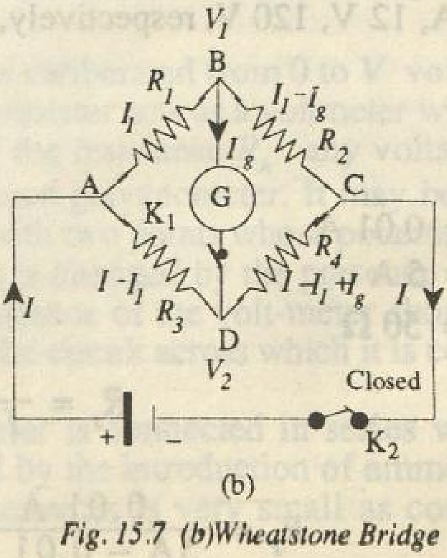
This is an arrangement of electrical circuit which forms the basis of most of the instruments used to determine an unknown resistance. It was deviced by Charles Wheatstone and hence it is known as Wheatstone bridge.

A wheatstone bridge consists of four resistances, $R_1$, $R_2$, $R_3$ and $R_4$ joined together to form a mesh ABCDA as shown in Fig. 15.7 (a).


B
$R_1$
$R_2$
A
G
$K_1$
C
$R_3$
$R_4$
D
E
Open
+
–
$K_2$

(a)

Fig. 15.7 (a)Wheatstone Bridge


$V_1$
B
$R_1$
$I_1$
$I_1 - I_g$
$I_g$
$R_2$
A
G
C
$K_1$
$R_4$
$I - I_1$
$R_3$
$I - I_1 + I_g$
$I$
$I$
D
$V_2$
Closed
+
–
$K_2$

(b)

Fig. 15.7 (b)Wheatstone Bridge

A battery of emf $E$ is connected between the points A and C, with a key $K_2$ and a sensitive galvanometer of resistance G and a key $K_1$ are connected between the points B and D.

If keys $K_1$, and $K_2$ are closed, a current will flow through the galvanometer. We can, however, arrange conditions under which no current flows through the galvanometer even though keys $K_1$ and $K_2$ are closed. In this condition the Wheatstone bridge is said to be balanced.

Let us consider the path of the current through different resistances of the bridge. The current $I$ starting from the positive terminal of battery reaches the point A. At A the current finds two paths, one through the resistance $R_1$ and the other through $R_3$. Let $I_1$ be the current flowing through $R_1$, then current through $R_3$ is $(I - I_1)$. When the current $I_1$ reaches B it again finds two paths to flow through, one through the galvanometer and the other through the resistance $R_2$. If $I_g$ is the current flowing through the galvanometer G, then the current through $R_2$ is $(I_1 - I_g)$. When this current $(I_1 - I_g)$ reaches the point C it is joined by the current flowing through ADC. The current $(I - I_1)$ passing through $R_3$ when reaches the point D, it is joined by $I_g$ the current through G. Thus the current passing through $R_4$ is $(I - I_1 + I_g)$. Thus the current at C is the sum of the current through ABC and ADC, i.e.,

$$I_1 - I_g + I - I_1 + I_g = I$$

When the bridge is balanced, then no current flows through the galvanometer. This means the potential at point B and D is the same i.e., $(V_1 = V_2)$. Hence,

$$I_1 R_1 = (I - I_1) R_3 \tag{15.4}$$

As no current flows through G, therefore, the current through $R_2$ is $I_1$ and that through $R_4$ is $(I - I_1)$, so

$$I_1 R_2 = (I - I_1) R_4 \tag{15.5}$$

Dividing Eq. (15.4) by Eq. (15.5) we get

$$\frac{I_1 R_1}{I_1 R_2} = \frac{(I - I_1) R_3}{(I - I_1) R_4}$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \tag{15.6}$$

This is the condition for the bridge to be balanced and is called the Wheatstone bridge principle.

Principle of wheatstone bridge is utilized to determine the unknown resistances by various modifications of the Wheatstone bridge. These are described below.

To determine unknown resistance $X$, we connect this resistance in the arm containing resistance $R_4$. The values of resistance $R_1$, $R_2$ and $R_3$ are so adjusted that the bridge is balanced and no current flows through the galvanometer. In this position

$$\frac{R_1}{R_2} = \frac{R_3}{X} \quad \text{or} \quad X = \frac{R_3}{R_1} R_2 \tag{15.7}$$

As the values of $R_1$, $R_2$ and $R_3$ are known, the value of $X$ can be calculated.

### 15.2.1 Post Office Box

It is a practical form of Wheatstone bridge. It measures resistance with sufficient accuracy. This was originally designed to measure resistance of telegraph Fig. 15.8 shows a P.O. box in its simplest form. The arms AB and BC contain resistances $P$ and $Q$ respectively. These arms are called ratio arms. Each of these arms contains resistances of 10,100 and 1000 Ω. The third arm AD corresponds to the third resistance $R$. In this arm, the resistances are arranged in such a way that a resistance of 1 to 10,000 ohm can be introduced in steps. The unknown resistance $X$ is connected between the terminals C and D. This forms the fourth arm of the Wheatstone bridge.

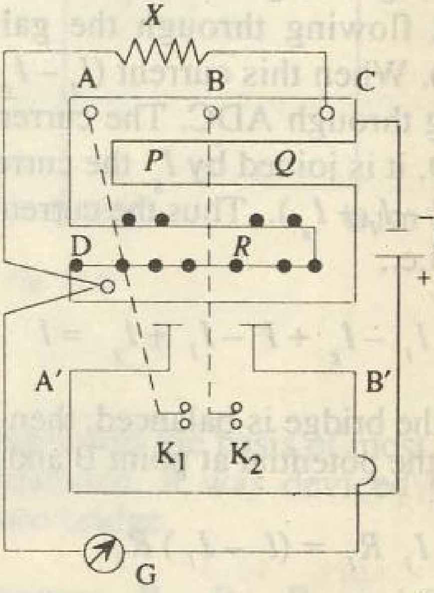


*Fig. 15.8 Post office box*

### 15.2.2 Slide Wire Bridge Or Metre Bridge

Slide wire bridge is another practical form of the Wheatstone bridge and it is used to determine the value of the unknown resistance.

It consists of a thin uniform metallic wire AC usually made of eureka. It is one metre long. It is stretched and fixed to two thick copper strips at A and C, and is placed along a metre scale over a wooden board. A jockey J is connected to the terminal D through a galvanometer G as shown in Fig. 15.9.

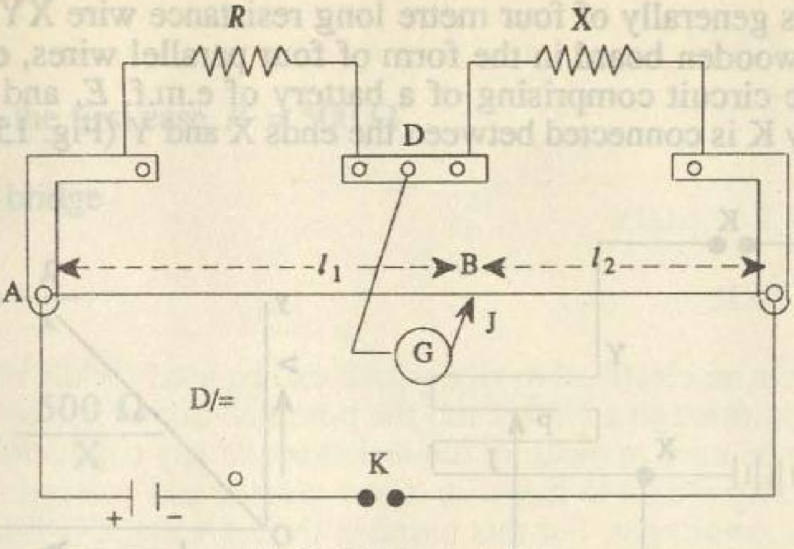


*Fig. 15.9 Slide wire bridge*

A resistance box is connected in the gap between A and D, so that a known resistance *R* can be introduced between A and D. The resistance across A and C through a key K.

To find the value of the unknown resistance, the key K is closed and a suitable resistance *R* is taken out of the resistance box. The jockey is moved along the wire till for a certain position, the galvanometer shows zero deflection. Let B be this position. Now the points A, B, C, and D correspond to the same points as on a Wheatstone bridge. Let the length AB $l_1$ and BC $l_2$ correspond to resistance *P* and *Q*, respectively. Applying the principle of Wheat-stone bridge

$$\frac{P}{Q} = \frac{R}{X}$$

Since the slide wire is of uniform thickness, therefore, the resistances *P* and *Q* must be proportional to the respective lengths $l_1$ and $l_2$. Hence

$$\frac{l_1}{l_2} = \frac{R}{X}$$

or $$X = \frac{l_2}{l_1} R$$

knowing $l_1$, $l_2$ and *R* the value of the unknown resistance can be calculated.

## 15.3 POTENTIOMETER

A very simple instrument which can measure and compare potentials without drawing any current from the circuit is known as potentiometer. This is described below.

It consists generally of four metre long resistance wire XY about 4 metre long fixed on a wooden board in the form of four parallel wires, each one metre long. An electric circuit comprising of a battery of e.m.f. $E$, and ammeter A, a rheostat $R_h$, a key K is connected between the ends X and Y (Fig. 15.10).

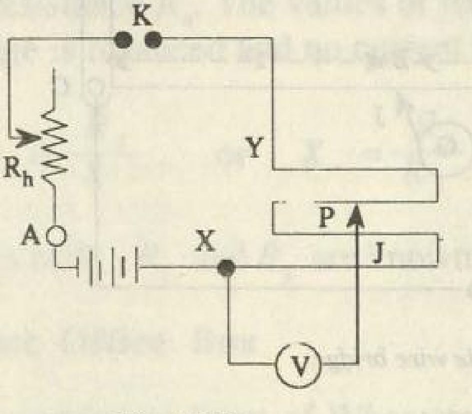


*Fig. 15.10*

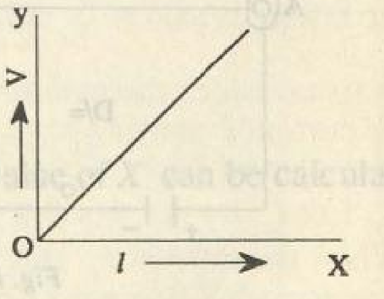


*Fig. 15.11*

This circuit is known as the auxilary circuit of the potentiometer. When the key K is closed, a constant current flows through the wire XY.

A voltmeter V is connected between the ends X and jockey J which can slide to and for along the wire XY. As the jockey is pressed, say at p, the reading of the voltmeter gives the difference of potential between the points X and P.

The potential difference across different lengths of wire are noted by pressing the jockey at different points on the wire. A graph is then plotted between the length $l$ of the wire and the potential difference $V$ across $l$. It will be a straight line (Fig. 15.11).

This is the fundamental principle of potentiometer and can be stated as given below:

"The potential difference across any length of a wire of uniform area of cross-section is directly proportional to its length when a constant current flows through it".

It is an arrangement by which the desired value of potential difference can be obtained.

**Example 15.3**

In a Wheatstone bridge, the ratio arms P and Q are approximately equal. When $R = 500\ \Omega$, the bridge is balanced. On interchanging P and Q, the value of $R$ for balance is $505\ \Omega$. Find the value of X and ratio of P and Q.

**Solution**

Here in the first case, $R = 500\ \Omega$

For a balanced bridge

$$\frac{P}{Q} = \frac{R}{x}$$

$$\frac{P}{Q} = \frac{500\ \Omega}{X}$$

In the second case, the arms P and Q are interchanged, and $R$ becomes equal to 505 Ω then

$$\frac{Q}{P} = \frac{505\ \Omega}{X}$$

Multiplying the last two equations we get

$$\frac{P}{Q} \times \frac{Q}{P} = \frac{500\ \Omega}{X} \times \frac{505\ \Omega}{X}$$

or $\quad X^2 = 500\ \Omega \times 505\ \Omega$

or $\quad X = \sqrt{500\ \Omega \times 505\ \Omega} = 502.5\ \Omega$

Putting

$$X = 502.5\ \Omega$$

in

$$\frac{P}{Q} = \frac{500\ \Omega}{X}$$

we get

$$\frac{P}{Q} = \frac{500\Omega}{502.5\Omega} = \frac{1}{1.005}$$

Thus P : Q :: 1 : 1.005

## 15.4 AVOMETER

Avometer is an electrical device which can be used for the measurement of the current passing through a circuit and the potential difference between any two points. It can also be used to measure the resistance of any conductor. Although an avometer is basically meant to measure direct current and voltages, it can also be used to measure a.c voltages. For this purpose the a.c voltage is first rectified and then provided to the avometer as a d.c voltage.

An avometer consists of a galvanometer provided with a series and parallel combination of resistances, all enclosed in a box. In order to use it as a direct current meter a shunt is brought in contact with it. This shunt is a precision resistor which is connected across the galvanometer movement to bypass a specific fraction of the circuit current around the meter movement. The combination of several shunts provides an extended range to the current meter.

Fig. 15.12 shows a series combination of shunt resistances $R_1$, $R_2$ and $R_3$ called Aryton or Universal shunt, connected in parallel with the meter movement, which can be used for the measurement of current in different ranges.

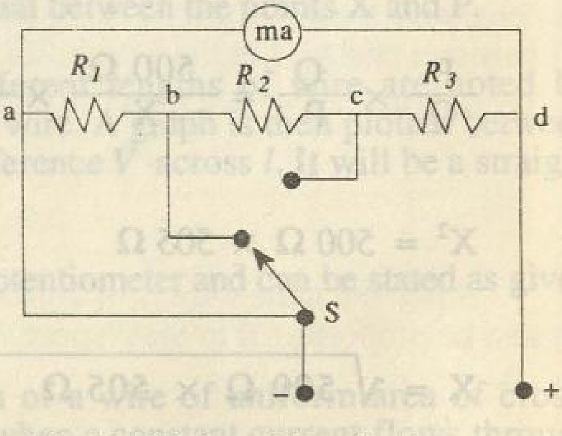


*Fig. 15. 12 Universal shunt for three ranges*

This circuit provides a safe method of switching between current ranges without any danger of excessive current through the meter movement. The wide contact in switch arm indicates that it makes the next connection before breakng the old contact. This shunt circuiting type switch protects the meter movement by providing shunt at all times during the switching over to various ranges.

In addition to its use as a current meter, an avometer can also be used for measuring voltage. This is done by the addition of a high resistance with the galvanometer. The series resistance must be much higher than the coil resistance to limit the current through the coil. This combination of the meter movement with its added series resistance thus forms a voltmeter. The series resistor, called a multiplier, is usually connected inside the voltmeter box.

An avometer, when used as voltmeter has several ranges. A range switch selects one multiplier for the required scale. The higher the voltage range, the higher the multiplier resistance.

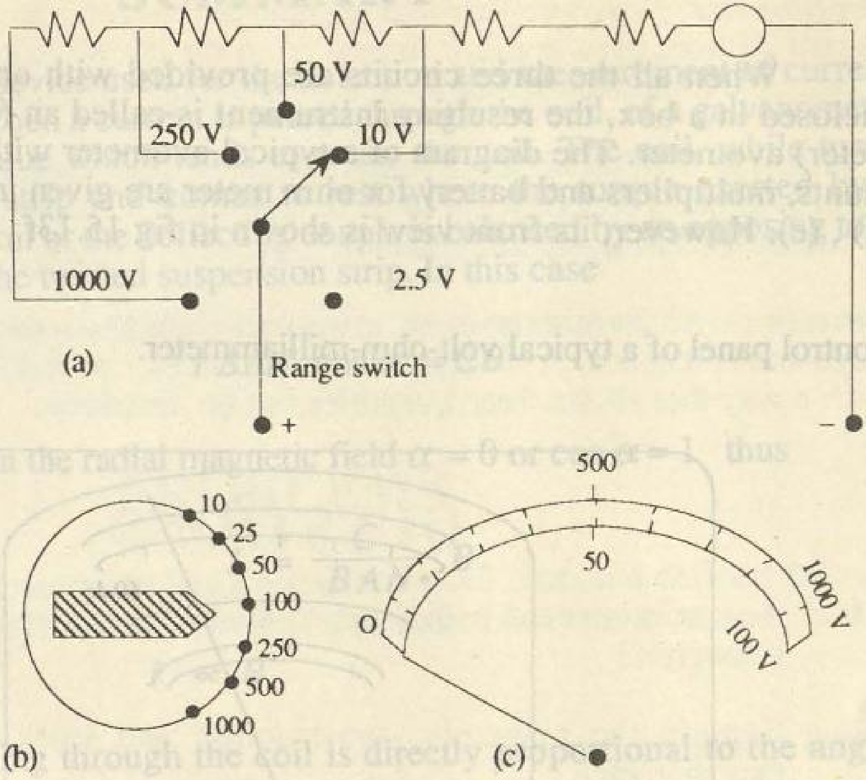


*Fig. 15 .13 (a) Typical voltmeter circuit (within avometer*
*Fig. 15 (b),(c) Range switch.*

Fig. 15.13a shows a typical multiple range switch arrangement.

The avometer can be used to measure resistance by connecting an internal battery with the galvanometer and the current limiting resistance. The circuit then called an ohm meter as shown in Fig. 15.13 (d) and 15.13 (e) The leads of the ohmmeter are connected across the resistance to be measured with power off in the circuit being tested. In this case only the ohmmeter battery supplies current to the galvanometer for deflecting its coil. Since the amount of current through the galvanometer depends upon the external resistance, we can calibrate the scale in ohms. The amount of deflection on the ohms scale indicates directly the magnitude of the resistance. The ohm meter reads up scale regardless of the polarity of the leads because the polarity of the internal battery determine the direction of current through the galvanometer. Commercial avometers provide resistance measurements from less than one ohm up to many megaohms.

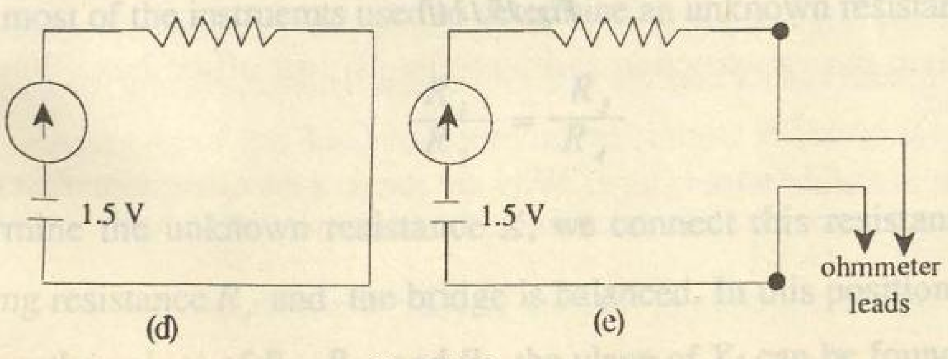


*Fig. 15.13*

When all the three circuits are provided with one galvanometer, and are enclosed in a box, the resulting instrument is called an (ampere, volt and ampere meter) avometer. The diagram of a typical avometer with back cover off to show shunts, multipliers and battery for ohm meter are given in fig. (15.13) (a), (b), (c), (d) , (e). However, its front view is shown in fig 15.13f.

Control panel of a typical volt-ohm-milliammeter.

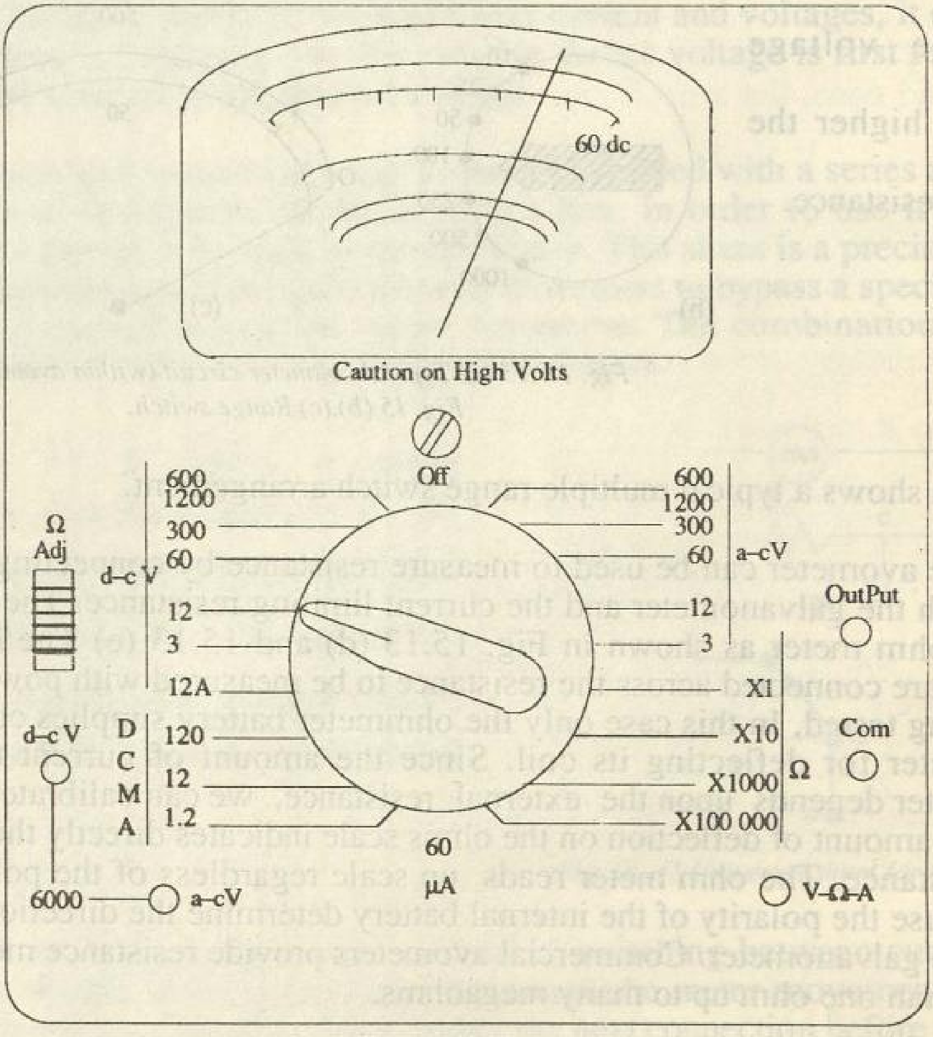

60 dc
Caution on High Volts
Off
Ω
Adj
d-c V
600
1200
300
60
12
3
12A
d-c V
D
C
M
A
120
12
1.2
6000
a-cV
60
μA
600
1200
300
60
a-cV
12
3
X1
X10
X1000
X100 000
Ω
OutPut
Com
V-Ω-A

Fig: 15.13 (f)

# SUMMARY

***Galvanometer :*** Any device used for the detection and measurement of current is called galvanometer. When a current is passed through the coil, of a galvanometer it is acted upon by a torque which tends to rotate the coil. The coil, while turning twists the suspention strip and comes to rest where the torque exerted by the magnetic field on the coil or the deflecting couple is balanced by an opposing torque due to the reaction of the twisted suspension strip. In this case

$$I\,BAN\ \cos\alpha = C\theta$$

As the coil is situated in the radial magnetic field $\alpha = 0$ or $\cos\alpha = 1$ thus

$$I = \frac{C}{BAN}\,\theta$$

or

$$I \propto \theta$$

Thus the current passing through the coil is directly proportional to the angle of deflection.

***Ammeter and Voltmeter :*** Ammeters and voltmeters are essentially galvanometer in which suitable modifcations have been made. To convert a galvanometer into an ammeter a suitable low resistance called shunt is connected in parallel to the galvanometer. The shunt resistance $R_s$ is given as below

$$R_s = \frac{I_g}{(I - I_g)}\,R_g$$

For the converstion of a galvanometer into a voltmeter reading upto a maximum of $V$ volts, a high resistance $R_h$ is connected in series with the galvanometer. The resistance $R_h$ is calculated by using the relation $R_h = \frac{V}{I_g} - R_g$, where $I_g$ is the current through the galvanometer and $R_g$ is the galvanometer resistance.

***Wheatstone Bridge :*** This is an electrical circuit arrangement which forms the basis of most of the instruemts used to determine an unknown resistance.

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

to determine the unknown resistance $X$, we connect this resistance in the arm containing resistance $R_4$ and the bridge is balanced. In this position $X = \frac{R_3}{R_1}\ R_2$

Knowing the values of $R_1$, $R_2$, and $R_3$ the vlaue of $X$· can be found.

***Potentiometer :*** A very simple instrument which can measure and compare potential without drawing any current from the circuit is known as potentiometer. The principle of potentiometer is:

The potential difference across any length of a wire of uniform area of across-section is directly proportional to its length when a constant current flows through it. It can be used as a potential divider.

***Avometer :*** An avometer is an instrument which is used to measure current, voltage and resistance. It consists of a galvanometer with different scales graduated in such a way that all the three quantities can be measured.

# QUESTIONS

**15.1** A wire is covered with a carpet, how will you find any current is passing through it?

**15.2** Explain the principle, construction and working of a moving coil galvanometer.

**15.3** Explain, what is meant by the sensitivity of a galvanometer?

**15.4** A sensitive galvanometer can not be stable. Explain, why?

**15.5** What is shunt resistance? Explain how it is used to convert a galvanometer into an ammeter?

**15.6** What is a voltmeter? Explain how a galvanometer is converted into a voltmeter.

**15.7** Why is a voltmeter connected parallel to the points between which the voltage is to be measured?

**15.8** Explain the principle of Wheatstone bridge.

**15.9** When the bridge is balanced, why does no current flow through the galvanometer?

**15.10** Explain the construction and working of post office box. Why is it so called?

**15.11** What is a slide wire bridge? Why are the lengths of the wire taken instead of its resistances while calculating the unknown resistance?

**15.12** Explain the principle of a potentiometer. Mention its uses.

**15.13** Describe how can the potential difference be measured with the help of a potentiometer.

**15.14** What is an avometer? Describe briefly its various functions.

**15.15** A voltmeter can not read the exact e.m.f of a cell. Why?

**15.16** Why should the potentiometer wire be long and uniform in diameter?

**15.17** Does an ordinary voltmeter measure the potential difference across a resistor correctly?

# PROBLEMS

**15.1** A certain galvanometer has a resistance of 40 ohm and deflects full-scale for a voltage of 100 mV across its terminals. How can it be made into 2-A ammeter?

[ Ans : Shunt resistance = 0.05 Ω ]

**15.2** An ammeter is connected in a circuit containing a battery and two resistors in series whose values are 150 Ω and 250 Ω. If the internal resistance of the ammeter is 50 Ω and it can use 3.0 A, calculate the actual current when the ammeter is absent.

[ Ans: 3.375 A ]

**15.3** A certain galvanometer has a full scale deflection for 0.003 amp current. How can this galvanometer be converted into 90–V voltmeter? The resistance of galvanometer is 20 ohm.

[ Ans: 29.980 Ω ]

**15.4** Two resistances each of 3.0 kΩ are connected in series with each other and then to a battery of e.m.f 6.0 V and internal resistance $r = 0.6\ \Omega$. An ammeter of internal resistance 0.30 Ω measures the current and at the same time a voltmeter with internal resistance 12 kΩ measures the voltage across one of the 3 kΩ resistors in the circuit. What do the ammeter and voltmeter read?

[ Ans: 1.11 mA, 2.664 V ]

**15.5** If the resistance in three successive arms of a balanced bridge be 1, 2 and 36 ohms, respectively, calculate the resistance of the galvanometer in the fourth arm.

[ Ans : 72 Ω ]

**15.6** A resistance of 10 ohm is placed in the right gap and one of unknown value is palced in the left gap of metre bridge. The null point is found to be at 40 cm. Find the unknown resistance.

[ Ans : 6.66 Ω ]

# 16

# ELECTRONICS

## 16.1 ELECTROMAGNETIC WAVES

It is known from Farady's law that a changing flux gives rise to an induced electromagnetic force. We consider magnetic flux $\phi$ changing at a rate $\frac{\Delta\phi}{\Delta t}$. The magnetic field due to $\phi$ is assumed to be uniform and restricted in an are $A$ as a simple case. The induced emf or potential difference in a conducting loop surrounding $A$ and concentric with this will be given by

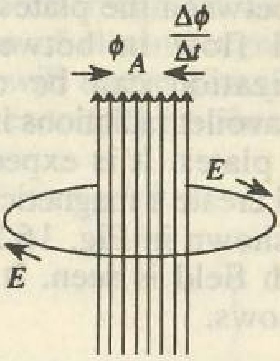


*Fig 16.1*

*The changing magnetic flux of induced electric field.*

$$e = \frac{\Delta\phi}{\Delta t} \qquad 16.1$$

This induced e.m.f will generate an electric field at each point of the loop which will be the same at each point as shown in Fig. 16.1. This field shown as $E$ will cause the electrons of the loop to move. Even if the loop is not present, any charged particle will experience a force due to this field and move in a circle. The direction of $E$ will be reversed if that of $\phi$ is reversed and also if $\frac{\Delta\phi}{\Delta t}$ changes sign.

Now if a unit positive charge is circulated once round the loop, the work done will be $2\pi r. E$ where $r$ is the radius of the loop. The work done is equal to the emf in the loop. Thus

$$e = 2\pi rE$$

Or

$$E = \frac{1}{2\pi r} e$$

Substituting from Eq. 16.1 we get

$$E = \frac{1}{2\pi r} \frac{\Delta\phi}{\Delta t} \qquad 16.2$$

**This equation shows that the electric field intensity will decrease inversely with the distance $r$ from the region of changing magnetic flux.**

In another case, consider an electric field $E$ between two plates charged positively and negatively as shown in Fig. 16.2. The plates are made circular for symmetrical and uniform field. If the gas/air in between the plates is ionized, a current will flow in between the plates. The ionization can be caused by allowing ultravoilet radiations into the space between the plates. It is expected that this current will create a magnetic field round the plates as shown in Fig. 16.2. But practically, no such field is seen. It can be explained as follows.

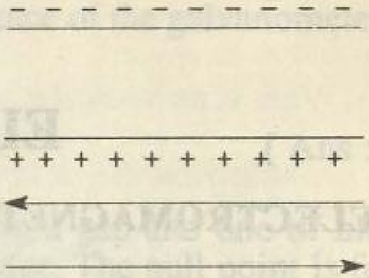


*Fig. 16.2 Magnatic flux due to ionization*
*Magnet flux due to changing electric flux*

The flow of current between the plates changes the electric flux due to electric field. The change of electric flux develops a magnetic field round the plates which being in opposite direction cancels the magnetic field due to the ionization current. The magnetic field can be calculated by considering the rate of change of electric flux to be equivalent to current $I$ given by Gauss's Law

$$I = \varepsilon_o \frac{\Delta\phi_e}{\Delta t} \tag{16.3}$$

where $\varepsilon_o$ is the permittivity of air and $\Delta\phi_e$ is the electric flux. The equivalent current is shown in Fig. 16.3, $\Delta\phi_e$ is uniform and restricted to an area. The intensity of the magnetic field $B$ at distance $r$ from current $I$ is given by Ampere's law. So,

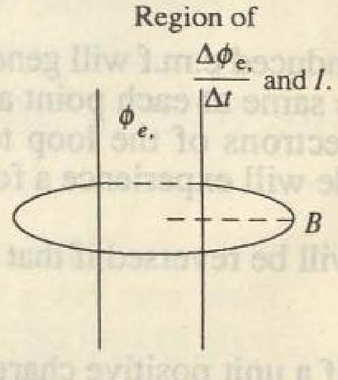


*Fig 16.3*

$$B = \frac{\mu_0}{2\pi r} . \mathrm{I} \tag{16.4}$$

where $\mu_0$ is permeability of air/vacuum Substituting from equation 16.3

$$B = \frac{\mu_0}{2\pi r} \times \varepsilon_0 \frac{\Delta\phi_e}{\Delta t} \tag{16.5}$$

Maxwell had proved mathematically that these two phenomenona lead to radiation of energy in space in the form of electromagnetic waves. It was much before its experimental discovery. He showed that the velocity of these waves is given by $\frac{1}{\sqrt{\varepsilon_o \mu_o}}$ in space.

One can conclude from the preceding discussion that a changing electric flux creates magnetic field which in turn creates electric field and electromagnetic waves are generated. Thus fundamental requirement for generation of electromagnetic waves is an electric charge with changing velocity since it will create changing electric flux. The velocity of an oscillating charge as it moves to and fro along a wire is always changing.

### 16.1.1 Production Of Electromagnetic Waves

A transmitting antenna is a good example of a system generating electromagnetic waves through an oscillating charge. An antenna is a long loop of wire charged by an alternating source of potential, say, of frequency $f$ and time period $T$ as shown in Fig. 16.4.

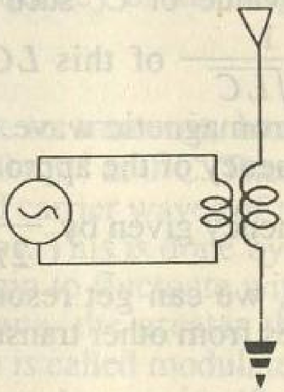


*Fig. 16.4 An antenna*

As the charging potential alternates, so does the charge on antenna. The dimensions of the antenna are adjusted for this purpose. If the charge on the top of the antenna is $+q$ at any time, the charge at this point will be $-q$ after time $\frac{T}{2}$. This reversal of charges occurs due to accelerated motion of charges which creates electric flux changing with frequency $f$. The changing electric flux gives rise to electromagnetic waves that propagate away from the antenna. The frequencies of such electromagneitc waves are equal to the frequencies of the sources generating them. The oscillating magnetic field produced by changing electric field is at right angle to it (the changing electric field). The direction of propagation of the waves is perpendicular to the electric field and magnetic field. (Fig. 16.5).

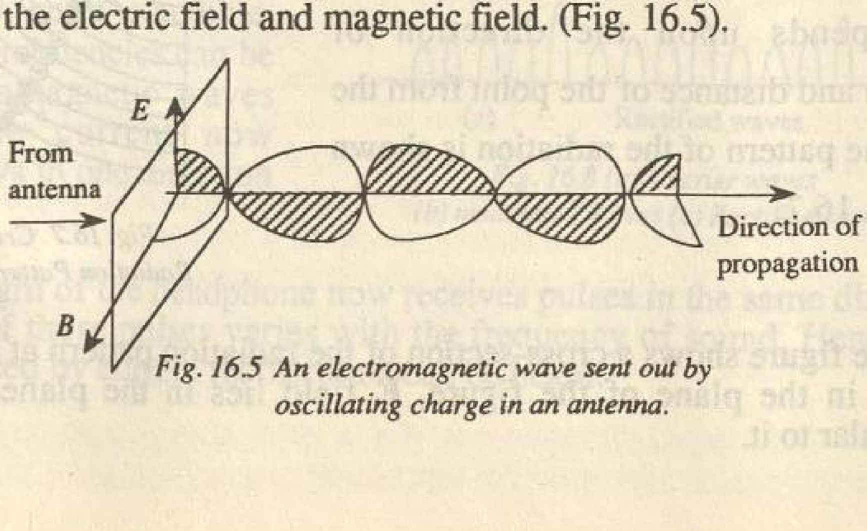


*Fig. 16.5 An electromagnetic wave sent out by oscillating charge in an antenna.*

If such a wave is intercepted by a conducting wire, the oscillating electric field will develop an oscillating voltage in it, (Fig. 16.6).

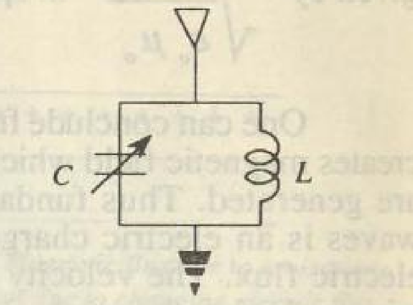


*Fig. 16.6 Oscillating electric field will develop oscillating voltage.*

The frequency of the voltage so developed will be equal to the frequency of the wave. There can be many transmitting stations producing electromagnetic waves of different frequencies simultaneously. We can make arrangements to receive one of these stations by connecting inductance $L$ and a variable capacitor of capacitance $C$ in parallel as shown in Fig. 16.6. We adjust the value of $C$ such that the natural frequency $\frac{1}{2\pi\sqrt{LC}}$ of this $LC$ circuit is equal to the frequency of the incoming electromagnetic wave. At this stage, resonance is said to be achieved between the frequency of the approaching wave and the $LC$ circuit. The response to a particular frequency given by $\frac{1}{2\pi\sqrt{LC}}$ is maximum at resonance. By changing the value of $C$, we can get resonance with other frequencies and receive electromagnetic waves from other transmitting antenna.

### 16.1.2 Radiation From Antenna

The waves from an antenna are expanding spheres which are concentric with centre at the source. At points away from the source, the vector **E** of the electric field and **B** of the magnetic field are perpendicular to each other and also to the direction of propagation from the source. The wave is transvervse. The intensity of the wave at a point depends upon the direction of generation and distance of the point from the source. The pattern of the radiation is shown in the Fig. 16.7.

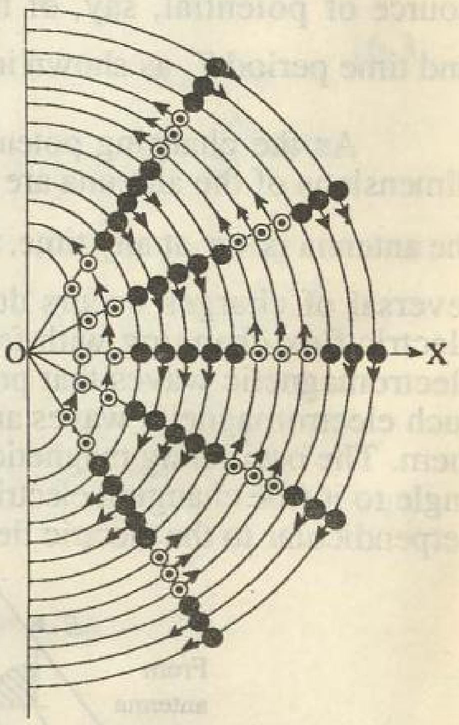


*Fig. 16.7 Cross-Section of Radiation Pattern at any instant*

The figure shows a cross-section of the radiation pattern at one instance. At all points in the plane of the figure, **E** field lies in the plane and **B** field is perpendicular to it.

The electromagnetic waves can be reflected by conducting surfaces. When the surface is large compared to the wavelength of the radiation, the reflection behaves like the reflection of light rays from a mirror. Large parabolic mirrors, several metres in diameter, are used as transmitting, as well as receiving antennas for microwave communication signals. The transmitting surface produces a wave that radiates in a narrow, well defined beam. The receiving surface gathers wave energy over its whole area and reflects it to the focus of the parabola. At the focus, there lies a detecting device.

### 16.1.3 The Transmission of Information and its Reception

The electromagnetic waves emitted by the *L C* cirucit of aerial of a station are of constant amplitude (Fig. 16.8 a). These waves have frequency of the range of $10^6$ Hz, travelling with the velocity of light i.e., $3 \times 10^8$ ms$^{-1}$

The sound waves, or the information that we want to transmit, have frequencies several thousands hertz and travel with a speed 330 ms$^{-1}$ at 0°C. These sound waves are superimposed on electromagnetic waves called carrier waves of an aerial, The sound waves are converted into electrical oscillations. This is done by a microphone. The amplitude of the electro magnetic waves begin to fluctuate with the frequency of the sound waves. The louder the sound wave, the greater the fluctuation in the amplitude of the electro magnatic wave which is called modulated wave (Fig. 16.8 b). The effect of sound wave, but not the sound wave itself, is carried by the electromagnetic waves which are therefore, called modulated waves.

The modulated electromagnetic waves falling on an aerial, induce a current of frequency equal to the frequency of the electromagnetic wave. If this current is passed through a headphone, the diaphragm of the headphone will not vibrate with this high frequency. Even if it does, normal human ear will not respond to it because only the frequency of the range 20 to $20 \times 10^3$ hertz is audible. In order that these frequencies can be audible, the electromagnetic waves are rectified. The current now fluctuates but it flows in one direction (Fig. 16.8 c).

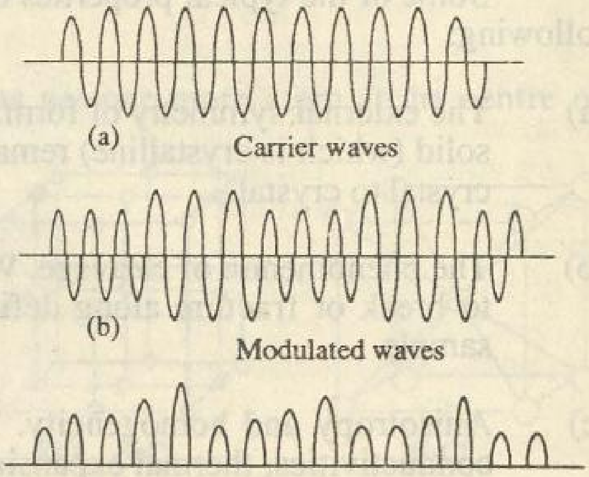


*Fig. 16.8 (a) Carrier waves*
*(b) modulated Waves (c) Rectified waves*

The diaphragm of the headphone now receives pulses in the same direction but the amplitude of these pulses varies with the frequency of sound. Hence, the sound waves produced by current rectification can be heard.

A crystal diode is used for rectification of the electromagnetic waves as shown in Fig. 16.9. The electromagnetic waves are received by an aerial of inductance $L_1$ coupled to an other tuning cirucit containing inductance $L_2$ and a variable capacitor $C_2$. The capacity of $C_2$ is varied so that the frequency of the tuning circuit $L_2$ $C_2$ is equal to the frequency of the incoming electromagnetic waves.

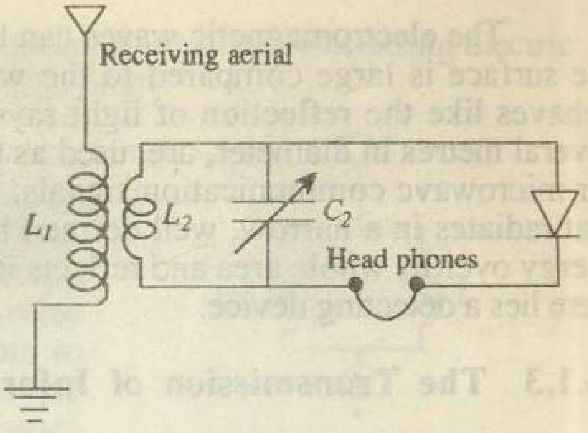


*Fig 16.9 Crystal diode used for rectification*

The current in the tuning circuit at this stage rises to maximum. The diode rectifies the current and makes it unidirectional and positive relative to the cathode and this current passes through the headphone. The amplitude of this current varies with the frequency of the sound wave. Thus the original sound is reproduced and heard.

## 16.2 CRYSTAL STRUCTURE

Matter is compsoed of atoms and molecules. In solids, they are tightly bound to positions close to each other and they are arranged in specific ways. This gives a solid, a form called crystal structure. The theory of crystal structure was worked out much before the discovery of x-rays which made possible the experiments on crystal structure.

Some of the typical properties of crystal structure, easily observed, are the following:

(a) The external symmetry of form. The angular positions of typical faces of a solid (which is crystalline) remain the same for a sample. It may vary from crystal to crystal.

(b) The phenomenon of cleavage. When a crystal is subjected to stress, it tends to break or fracture along definite direction which is characteristic of a sample.

(c) Anisotropy and homogeneity. The properties like thermal and electrical conductivities, thermal expansion and velocity of propogation of light etc; within a crystal depend upon the direction considered. These properties are different in different directions but the same in direction parallel to each other. This is called anisotropic characteristic of a single crystal. It is due to this property that the size or shape of a lattice is not important. In case the density of atoms or moecules does not vary from direction to direction, the crystal is said to be homogeneous and the property is known to be homogneity.

In the internal arrangement of a crystal, a minimum number of particles is always arranged in a form which represents the entire crystal. The whole crystal is built up by the repetition of this basic structure. This fundamental pattern is called unit cell or lattice unit. While repeating the pattern there is a regular distance, called space lattice, between the atoms, molecules or their groups. In a space lattice, all the points may be included in a set of parallel and equally spaced planes known as lattice planes.

It has been found from symmetry considerations that there are 14 possible ways of space lattice. Only three simple ones are shown in Fig. 16.10 (a)

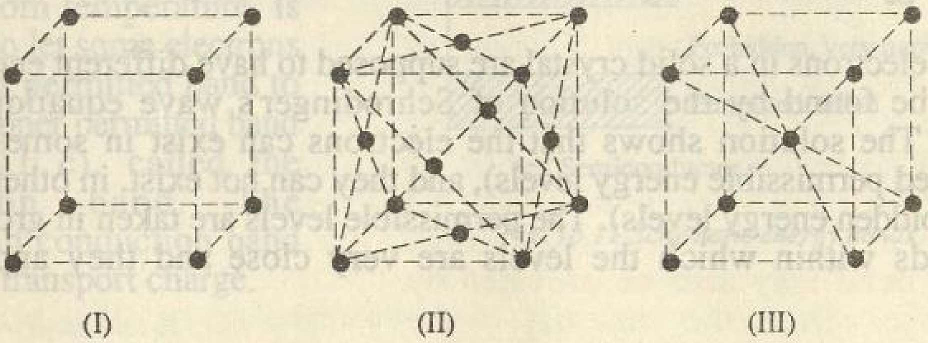


*Fig 16.10 (a)*

*(i)* *Simple Cubic*

Here one atom or molecule lies at each of the eight corners of a cube.

*(ii)* *Face Centred Cubic*

It is a cubic pattern having one extra atom or molecule at the centre of each of the six faces of the cube.

*(iii)* *Body Centred Cubic*

In this case the pattern has got one more atom at the centre of a simple cube.

The examples of these simple crystals are those of KCl, NaCl and CsCl as shown in Fig. 16.10 (b).

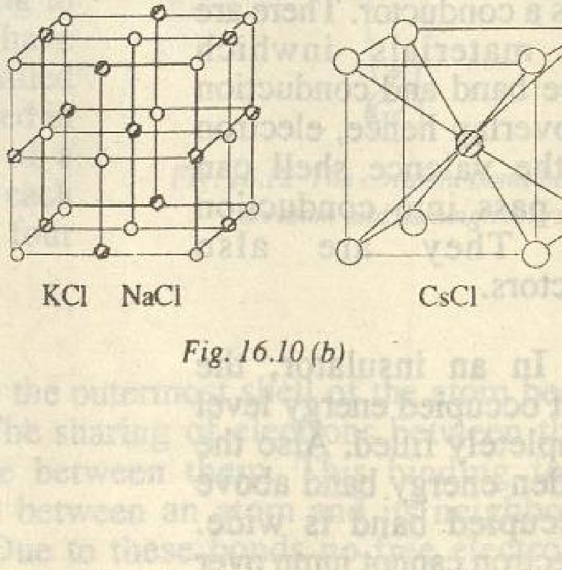


*Fig. 16.10 (b)*

### 16.2.1 Classification of Solids

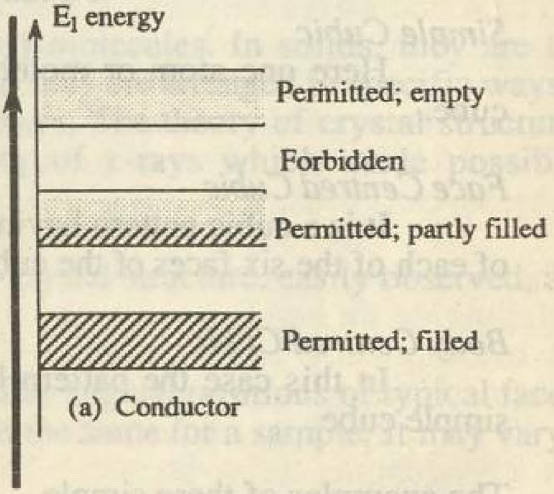
### (a) Free Electron Theory.

It has been discussed in the above section that atoms and molecules of a solid are arranged in a specific way inside a crystalline solid. These atoms have electrons revolving around the nucleus of the atom. The electrons in the outer most orbits of atoms in a metallic conductor are loosely bound with the nucleus as they experience the least attractive force to the nucleus. A small amount of energy shell let these electrons leave the atom. These electrons can 'wander' in the solid and are called free electrons. When a potential difference is applied to a solid (metal), these electrons travel along a directed path, and a current begins to flow. This theory could not explain completely the behaviour of conductors, insulators and semiconductors. It is for the explanation of conduction in the three categories of solids that band theory had been developed.

### (b) Energy Band Theory

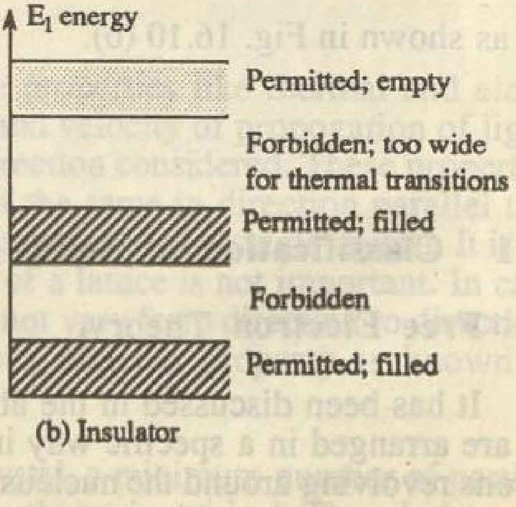
The electrons in a solid crystal are supposed to have different energy levels which can be found by the solution of Schrodinger's wave equation in wave mechanics. The solution shows that the electrons can exist in some ranges of energy (called permissible energy levels), and they can not exist, in other ranges of energy (forbidden energy levels). The permissible levels are taken in groups called energy bands within which the levels are very close and they appear to be continuous.

The energy bands give us an easy and satisfactory explanation of the electrical classifcation of solids. In case the highest energy band is unfilled (Fig. 16.11 a) these electrons can be excited from lower to higher energy level within the band. This is possible on application of even weak electric fields. A material with such an energy band is a conductor. There are some materials inwhich valence band and conduction band overlap hence, electron from the valence shell can easily pass into conduction band. They are also conductors.



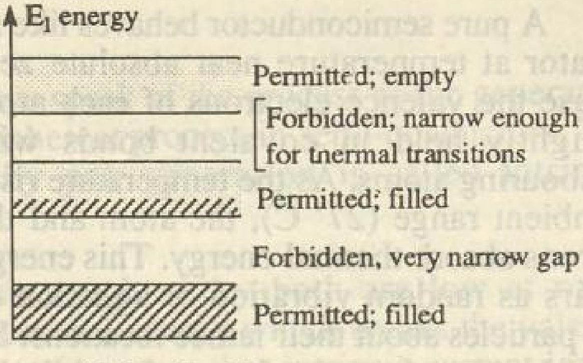
(a) Conductor

In an insulator, the highest occupied energy level is completely filled. Also the forbidden energy band above the occupied band is wide. The electron cannot jump over from lower to upper permissible energy levels ordinarily (Fig. 16.11 b).



(b) Insulator

Another category of solids is semi-conductor. Here the gap between the filled energy band and next higher permitted energy band is small. The themral energy, even at room temperature, is sufficient to let some electrons from lower permitted band to go to the upper permitted band (Fig. 16.11 c), called the conduction band. The electrons in conduction band are free to transport charge.


$E_1$ energy
Permitted; empty
Forbidden; narrow enough for thermal transitions
Permitted; filled
Forbidden, very narrow gap
Permitted; filled

(c) Semiconductor

*Fig. 16.11 Schematic energy bands*

## 16.3 SEMICONDUCTORS

The substances with resistivity of the order of $10^{-4}$ ohm – metre are good conductors of electricity. The value of resistivity for insulator is of the order of $10^{8}$ ohm – metre. There are certain substances which have intermediate values of resistivity. They are known as semi-conductors. The examples of such substances are the crystals of Germanium and Silicon which belong to fourth group of periodic table. They have four electrons in their outermost shell called valence electrons. These elements are used in semi-conductor devices (in the form of a single crystal). The valence electrons of each of these atoms are shared in pairs with four adjacent atoms as shown in Fig. 16.12.

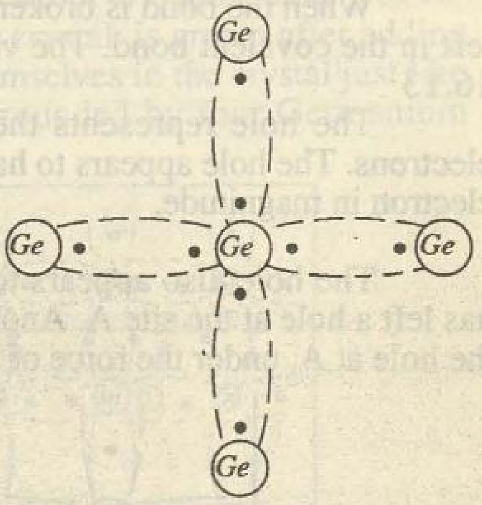


*Fig. 16.12 The covalent bond between an atom and its neighbours*

Thus the number of electrons in the outermost shell of the atom becomes eight to form a stable configuration. The sharing of electrons between the two atoms gives rise to the binding force between them. This binding force is responsible for forming covalent bond between an atom and its neighbours as shown by dotted lines in Fig. 16.12. Due to these bonds no free electrons are available as are available in metals. Thus the value of conductivity of the crystal is very low. It can be increased by doping these crystals with atoms of group III and V as explained in the following.

### 16.3.1 Conduction in Silicon and Germanium (Electrons and Holes)

A pure semiconductor behaves like an insulator at temperature near absolute zero because the valence electrons of each atom are tightly held in covalent bonds with neighbouring atoms. As the temperature rises to ambient range (27° C), the atom and the electrons absorb thermal energy. This energy appears as random vibration or agitation of these particles about their lattice locations. So some electrons become free and mobile by acquiring sufficient energy to break the covalent bond. The energy required to break a covalent bond may also be provided by high voltage across the material or by exposing the material to photons of proper wavelength.

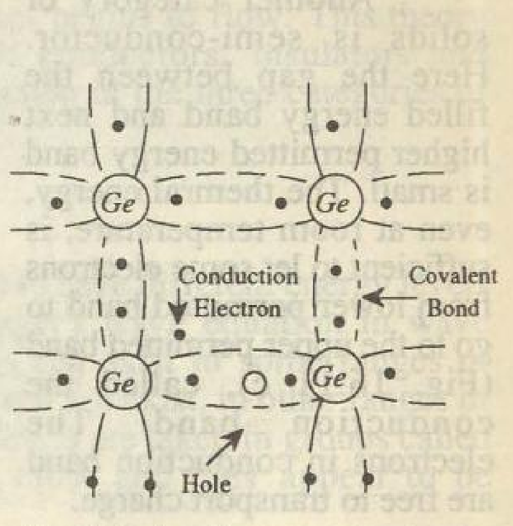


*Fig. 16.13 Breaking of covalent bond in a germanium crystal*

When the bond is broken and an electron is free thereby, electron vacancy is left in the covalent bond. The vacant electron site is called a hole as shown in Fig. 16.13

The hole represents the absence of negative charge and is attractive to electrons. The hole appears to have a positive charge equal to the negative charge of electron in magnitude.

The hole also appears to be mobile. In Fig. 16.14 a broken covalent bond has left a hole at the site A. Another electron may move from a bond at the site B to the hole at A, under the force of a small voltage applied across the material.

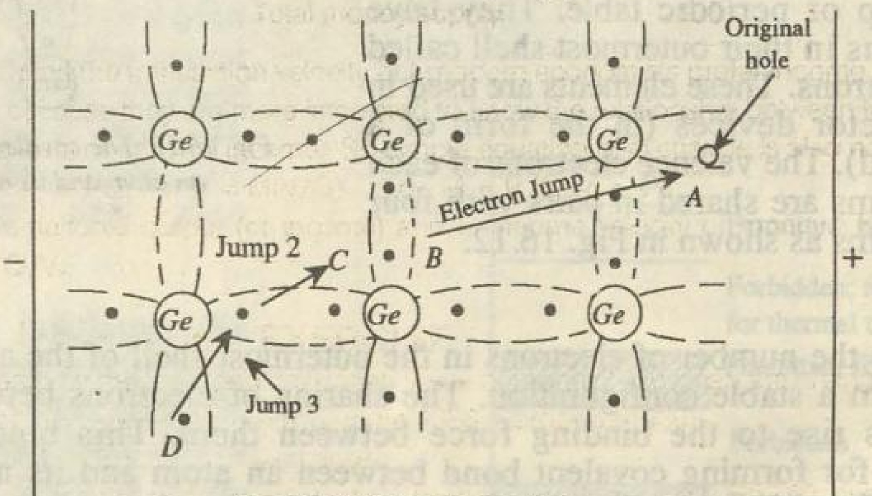


*Fig. 16.14 Holes are moving*

When the electron fills the hole at the site A, then there is now a hole at the site B. Then another electron may move from a bond at the site C to the hole at the site B and the hole thus moves to the site C. Similarly, an electron may move from a bond at the site D to hole at the site C and so the hole has moved from A to D.

The holes and electrons are both mobile charges that can take part in electrical conduction in semi-conductor materials. The movement of holes corrsponds to conventional current.

When a covalent bond is broken, we speak of the process as the generation of electron-hole pair. This is natural and inherent process in semi-conductors. The conduction due to charges produced by pair generation is called intrinsic conduction.

The pair production and intrinsic conductivity both are low at room temperature because of the low value of thermal energy which breaks the valence bonds. The process is so sensitive to the temperature that intrinsic current levels may be expected to double for every increase of 10° C limiting the use of silicon devices to 200° C and germanium devices to 90° C.

### 16.3.2 n and p Type Substances

In order to obtain the desired conduction properties, the pure Silicon or Germanium is doped with minute amount of selected elements as controlled impurities. The impurities are added at the rates of one atom of doping elment to $10^5$ to $10^8$ atoms of semi-conductor. When a Germanium crystal is grown after adding the Antimony impurity, the impurity atoms lodge themselves in the crystal just like any other atom of Germanium. It will also be surrounded by four Germanium neighbour atoms as shown in Fig. 16.15 (a).

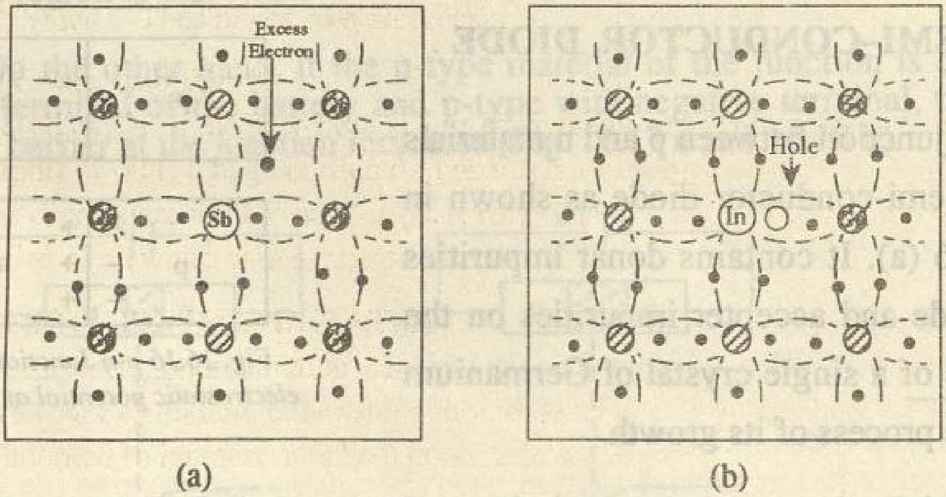


*Fig. 16.15 n – type and p – type substances*

As there is room for only four valence electrons in the covalent bond, the fifth electron of Antimony is left over as shown in Fig. 16.15 (a).

The binding force of this electron to its nucleus is slight and the thermal agitation at room temperature is sufficient to make this electron free. By addition of pentavalent impurities such as Antimony, Phosphorus or Arsenic, we have created a material having only electrons as free and mobile charges. Such a semi-conductor is said to be n-type material. The pentavalent impurities which have donated the electrons are called donar impurities.

In an n-type semiconductor the conduction will be predominantly by electrons and the elctrons are called the majority carriers. There will be a few holes present in n-type material due to electron hole pair generation at usual ambient temepratures. The holes present in an n-type material are known as the minority carriers.

When a trivalent element such as Boron, Indium, or Gallium is added to a pure Germanium or Sillicon as impurity, the three valence electrons of the impurity atom enter the covalent bond with electrons from three neighbouring silicon or Germanium atoms but an electron vacancy or hole is left in the bond to a fourth neighbour atom as shown inFig. 16.15 (b).

At room temeprature, the built-in-hole can be filled by an electron from a nearby bond and this process creates a mobile hole in the adjacent area.

The use of trivalent impurity elements has created a material with free holes, so conduction occurs by hole transfer, and impurity semiconductor is said to be p-type material. The trivalent impurity atoms accept electrons to fill their bond vacancies and are called acceptor atoms and the impurities as acceptor impurities.

The conduction in a p-type material will be primarily due to holes and they are known as majority carriers. There will be electrons present in a p-type material due to thermal pair-generation. These electrons in a p-type material are known as minority carriers.

## 16.4 SEMI–CONDUCTOR DIODE

A junction between p and n materials forms a semi-conductor diode as shown in Fig. 16.16 (a). It contains donar impurities on one side and accepter impurities on the other side of a single crystal of Germanium during the process of its growth.

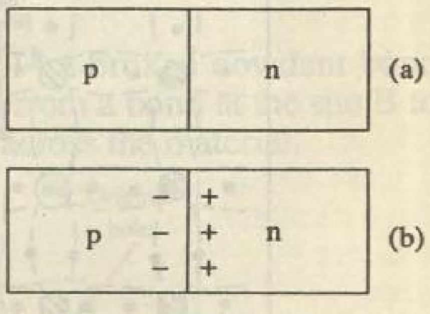


*Fig. 16.16 p-n Junction and the electrostatic potential at its junction*

Initially there are holes (majority carriers) to the left of the junction and free electron to the right (Fig. 16.16 a). A diffusion of electrons and holes takes place across the junction which stops after setting up a potential difference at the junction, the n-type being at a positive potential with respect to p-type as shown in Fig. 16.16 (b).

In the region of the junction, there are no mobile carriers except those that get swept immediately to the opposite side. The p-region holes are annihilated by electrons and n-regions electrons have migrated to p side. Since the charge carriers are removed, this is called depletion region. It is also called barrier region because it acts as a barrier to further diffusion of charges. The width of the depletion region depends on how heavily the p and n materials have been doped.

The semi-conductor diode has the property of one way conduction, i.e., it allows the current to flow only in one direction. The flow of current is practically zero in the opposite direction; it acts as a diode. This can be understood by considering an example of water flow as shown in Fig. 16.17. (a,b) It is clear that the reduction in the height of obstruction or barrier increases the water flow and vice-versa. Thus, if we want to send the current across a p-n junction, we have to reduce the height of the potential barrier at the junction (Fig. 16.17 a).

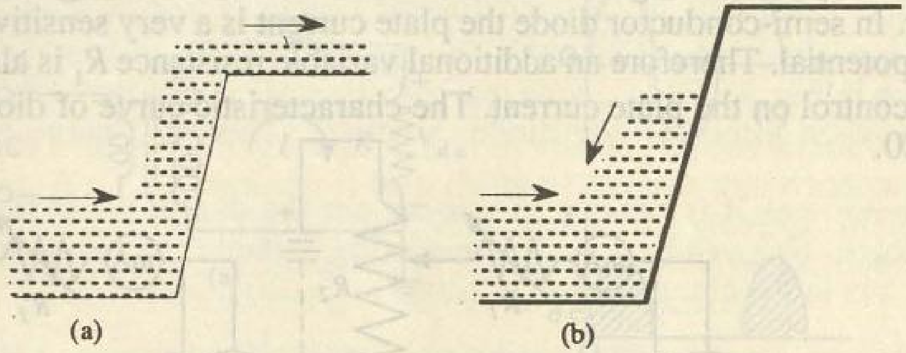


*Fig. 16.17*

This can be done by connecting the p-type material of the junction with positive terminal of the battery and n-type with negative terminal as shown in Fig. 16.18 (a). The height of potential barrier has been reduced and it gives easy way to flow of electric current. In this case, the diode is said to be forward biased. The depletion region is narrowed.

On the other hand, if the n-type material of the junction is connected to positive terminal of the battery and p-type with negative terminal, the height of potential barrier at the junction increases (Fig. 16.17 a and 16.18 b).

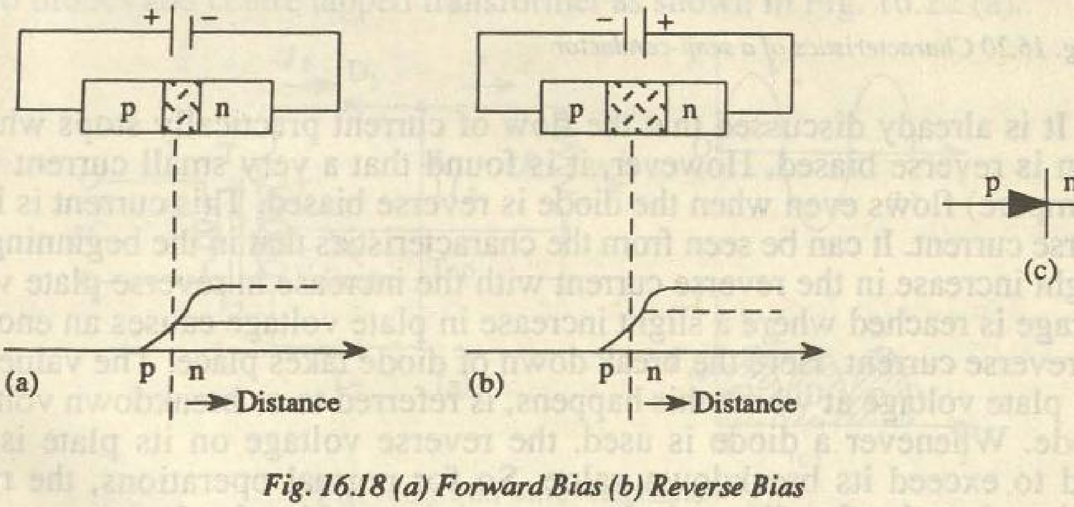


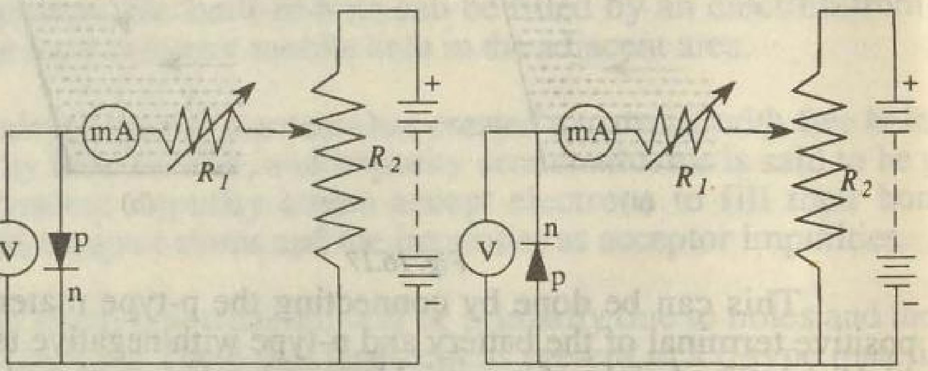
*Fig. 16.18 (a) Forward Bias (b) Reverse Bias*

This makes the flow of current quite difficult across the p-n junction. In this state the diode is said to be reverse biased and practically no current flows in the circuit. The depletion region is widened in this case.

The semi-conductor diode p-type end is usually referred as plate or anode and the n-type end as cathode, although there are no plate and cathode in a semi-conductor diode. The anode is represented by the arrow of the diode symbol and is internally connected to the p-region, while, the cathode is the bar of the symbol and is connected to n-region (Fig. 16.18 c).

### 16.4.1 Characteristic Curve of a Diode

The characteristic of a diode is simply the plot of current as a function of plate potential. The circuits used for the required purpose are given in Fig. 16.19 (a) and 16.19 (b) showing the forward and reverse biasing of the diode, respectively. In semi-conductor diode the plate current is a very sensitive parameter of the plate potential. Therefore an additional variable resistance $R_1$ is also needed to have a fine control on the plate current. The characteristic curve of diode is shown in Fig. 16.20.



(a) (b)

*Fig. 16.19*

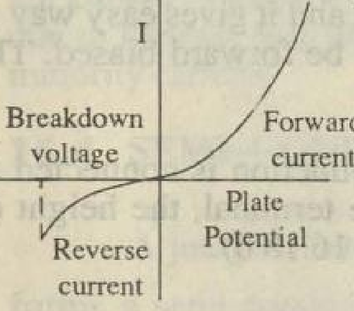


*Fig. 16.20 Characteristics of a semi-conductor*

It is already discussed that the flow of current practically stops when the junction is reverse biased. However, it is found that a very small current (a few microampere) flows even when the diode is reverse biased. This current is known as reverse current. It can be seen from the characteristics that in the beginning there is a slight increase in the reverse current with the increase in reverse plate voltage but a stage is reached where a slight increase in plate voltage causes an enormous rise in reverse current. Here the break down of diode takes place. The value of the reverse plate voltage at which this happens, is referred to as breakdown voltage of the diode. Whenever a diode is used, the reverse voltage on its plate is never allowed to exceed its breakdown value. So for normal operations, the reverse current is so low that for all practical purposes it is considered to be zero.

### 16.4.2 Diode as a Rectifier

In the previous section, we have studied that the diode easily passes current in the forward direction, i.e., when it is forward biased. The flow of current is practically zero in the reverse direction, i.e., when it is reverse biased. It means the diode offers very small resistance (zero in ideal case) in the forward direction but very large resistance in the reverse direction (infinite in ideal case). Thus the diode can act as a switch. It is due to this important property of diode, that it can be used for rectification, i.e., to convert alternating current into pulsating direct current. In most of the electronic devices (Radio, T.V. etc.) the diode serves as a rectifier. The circuit in Fig. 16.21 (a) is used for half-wave rectification.

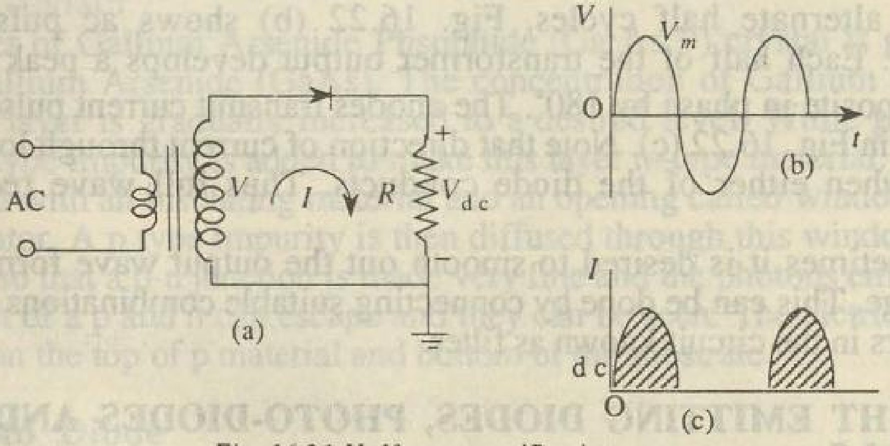


*Fig. 16.21 Half wave rectification*

During the positive half cycle of tranformer voltage, the diode anode is positive and diode acts as a switch. The voltage '*V*' of the transformer Fig. 16.21 b ($V_m$ is maximum) *V* is then directly connected to the load and a current flows in the circuit as a pulse of half-sine wave form as shown in Fig. 16.21 (c). During the next half cycle of the transformer voltage, i.e., negative half cycle, the anode of the diode is negative, and thus it is reverese-biased and behaves like an open circuit. No current exists in the load as shown in Fig. 16.21 (c) and therefore no potential difference appears on the load. The full-wave rectification can also be obtained by using two diodes and centre tapped transformer as shown in Fig. 16.22 (a).

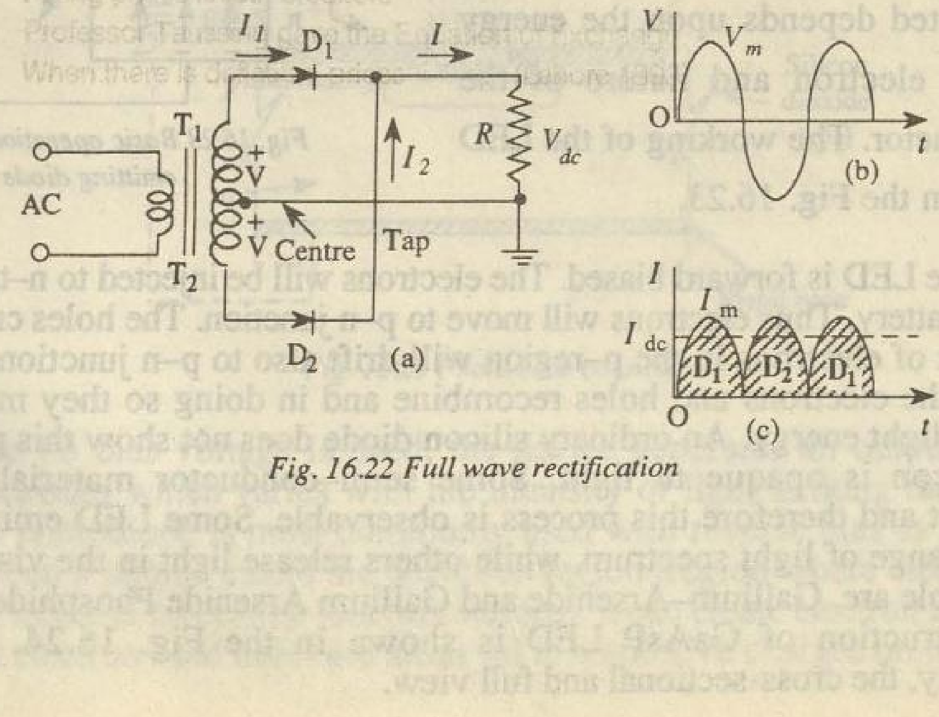


*Fig. 16.22 Full wave rectification*

The end of the transformers winding are connected to two diodes. In the first half cycle, diode $D_1$, is forward biased from $T_1$ and conducts a current pulse to the load. During this time diode $D_2$ has reverse voltage from $T_2$ and appears as an open cirucit. In the second half cycle, the transofrmer polarity reverses and diode $D_2$ is forward biased from $T_2$. It allows a current pulse to the load. At this time diode $D_1$ is reverse biased and open. Thus positive voltages are supplied to the diode anodes on alternate half cycles. Fig. 16.22 (b) shows ac pulses from the transofrmer. Each half of the transformer output develops a peak voltage $V_m$, which is opposite in phase by 180°. The anodes transmit current pulses to the load $R$ as shown in Fig. 16.22 (c). Note that direction of current through load R remains the same when either of the diode conducts. Thus full wave rectification is obtained.

Sometimes it is desired to smooth out the output wave form to a steady direct voltage. This can be done by connecting suitable combinations of capacitors and inductors in the circuit known as filter.

## 16.5 LIGHT EMITTING DIODES, PHOTO-DIODES AND SOLAR CELLS

A light emitting diode basically is a p–n junction diode. Energy is released when electric current passes through such a junction. If the energy released is in form of light (photons), it is called a light emitting diode (LED). The wave length of light emitted depends upon the energy levels of electron and nature of the semiconductor. The working of the LED is shown in the Fig. 16.23.

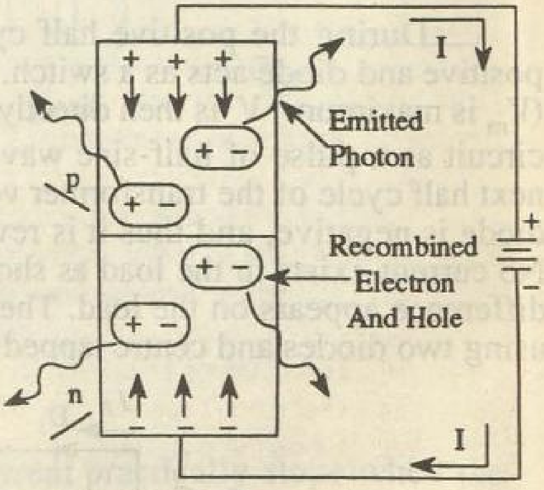
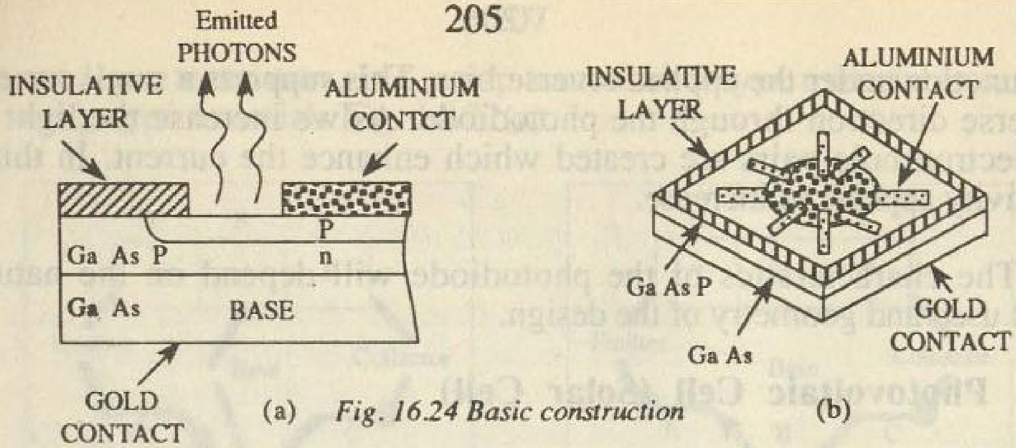


*Fig. 16.23 Basic operation of a light emitting diode*

The LED is forward biased. The electrons will be injected to n–type material from the battery. Thus electrons will move to p–n junction. The holes created by the movement of electrons in the p–region will drift also to p–n junction. At the p–n junction, the electrons and holes recombine and in doing so they may release a photon of light energy. An ordinary silicon diode does not show this phenomenon since silicon is opaque to light. Some semi–conductor materials are semi-transparent and therefore this process is observable. Some LED emit light in the invisible range of light spectrum, while others release light in the visible regions. The example are, Gallium–Arsenide and Gallium Arsenide Phosphide, (Ga As P.) The construction of GaAsP LED is shown in the Fig. 16.24. (a) and (b) respectively, the cross-sectional and full view.


Emitted
PHOTONS
INSULATIVE LAYER
ALUMINIUM CONTCT
P
Ga As P
n
Ga As
BASE
GOLD CONTACT
INSULATIVE LAYER
ALUMINIUM CONTACT
Ga As P
Ga As
GOLD CONTACT

(a) *Fig. 16.24 Basic construction* (b)

A layer of Gallium Arsenide Phsophide (Ga As P) crystal is grown on a surface of Gallium Arsenide (GaAs). The concentration of Gallium Phosphide, (GaP), in the layer is gradually increased to a desired level. While growing the crystal, an n–type impurity is added to make this layer n–type material. The grown layer is coated with an insulating material and an opening called window is etched into the insulator. A p type impurity is then diffused through this window on to n–type material so that a p-n junction is made very fine and the photons emitted on the recombination of a p and n can escape and they can be seen. The elctrical contacts are provided on the top of p material and bottom of the substrate.

### 16.5.2 Photo Diode

A photodiode is a semi-conductor device ususally made from silicon. One type of such a diode uses p–n junction as shown in Fig. 16.25.

A p-type material is diffused on to a n-type substrate. The diffusion occurs through a round window etched into a silicon dioxide layer formed on the top of a n-type material. This window makes electrical contact with the p-type region and serves as an electrode. The window controls the area receiving the light. A metal base is also formed at the bottom of n-type material. This metal layer serves as second electrode.

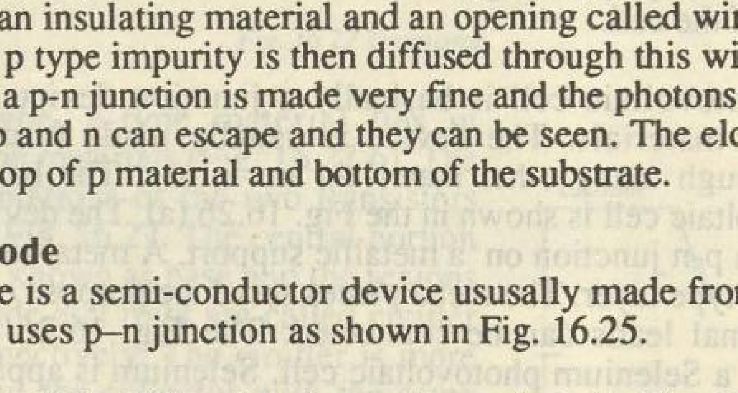

REVERSE BIAS
–
+
Light
Metal window
P
Silicon dioxide layer
n
Metal base

*Fig. 16.25 Photodiode construction*

In case no bias voltage is used, the device generates an output voltage across its electrodes which varies with the intensity of light striking the p-layer. However, the photodiode is most commonly used with reverse bias as shown in Fig. 16.25. When a photon enters the depletion region (region where electron of n and holes of p material meet at np junction surface), it can create electron hole pairs. The separated electrons and holes are attracted to respective charges on the side of the pn junction under the applied reverse bias. This supports a small current flow in the reverse direction through the photodiode. As we increase the light intensity, more electron hole-pairs are created which enhance the current. In this way the conductivity appears to increase.

The characterstics of the photodiode will depend on the nature of the material used and geometry of the design.

### 16.5.3 Photovoltaic Cell (Solar Cell)

It is a device which converts the light energy into electrical energy. When light is allowed to fall on this cell, the cell generates a voltage across its terminals. This voltage increases with increase in the light intensity. The cell is so designed that a large area is exposed to light which enhances the voltage generation across the two terminals of the cell.

The photovoltaic cell is basically a junction device that is made from semiconductor materials. The device is usually made from either Silicon or Selenium although many other materials are in use. The basic structure of the Silicon photovoltaic cell is shown in the Fig. 16.26 (a). The device has p and n type layers making a p-n junction on a metallic support. A metal ring is attached to the outer edge of p-type layer. The metal pieces serve the purpose of electrical contacts to which external leads can be connected. The Fig. 16.26 (b) shows that the construction of a Selenium photovoltaic cell. Selenium is applied to a metal base plate which becomes the positive terminal of the cell. The light sensitive area of this cell will always be negative element. The junction reaction in this case is that between Selenium and the metal base (two dissimilar metals).

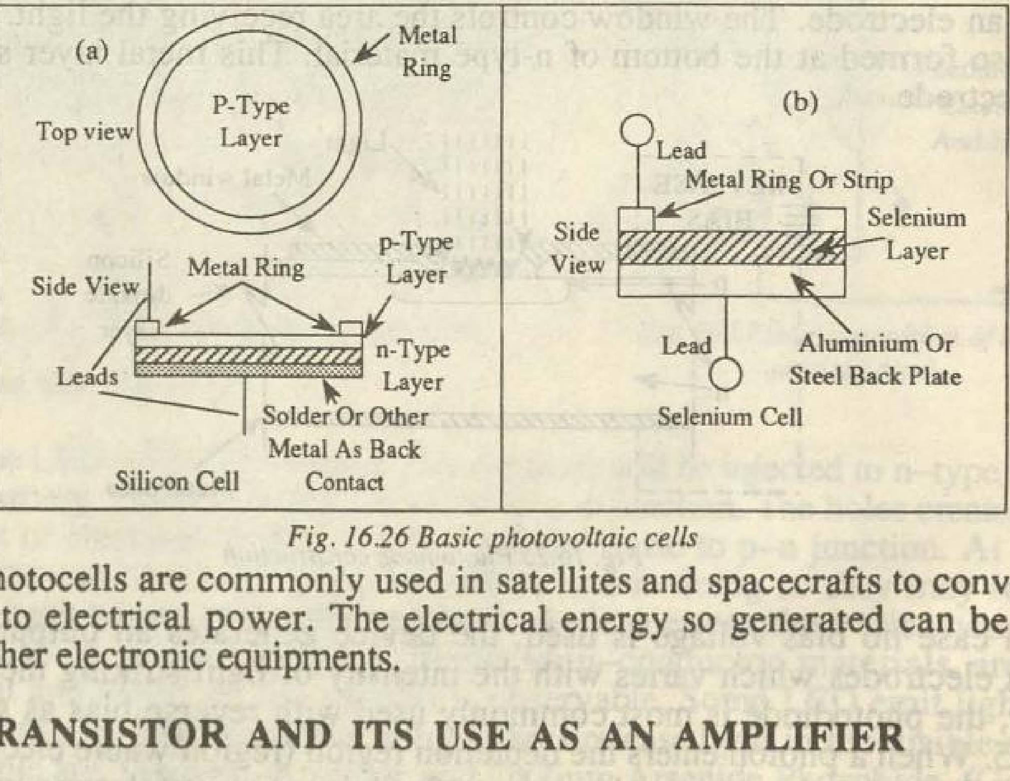

(a)
Metal Ring
P-Type Layer
Top view
Side View
Metal Ring
p-Type Layer
n-Type Layer
Leads
Solder Or Other Metal As Back Contact
Silicon Cell
(b)
Lead
Metal Ring Or Strip
Selenium Layer
Side View
Lead
Aluminium Or Steel Back Plate
Selenium Cell

*Fig. 16.26 Basic photovoltaic cells*

Photocells are commonly used in satellites and spacecrafts to convert solar energy into electrical power. The electrical energy so generated can be used to operate other electronic equipments.

## 16.6 TRANSISTOR AND ITS USE AS AN AMPLIFIER

In the previous section we discussed the diode and its action as rectifier. We shall now consider another important device, the transistor. They are of two types i.e., p-n-p or n-p-n transistors. In a p-n-p transistor, n-type material is sandwiched between two p-type materials Fig. 16.27(a).

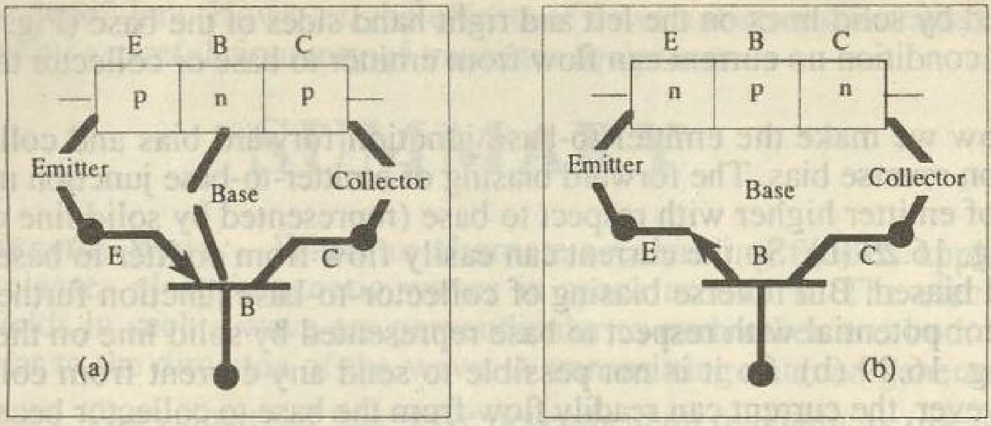

E
B
C
p
n
p
Emitter
Base
Collector
E
C
B
(a)
E
B
C
n
p
n
Emitter
Base
Collector
E
B
C
(b)

*Fig. 16.27 Transistor*

In the second type, p-type material lies in between two n-type materials (Fig. 16.27 b). The symbolic representations of the two transistors are also shown in Fig. 16.27. The central portion of the transistor is known as base and the regions on the left hand side and right are called emitter and collector, respectively. The emitter is more heavily doped than the collector with the same impurities. These three portions of the transistor form two junctions i.e., emitter-base junction (also knows as emitter junction) and collector-base junction (also known as collection junction). For normal operations the emitter base junction is forward-biased and collector base junction is reverse-biased as shown in Fig. 16.28.

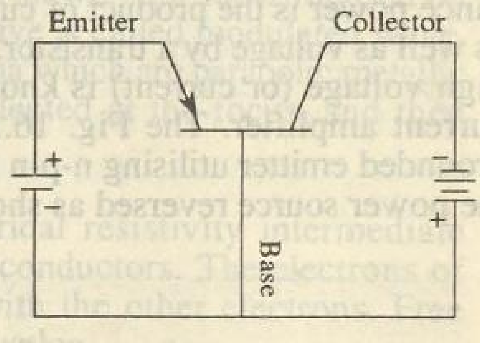

Emitter
Collector
+
−
−
+
Base

*Fig. 16.28 Forward biased and Reverse biased junctiŏn*

The potential barriers (potential hills ) at the two junctions are shown in Fig. 16.29.

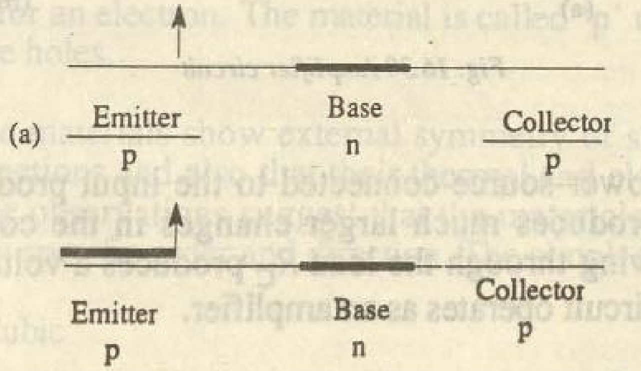
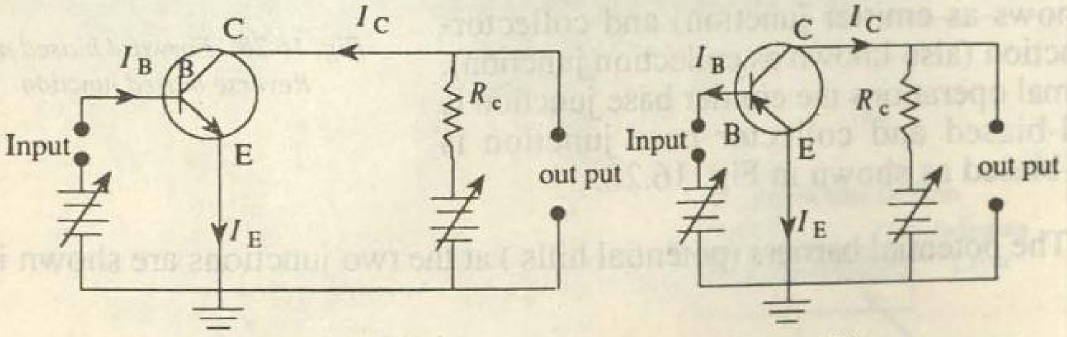

(a)
Emitter
p
Base
n
Collector
p
Emitter
p
Base
n
Collector
p
(b)

*Fig. 16.29 Potential barrier*

In Fig. 16.29 (a) potentials of emitter and collector are shown with respect to base. It is clear that both are at lower potentials with respect to base as represented by solid lines on the left and right hand sides of the base (Fig. 16.29 a). So on this condition no current can flow from emitter to base or collector to base.

Now we make the emitter-to-base junction forward bias and collector-to-base juction reverse bias. The forward biasing of emitter-to-base junction makes the potential of emitter higher with respect to base (represented by solid line on left of base in Fig. 16.29 (b). So the current can easily flow from emitter to base when it is forward biased. But reverse biasing of collector-to-base junction further lowers the collector potential with respect to base represented by solid line on the right of base in Fig. 16.29 (b). So it is not possible to send any current from collector to base. However, the current can readily flow from the base to collector because now it is to slip down the potential hill. As the emitter current enters into the base region, nearly the whole of it slips down the base to collector potential hill. Just as water gains power when flowing down a slope, similarly the current when it slips down the potential hill between the base and collector, gains considerable electrical power. Since power is the product of current and voltage, we can amplify both the current as well as voltage by a transistor. The process to convert low voltage (or current) to high voltage (or current) is known as amplification. The transistor is basically a current amplifier. The Fig. 16.30 (a) shows a circuit for an a-c amplifier with grounded emitter utilising n-p-n transistor. However, a p-n-p transistor would have the power source reversed as shown in (Fig. 16.30 b).



(a) (b)

*Fig. 16.30 Amplifier circuit*

A small ac power source connected to the input produces changes in the base current. This produces much larger changes in the collector current. The collector current flowing through the load $R_C$ produces a voltage drop. Under this condition the above circuit operates as an amplifier.

Transistor can be used as oscillators, switches, memory, units and perform many other useful functions. Their characteristics are compactness, reliability, including ruggedness and very long lifetimes, and low power consumption. They have caused a major revolution in computers and electronic control circuits.

The only draw back with transistor is that its characterisitcs are strongly temperature dependent. However, the effects of these variations in transistors can be decreased by a careful circuitory of transistor equipment.

# SUMMARY

***Electromagnetic Wave:*** When an alternate accelerating field is applied to a charge it releases electromagnetic waves in space around it. The electric and magnetic fields in such a wave are perpendicular to each other and both are also perpendicular to the dirceiton of the wave. A transmitting antenna generates such waves in space. The velocity of this wave depends upon magnetic permeability $\mu_o$ and electric permittivity $\varepsilon_o$ of the medium. It is equal to $\frac{1}{\sqrt{\mu_o \varepsilon_o}}$ which is equal to velocity of light. Sound waves, which travel much slower, are superimposed on these waves called carrier wave, and the resultant wave is called modulated wave. The modulated waves are received on receiving antenna which are parabolic metallic surface in nature. The waves are reflected and collected at the focus, and then detected electronically.

***Semiconductor:*** Some substances having electrical resistivity intermediate between conductors and insulators, are called semiconductors. The electrons of atoms in such a substance make covalent bonds with the other electrons. Free electrons and holes are responsible for electrical conduction.

***n and p Materials:*** When a pentavalent material like As, is added to tetravalent material i.e., Ge, the outer most electron of the atoms form bonds leaving one electron in excess. This is called n-type material. The electrons are majority carriers in this case.

When a trivalent material like Indium or Galium is added to Ge, three bonds are complete but in the fourth there is deficiency of an electron which gives rise to a 'hole' i.e., empty place for an electron. The material is called 'p' type. The majority carriers in such cases are holes.

***Crystal Structure:*** The materials show external symmetry of shape, tendency to break along definite directions and also that their thermal and electrical properties are directional. All these observations suggest that the materials have their atoms and molecule arranged in specific order and direction. The simplest crystals are

(i) Simple Cubic

(ii) Body Centred Cubic

(iii) Face Centred Cubic

***Conduction Band:*** The solids have different electronic energy bands. There are energy bands which are permitted to be occupied by electrons and there are those which are forbidden.

In case one permitted band is partially filled with electrons and upper permitted band is empty with a forbidden band in between, the electrons can fill in the empty places in the lower permitted band. In case the forbidden band is too wide, the electrons from the lower energy band cannot jump easily to upper empty band; the material is an insulator.

In semi-conductors, the forbidden energy band between two permitted bands, i.e., lower filled and upper empty, is narrow. The electrons can easily move from lower to upper energy bands on gaining small energy. It is due to this property that the electrical resistivity in semi-conductors decreases on rise of temperature which is contrary to the case of other metallic solids.

***Semi-Conductor Diode:*** A junction of n and p type materials together make a semi-conductor diode. There appears a potential barrier across the junction. This potential barrier is increased if p material is connected to –ve and n material to +ve terminal of a battery. This is reverse bias of the diode and flow of current across the junction is difficult. On the other hand, if p is connected to +ve and n to –ve the barrier is reduced. The flow of current across the junction is enhanced. This is forward bias.

***Diode as Rectifier:*** It is simply a semi-conductor diode with a series resistance connected across an a-c supply. A current flows in the resistance when end 'p' of the diode is positive and it is nearly stopped when this end is negative. We get a half wave rectification. The –ve half cycle of the a-c are removed. If, however, we use two diodes and a centre tapped transformer as shown in Fig. 16.22, we get full wave rectification.

***Light Emitting Diode (L.E.D):*** It is basically a forward biased p-n semi-conductor diode. The electron of n and holes of p drift to the junction where they recombine and release photons of energy in the visible region of the spectrum.

***Photodiode:*** It is generally a reverse biased p-n junction in which light is allowed to fall on the p-layer through a window provided for this purpose. When a photon enters the region where electrons and hole meet (depletion region), it creates electron and hole pairs which drift to the respective charges on the sides of p-n junction. The current flow is small but it rises as the light intensity, window area or both are increased and it falls if the intensity or window area are decreased.

***Photovoltaic Cell:*** It is a junction device made from semi conductor materials. When light falls on this cell, it generates a voltage across the ends. The generation of voltage can be changed by increasing the intensity of light or area exposed to light.

***Transistor:*** These devices are of two kind i.e., p-n-p and n-p-n materials. We have three sections and two junctions in each case. The end materials are called emitter and collector and the central material is called base as shown in Fig. 16.27 (a,b). The emitter base junction is forward biased and collector base junction is reverse biased for the semi conductor device to work as an amplifier. When the potentials of emitter and collector are less than that of base, there is no current flow in either direction both reverse biased., If, now emitter base junction is forward biased, current will flow. The characteristics of a trnasistor can be studied by using suitable electric circuits. This device can be used for both current and voltage amplification and also for oscillators. It has replaced vacuum tubes due to its compactness, reliability, long life and low power consumptions.

# QUESTIONS

**16.1** What is meant by amplitude modulation? Explain how an information is carried by electromagnetic waves?

**16.2** What are holes in semi-conductors? Explain the electron-holes pair generation in semi-conductor materials.

**16.3** How the conductors, insulators and semi-conductors can by distinguished using band theory of solids? Explain.

**16.4** What is meant by forward and reverse biasing of a semiconductor diode? Why a semi-conductor diode passes current in forward direction but not in the reverse direction?

**16.5** What is meant by "rectification"? Explain the action of semi-conductor diodes as half-wave and full wave rectifier.

**16.6** What are photodiodes? For what purposes they are used?

**16.7** Explain the construction and working of a solar cell.

# PROBLEMS

**16.1** The induced electric field at a distance of 30 cm from the magnetic flux is 90 N/Coulomb. Calculate the rate of change of magnetic flux.

[ Ans : 54 $\pi$ webers ]

**16.2** The rate of decreas of magnetic flux is 100 webers per second. Calculate the induced electric field at the distance of 15 cm from the centre of circular area containing flux.

[ Ans : 106.15 N/C ]

**16.3** A wire carries a current of 10 amperes. At what distnace from the wire magnetic field will be equal to the earth magnetic field? The earth magnetic field is $5 \times 10^{-5}$ T.

[ Ans : $4 \times 10^{-2}$ metres ]

**16.4** Calculate the resonance frequency of a tuning circuit in which an inductance of 0.9 h and capacitor of 4p F are connected in parallel.

[ Ans : 83.89 k Hz ]

**16.5** An Inductor having L = 40 mH is to be combined with a capacitor to make an L-C circuit with natural frequency $2 \times 10^{6}$ Hz. What value of capacifance should be used?

[ Ans : 0.158 pF ]

# 17

# ADVENT OF MODERN PHYSICS

The aim of the most part of this textbook has been to introduce the student to classical physics. The most fundamental to classical physics are the laws of Newtonian mechanics. Upto the end of the nineteenth century, principles of Newtonian mechanics formed the basis of a physical theory. Electricity and magnetism still presented problems. The electron had been discovered but its charge had only just been measured. The classical physics thus comprised the study of mechanics of chargeless bodies and motion of charged particles. The great success of Newton theory was the formulation of diverse scientific observations into certain set of laws. Newton considered the motion of a set of point masses under the influence of their natural attractive gravitational force. Given the knowledge of the initial position and momentum of each particle the whole evolution of the system can be determined by the solution of Newton's equations of motion. This theory gives an insight into the nature of universe.

Throughout the nineteenth century steady progress was made in the study of classical physics. During the nineteenth century Michael Faraday, Coulomb, Cavendish, Oersted, Ohm, Ampere and Henry established the relationship between electricity and magnetism. Faraday's strong intuition also helped him to propose the idea of "lines of force" filling the space around the charged and magnetised bodies. His student James Clark Maxwell, a young Scottish mathematician described the Faraday's ideas of lines of force in a mathematical language. He not only showed the relationship between electrical currents, magnets and charged bodies, which were found experimentally by Faraday, but also derived a set of equations from which the existence of electromagnetic waves could be predicted. One of the greatest successes of classical physics, therefore, was the experimental confirmation of the electormagnetic waves by Heinrich Hertz and the resolution of long standing controversy about the nature of light between Newton's and Huygen's school of thought. Maxwell's equations, therefore, expalined light wave as an electromagnetic effect.

Maxwell still thought of his waves as analogous to material waves such as sound waves which required a material medium for their propagation. That is to say, he thought of electormagnetic waves as propogating through a medium called "Ether" which existed everywhere, even in interstellar space. So electromagnetic theory of Maxwell successfully employed to expalin phenomena involving electric and magnetic effects. In the beginning of twentieth century, a series of new experiments were carried out whose results and their explanations shatterted basic principles of classical physics. Most of these experiments were related to events happening on microscopic scale. Ideas presented by Michelson and Morley, Einstein, Schrodinger, de Broglie, Bohr, Heisenberg and others challenged the old ideas and principles of classical physics. These ideas brought shockwaves whose reverberations could be felt throughout the twentieth century. For there was to become a new physcis called modern physics consisting of Einstein's special theory of relativity and quantum mechanics. In the proceeding paragraphs we shall discuss these novel experiments, the fundamental issues invovled in them and construct a striking new physical set of ideas. This chapter, however, is not intended to cover twentieth-century physics.

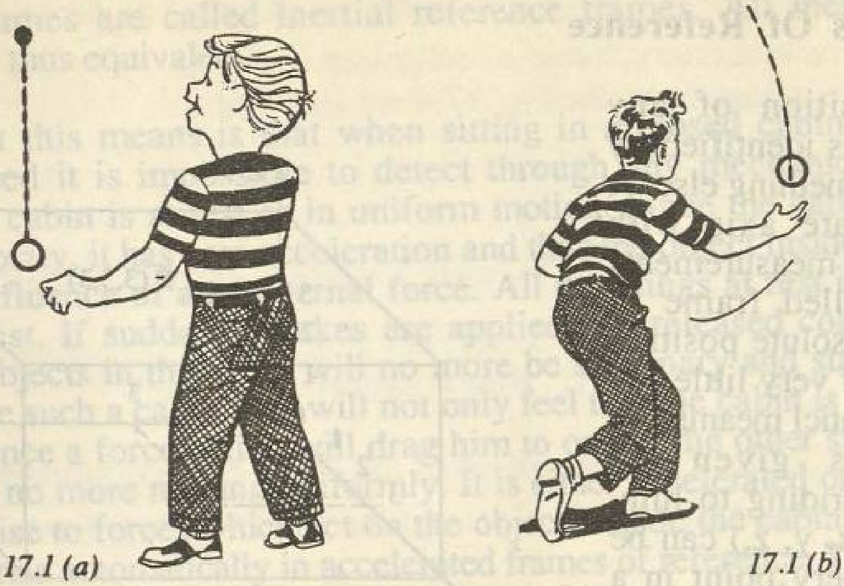
## 17.1 RELATIVE MOTION

Newton's laws of motion invoked two important issues relating to space and time. These were absolute position or absolute space and absolute time. Both Galileo and Newton believed in absolute time. That is the interval of time between two events occurred in the same postions could be measured unambigously and that this postition and time would be the same whoever measured it. The space assumed by Newton to formulate his laws was infinite, homogeneous (the same from one point to another), isotropic (the same in all direction). This space possesses no preferred points, no preferred direction. Time flows as if governed by some sort of absolute world timepiece, counting off the seconds for the entire unvierse. What is more, this timepiece cannot be altered by matter or the character of its motion. Time was completely separate from and independent of space. Thus, it was view of absolute space and absolute time that leads so naturally to the first law of motion. An object moving through a directionless infinite space continues to move with unifrom speed in a straight line in the absence of forces. Why should it do otherwise?

In the words of Newton the absolute space, in its own nature, without relation to anything external, remains always similar and immovable. Up to the start of this century that was the unshakable view of space and time. However, we have had to change our ideas about space and time.

One finds that the statement of Newton's first law of motion is not only the seed but also the destruction of the concept of absolute space. It also implies the lack of absolute position. For one important aspect associated with motion of objects is the physical experience about the relativity of motion. One can not say simply that an object is in absolute motion. That is to say that we have no way of detecting our true or our absolute motion. One has to specify with respect to what object it is moving i.e., with respect to a certain observer (We shall call these observers "frames of reference" and discuss in detail in the next section).

It is a common experience that the apparent motion is, of course, different in various reference systems. The walls of a moving train cabin are stationary with respect to the reference system of the passengers sitting inside. On the otherhand, the same cabin walls are in motion to the observer stationary on the Earth. The flight path of a vertically falling stone looks different to a stationary observer and to a rapidly moving observer (Fig. 17.1). Thus the velocity, the distance travelled by objects and trajectory are all relative.



17.1 (a) 17.1 (b)

*Fig 17.1 (a) Astationary observer, (b) A running observer.*

The examples discussed in the above paragraph lead one to conclude that an absolute standard of rest is non-existent. In other words, it means that one can not give an event an absoulte position in space. The position of events and the distances between them would be different for person on the moving train and one on the plateform. The lack of absolute position or absolute space even though worried Newton, he in fact, refused to accept it.

As discussed in the introduction, the light waves were assumed to travel through the ether at a certain fixed speed as sound waves travel through air. Their speed should, therefore, be relative to the ether. Thus it seemed that it ought to be possible to detect the presence of ether. Since our earth is moving on its orbit at a speed of 30 kms$^{-1}$, and if the earth is moving through the ether, then from the earth the ether should appear to be moving past the earth. We would experience an "ether wind" blowing in the dirction opposite to our motion in the same way that speeding motorcyclist on a calm day, a strong "air wind" appears to be blowing into his face. Actually it is the motorcyclist, not the air, that is "moving" but he feels the relative motion as a wind. The American scientists A. A. Michelson and E. W. Morley tried to measure that speed in 1887, using the Michelson interferometer, discussed in Chapter 9 of the Textbook on Physics for class XI.

The principle of the Michelson interferometer is based on the analogy of sound waves. If an observer is moving in the same direction as a sound wave the velocity of the wave seems to be less. If he is moving in the opposite direction, the apparent velocity of the sound wave is increasecd. Using the interferometer, Michelson and Morley performed their experiment very accurately to measure the "ether wind". But their results were completely negative. There was no "ether wind" whatsoever. The observed velocity of light was completely constant and independent of the motion of the observer. Albert Michelson was the first American to receive the Nobel prize for physics for this experiment.

### 17.1.1 Frames Of Reference

The position of any object in space is identified by referring it to something else. A set of coordinate axis with respect to which measurements are made is called frame of refrence. The absolute position of an object is of very little use to us. A mathematicl meaning to this idea was given by Descartes. Accoriding to him, three numbers (x, y, z,) can be associated to every point in a three-dimensional space to determine its position in space at a certain time. The three dimensions of space are laid out as perpendicular axes, x, y, and z.

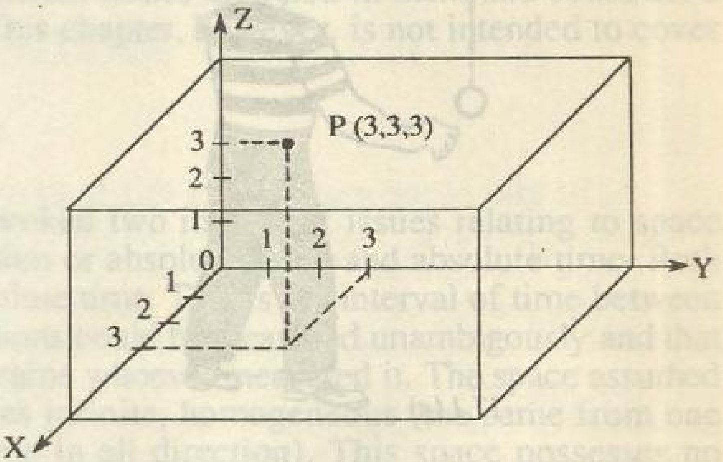


Fig. 17.2 A set of Cartesian coordinates.

A point P from the origin O, which is assumed to be (0,0,0) position, is located by saying, for example, "three along x-axis, three along y-axis and three along z-axis" or by a set of three numbers (3, 3, 3) corresponding to x, y and z (Fig. 17.2). The three perpendicular axes x, y and z constitute Cartesian coordinate system and they form a suitable frame of refrence to locate the position of any object in space.

Now, if we wish to locate an event, it will be a point in space and time. The event can be specified by the three dimensions and a clock. It can be located at the point P in space P (x, y, z) and the time at which it occurs as shown in Fig. 17.3.

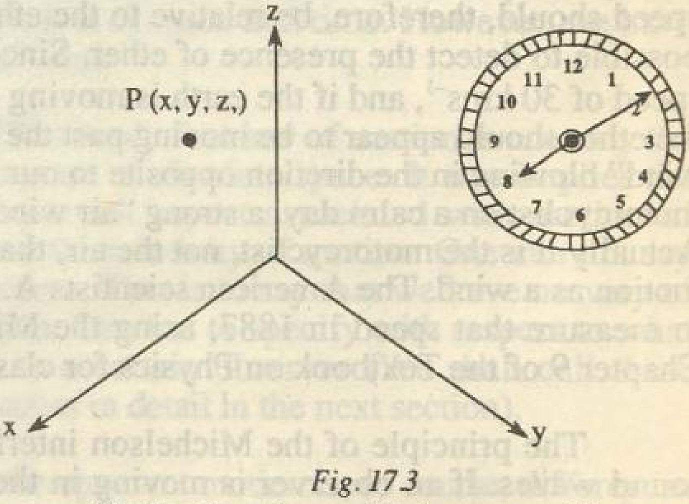


*Fig. 17.3*
*The location of any point p in space and time.*

### 17.1.2 Inertial Frame of Reference

Ordinary objects can be seen to move at different speeds if they are observed from different reference frames in relative motion. Thus every motion is relative. However, there is something that does not depend on the choice of the reference system and are true in thier own frame of refrence the Newton's laws of motion themselves. In all frames of reference that move with uniform velocity with respect to one another, these laws of motion are identical. That is to say, Newton's laws of motion are valid when observed in unaccelerated reference frames. Such reference frames are called inertial reference frames. All inertial systems of reference are thus equivalent.

What this means is that when sitting in a closed cabin moving with a uniform speed it is impossible to detect through any mechanical experiments, whether the cabin is at rest or in uniform motion. Since the cabin moves with a uniform velocity, it has zero acceleration and thus the object inside the cabin is not under the influence of any external force. All the things at rest in this cabin will remain at rest. If suddenly brakes are applied and released continually, all the stationary objects in the cabin will no more be stationary and start jostling. Now sitting inside such a cabin, one will not only feel that the cabin is in motion but he will experience a force which will drag him to one or the other side. We find that the cabin is no more moving uniformly. It is either accelerated or decelerated and thus gives rise to forces which act on the objects inside the cabin. This shows that the forces arise automatically in accelerated frames of reference. The Newton's first two lasws do not hold in an accelerated frame of reference and must be corrected to account for the forces which are generated automatically in such frames.

It is interesting to remark that a laboratory fixed on the surface of the earth does not provide a good inertial frame of reference. Therefore, the Newton's second law of motion $F = ma$ should be corrected to allow for the acceleration of the laboratory resulting from the daily rotation of the earth about its axis.

As discussed above the Newton's laws of motion do not change in any inertial frame of reference. If the force, $F = ma$ is valid in an inertial system moving with a uniform velcoity at time $t = 0$, then the equation also holds in another frame of reference $S'$, moving with a constant velocity with respect to it (Fig. 17.4). Also imagine that at $t$, = 0, the x and y axes of the two frames of reference coincide. Since the inertial frame of reference $S'$ moves with a uniform velocity $v$ in the x-direction, after time $t$ it will be at a distance $vt$ from $O$ (the origin of the stationary frame of reference). Suppose an experiment is performed to measure

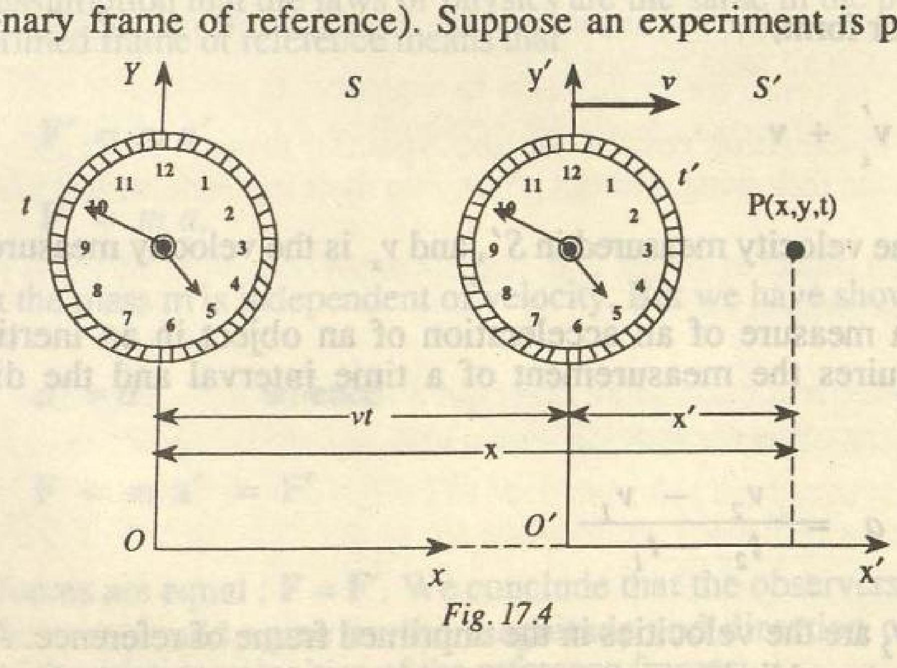


Fig. 17.4

$F = ma$ in space and time at a point P. It would be recorded by the point P labelled as P (x, y; $t$) in the stationary frame of reference and P (x′, y′; $t'$) in the moving frame of reference, where from the Fig. 17.4 we find that

$$x' = x - vt$$

$$y' = y$$

$$t' = t \qquad (\therefore \text{ time is assumed absolute})$$

This is the Galilean transformation of the space-time coordinates of a point between the primed coordinates (x′ , y′ ; t) and the unprimed coordinates (x, y; t).

The immediate consequence of Galilean transformation is the law of velocity addition:

$$v_x = \frac{x}{t}$$

$$= \frac{(x' + vt)}{t}$$

$$= \frac{x'}{t'} + v \quad (\therefore t = t')$$

$$v_x = v'_x + v$$

or, in the vector form,

$$\mathbf{v}_x = \mathbf{v}'_x + \mathbf{v}$$

where $v_x'$ is the velocity measured in $S'$, and $v_x$ is the velocity measured in $S$.

Now a measure of an acceleration of an object in an inertial frame of reference requires the measurement of a time interval and the difference of velocities:

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

where $v_1$ and $v_2$ are the velocities in the unprimed frame of reference. According to the law of velocity addition:

$$v'_2 = v_2 - v$$

$$v'_1 = v_1 - v$$

where $v'_2$ and $v'_1$ are the velocities measured in the primed frame of reference.

Therefore, the new acceleration $a'$ in the primed frame $S'$ is

$$a' = \frac{v'_2 - v'_1}{t'_2 - t'_1}$$

Since time is an absolute quantity, therefore,

$$t' = t$$

and $$a' = \frac{(v_2 - v) - (v_1 - v)}{t_2 - t_1}$$

$$a' = \frac{v_2 - v_1}{t_2 - t_1} = a$$

Thus, the acceleration measured in the moving frame of reference is exactly equal to that measured in the stationary frame.

The assumption that the laws of physics are the same in the primed as well as in the unprimed frame of reference means that

$$\mathbf{F}' = m\,a'$$

if $$\mathbf{F} = m\,a,$$

provided that the mass m is independent of velocity. But we have shown that

$$a' = a \quad \text{whence}$$

$$\mathbf{F} = m\,\mathrm{a}' = \mathrm{F}'$$

and thus the forces are equal : **F** = **F**′. We conclude that the observers in all inertial frames of reference would agree on the magnitude and direction of the force **F** independent of the relative velocities of the reference frames.

The above discussion is true only for the description of ordinary mechanical processes. The principle of Galilean transformation fails when applied to the electrical phenomena. Since velocity appears to be different in different frames of reference in relative motion, it is natural to assume that the speed of light and hence the Maxwell's electromagnetic equations should be different in different inertial frames. This conclusion, of course, appears to violate the experimental observation of Michelson and Morley about the constancy of speed of light. This dilemma along with some other were resolved by the Einstein's special theory of relativity.

## 17.2 EINSTEIN'S POSTULATES OF SPECIAL THEORY OF RELATIVITY

In 1905, Einstein proposed a new theory of relativity based upon two postulates which are as follows:

(i) All laws of physics are the same in every inertial reference frame.

(ii) The speed of light in a vacuum, measured in any inertial reference frame, always has the same value of *c*, no matter how fast the source of light and the observer are moving relative to each other.

In the light of the above made discussion about the inertial reference frame it is not difficult to accept the first postulate of special theory of relativity, for example, if an abserver standing on the earth records the location and time of an event of a shuttle in earth-based reference frame the same event is observed by another observer on board an aeroplane using a plane-based reference frame, both the observers will find their data consistent with Newton's laws of motion. Furthermore, both the observers will find that the behaviour of electronic gadgets on board the shuttle is described by the laws of electromagnetism, such as Faraday's law of electromagnetic induction. Then, it may be concluded that any inertial referance frame is as good as any other for expressing the laws of physics, because the laws are the same in all such frames.

Some interesting results of the special theory of relativity can be summarized in the following without going into their mathematical derivations.

When an object is at rest with respect to an observer, the mass of the object is called the rest mass $m_o$.

1. If $m_o$ is the rest mass of an object in a frame of reference, then, the mass $m$ as measured by an observer moving with a velocity **v** relative to the object is

$$m = \frac{m_o}{\sqrt{1-(v/c)^2}}$$

This equation shows that mass $m$ changes with velocity. The mass $m$ here is known as relativistic mass to distinguish it from the rest mass since $\dot{v}$ is less than $c$, the term $\sqrt{1 - v^2 / c^2}$ in above expression is less than 1, and the relativistic mass $m$ is greater than the rest mass $m_o$.

2. If $l_o$ is the length of an object in a stationary frame of reference, then the length $l$ of the same object as viewed by the observer in frame of reference moving with a velicty $v$ relative to the object, is given by

$$l = l_o \sqrt{1 - (v/c)^2}$$

since $v/c < 1$, therefore $l < l_o$. There is a shortening of length. This is known as the Lorentz contraction of length. For average velocities the contraction in length is negligible as observed in day-to-day observations.

3. Time in special theory of relativity is not an absolute quantity. If $t_o$ is the instant of time of an event as measured by an observer in a moving frame of reference, the time instant $t$ as it appears to a stationary observer with respect to the moving one is given as

$$t = \frac{t_o}{\sqrt{1 - (v/c)^2}}$$

The time interval $t_o$ of the moving clock as viewed by the stationary observer is longer than the interval of the clock in the stationary frame of reference. Thus, a moving clock runs slower than an identical clock that is stationary in his frame of reference. This is called the time dilation.

It is interesting to remark that in all ordinary situations the velocity of material objects is much smaller than the speed of light and, therefore, the ratio $v/c \cong 0$ in the factor $\sqrt{1-(v/c)^2}$. For example, at a typical planetary speed $\approx 30$ km s$^{-1}$ $v^2/c^2 = 10^{-8}$. This results in an increase of mass of about one part in a hundred million which is negligible. Thus we find that the above, expressions for $m$, $l$ and $t$ in this case reduce to:

$$m = m_o$$
$$l = l_o$$
and
$$t = t_o$$

However, on the subatomic level we have particles with extremely small masses moving with velocities near that of light. For such particles the factor $\sqrt{1 - (v/c)^2}$ becomes significant. It is not difficult to see some of the most important qualitative consequences of $m = m_o / \sqrt{1 - v^2 / c^2}$ . As the speed $v$ of the object approaches that of light, then $v^2/c^2$ approaches unity: $\sqrt{1 - v^2 / c^2}$ become very small and the mass which is essentially the object's inertia, grows large. Thus as the speed of infinite as v becomes equal to c. Since inertia is a measure of the resistance of the particle to a change of motion, the greater the inertia, the greater force will be required to accelerate it. As the particle approaches the speed of light, it becomes more and more difficult to change its speed or to give it an acceleration. Thus we obtain the important result that it is impossible to accelerate a particle to speed equivalent to that of light. Only light, or other waves that have no intrinsic mass, such as photon, can move at the speed of light.

**Example 17.1**

An electron whose rest mass $m_o$ is $9.11 \times 10^{-31}$ kg is accelerated in a particle accelerator to a speed $v$ equal to 0.999c. Find the increase in mass (relativistic mass) of the electron.

**Solution**

The increased mass (relativistic mass) $m$ of the electron can be determined using

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_o}{\sqrt{1-\left(\frac{0.999}{c}\right)^2}} \cong 224\ m_o$$

$$m = 2.24 \times 10^2 \times 9.11 \times 10^{-31}\ \text{kg}$$
$$= 2.04 \times 10^{-28}\ \text{kg}$$

Thus, the relativistic mass of the electron moving with speed equal to $0.999c$ is 224 times greater than its rest mass.

Now the idea that all observer should measure the same speed of light, no matter how fast they are moving has some astonishing results. Perhaps the best known are the equivalence of mass and energy, summed up in Einstein's famous equation

$$E = mc^2 = \frac{m_o c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

(i) where E is total energy of an object that is moving at a speed $v$ relative to the observer, $m$ is relativistic mass, $m_o$ is the rest mass and c is the speed of light.

(ii) the law that nothing may travel faster than the speed of light;

and

(iii) the end of the idea of absolute time and absolute space:

(iii) the end of the idea of absolute time and absolute space:

**Example 17.2**

A 5 kilogram ball is travelling at 1000 $ms^{-1}$. How much mass does the kinetic energy have?

**Solution**

This example requires us to find the amount of mass involved in a certain amount of energy, therefore, the proper formula is

$$E = mc^2$$

Here
$$E = ?$$
$$m = ?$$
$$c = 3 \times 10^8 \text{ kms}^{-1}$$

Out of three variables in the above equation two are unknown. Therefore, we look for more of Newton's equations. Since the speed is low compared to the speed of light, we can use the old formula for kinetic energy:

$$KE = \frac{1}{2} m_o v^2$$

in which

$m_o$ = 5 kg (the mass of the ball)

$v$ = 1000 $ms^{-1}$

then
$$E = \frac{1}{2} \times 5 \times (1000)^2 \text{ kg} \times \text{ m}^{-2} \text{ s}^{-2}$$
$$= 2.5 \times 10^6 \text{ kg m}^{-2} \text{ s}^{-2}$$

Substituting the value of $E$ and $c$ in $E = mc^2$ and solve:

$$2.5 \times 10^6 \text{ kg} \times \text{m}^2 \text{ s}^{-2} = 9 \times 10^{16} \times \text{m}^2 \text{ s}^{-2}\, m$$

$$m = \frac{2.5 \times 10^6}{9 \times 10^{16}} \text{ kg} = 0.28 \times 10^{-10} \text{ kg}$$

We find that the increase in the mass of the ball caused by a velocity of 1000 $ms^{-1}$ is insignificantly small.

**Example 17.3**

How fast will a ruler have to move in the direction of its length in order to appear to a stationary onlooker, to be half its proper length?

**Solution**

We are dealing with the Lorentz length contraction equation:

$$l = l_o \sqrt{1 - v^2/c^2}$$

Here $l_o$ = proper length at rest.

$l = l_o/2$ = shortended length as observed from stationary system.

$v = ?$

$c = 3 \times 10^8 \text{ ms}^{-1}$

Substituting these values and solve.

$$\frac{l_o}{2} = l_o \sqrt{1 - v^2/c^2}$$

$$\frac{1}{2} = \sqrt{1 - v^2/c^2}$$

Squaring both sides :

$$\frac{1}{4} = 1 - \frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\frac{v}{c} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$v = \frac{1}{2}\sqrt{3}\,c = \frac{1}{2}\,(1.73)\,c$$

$$v = 0.87\,c$$

$$= 0.87 \times 3 \times 10^8 \text{ ms}^{-1} = 2.6 \times 10^8 \text{ ms}^{-1}$$

Notice that the result is close to the speed of light when it shows this much contraction.

## 17.3 BLACK BODY RADIATION

The beginning of the twentieth century was witnessing a series of dramatic developments in the world of science. Einstein's theory of relativity gave revolutionary ideas and concepts about space and time. Thomson had discovered the existence of electron. About the same time Planck, a German scientist and others were trying to solve the unsolved problem about the distribution of wavelengths of the radiation from the incandescent solids. The analysis of this rather obscure phenomenon set the foundation for a new mechanics, known as quantum theory.

Suppose a lump of iron is heated in a furnace. At first the iron looks just as it did but without touching it one can feel the heat radiated from it. So after moderate heating the iron emits infra-red or heat waves. As the temperature rises the appearance of the lump of iron changes. Soon it is glowing dull red: a portion of the electromagnetic waves are now being produced as light. Raise the temperature still further and the light becomes whiter until eventually it matches the white light of the sun or modern incandescent light bulb. Thus depending upon the temperature, radiation are emitted with a continuous range of wavelengths.

In fact, all bodies, no matter how hot or cold, continuously radiate electromagnetic waves. The cooler objects, however, emit visible light waves only weakly and as a result, do not appear to be glowing. Certainly, the human body, at only say 310K, does not emit enough visible light to be seen in the dark with the unaided eye. But the body does emit electromagnetic waves in the infrared region of the spectrum and these can be detected with infrared sensitive detectors.

The radiation emitted from such bodies with different wavelengths can be shown in the form of a curve for each temperature (Fig. 17.5). All the curves have similar shape with a maximum which shifts to smaller wavelengths as the temperature increases.

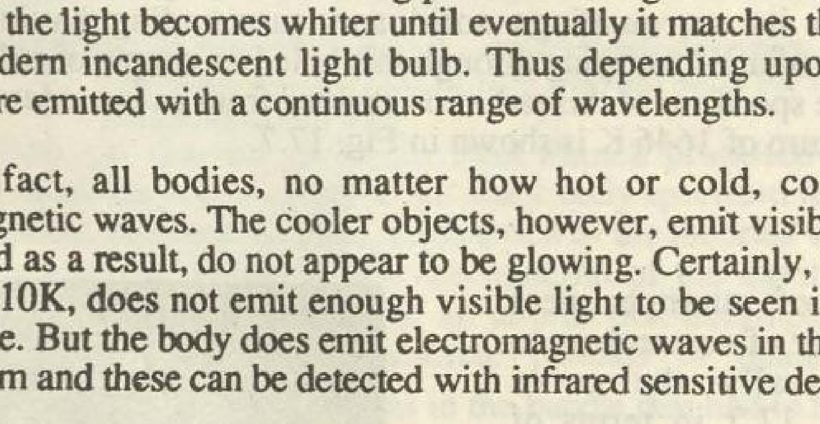


*Fig. 17.5 The spectrum of the radiation from a hot body for different temperatures. For a given temperature the amount of radiation is found to be maximum for certain wavelength λ. This maximum shifts to smaller values of λ for higher temperatures.*

In order to understand the distribution of radiation emitted from a hot body, we consider a solid that has a hollow cavity within it. It has a small hole and the radiation can escape only through this hole. We evacuate the cavity and then heat the solid to the temperature T. The atoms of the walls of the cavity constantly emit and absorb radiation, and so at equilbrium we will have a cavity filled with electromagnetic radiation. Such a cavity is called a blackbody cavity and the radiation within the cavity is called blackbody radiation. It is a little oven which is so designed that any radiation emitted from its inside surface bounces back and forth many times before it leaves through the small hole in it (Fig. 17.6). Some heated solids, for example, electrically heated wire, filaments radiate essentially as a blackbody. To make the situation simple, let us bring the solid to thermal equilibrium with its surroundings solid and surrounding having the same temperature. The radiation escaping through this hole is a sample of radiation within the enclosure. The spectrum of this radiation emitted from an incandescent tungsten tube at a temperature of 1646 K is shown in Fig. 17.7.

A series of attempts were made by a number of researchers to explain theoretically the curve obtained in Fig. 17.7 in terms of classical electromagnetic theory. The notable among these attempts were the two; one made by Max Carl Wien and the other by Lord Rayleigh. The predictions of these theories have been shown in Fig. 17.7 for comparison with experimental results. It is evident that neither of the two is a satisfactory representation of the experimental data. Rayleigh Jeans theory is incomplete disagreement with the experimental curve in the short wavelength region and agrees with experiment only in the longer wavelength region. On the other hand, one sees a complete disagreement between the predictions of Wien's theory and the experimental data in the long wavelength region.

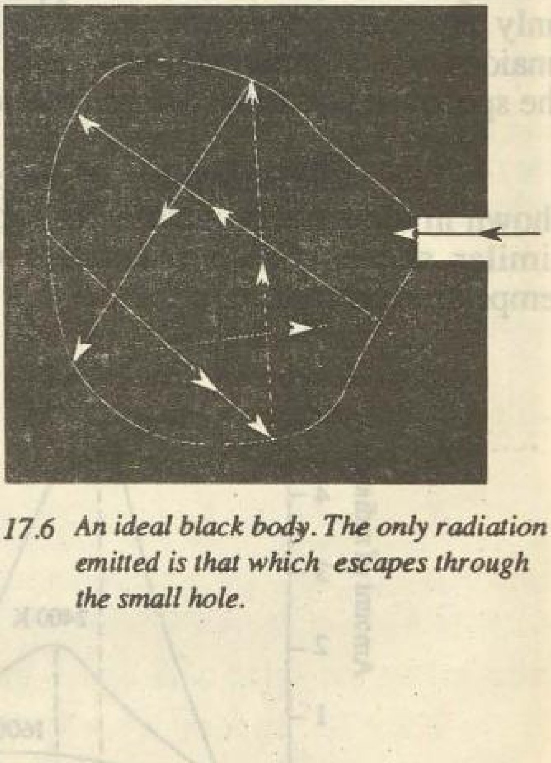


*Fig. 17.6 An ideal black body. The only radiation emitted is that which escapes through the small hole.*

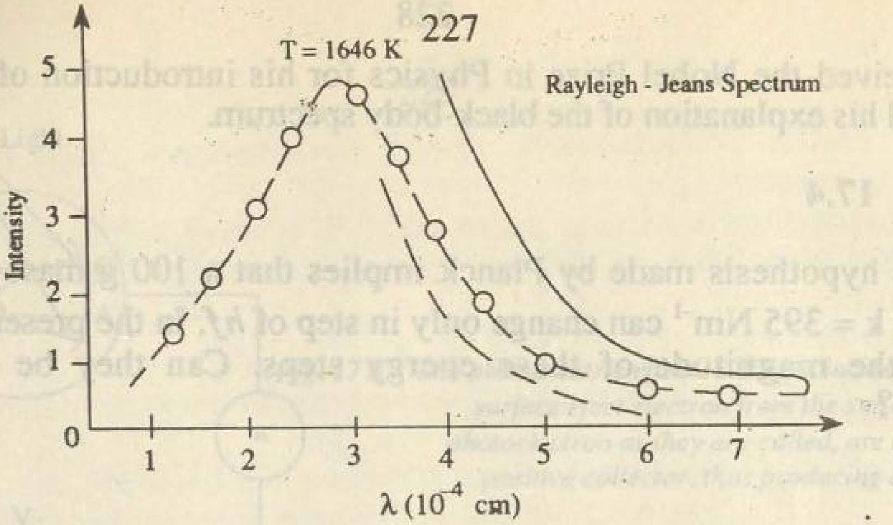


*Fig. 17.7 Spectrum of cavity radiation. -- 0 -- represents experiment, solid line is the theoretical results of Rayleigh Jeans and dotted line shows the theoretical curve of Wien.*

In 1900, the German physicist Max Planck proposed a formula which explained in detail the whole shape of the black body spectrum for all wavelengths. In all the previous attempts the basic assumption of the theories was that the energy exchange between the radiation and the material of the cavity took place in a continuous way. Planck made instead an unorthodox assumption. According to him, energy exchange takes place in discrete "bundles" or "quanta". The higher the frequency $f$, the more energy there was in the bundle that had to be emitted, that is,

$$E = hf$$

where $f$ is the frequency of vibration (in hertz) of atoms which emit radiation, and $h$ is the proportionality constant, now known as Planck's constant. Note that for our purpose, it is not the problem of the blackbody radiation itself that is important; which has been discussed only briefly and incompletely, but the fact that its solution led Planck to discover the discrete nature of the electro-magnetic field. The value of $h$ is $6.63 \times 10^{-34}$ joule-second. This new fundamental constant is as important in physics as the constant $c$, the speed of light in vacuum.

Planck's theory changed our concept about electromagnetic waves. Instead of regarding electromagnetics radiation as a steady flow of waves we now talk in terms of bundles of waves that is, a kind of graininess has been introduced in the otherwise a steady flow of waves. Since the value of the Planck's constant is sufficiently small, these bundles or quanta of energy are of small amount.

It is interesting to recall that most physicists regarded Planck's method for the reproduction of the blackbody spectrum as an interesting trick of no fundamental importance and continued to believe that all physical processes were continuous. Planck's idea remained in limbo for some years before Albert Einstein in 1905, took the idea a stage further. Planck had assumed that the graniness of light was due to some property of the atoms producing it. But Einstein extended Planck's idea and postulated that the packets or bundles of energy are integral part of all electromagnetic radiation and that they could not be subdivided. These indivisible bundles of electromagnetic energy he called "Photon". In 1918, Max

Planck received the Nobel Prize in Physics for his introduction of the quantum concept and his explanation of the black-body spectrum.

**Example 17.4**

The hypothesis made by Planck implies that a 100 g mass attached to a spring with $k = 395\ \text{Nm}^{-1}$ can change only in step of $hf$. In the present case $f = 10$ Hz. Find the magnitude of these energy steps. Can they be observed by experiment ?

**Solution**

The size of energy step is

$$\Delta E = hf = (6.63 \times 10^{-34}\ \text{J.s}) \times 10\ \text{Hz} = 6.63 \times 10^{-33}\ \text{J}$$

The total energy of this mass-spring system when it oscillates with an amplitude $x$ as small as wavelength of light is $60 \times 10^{-7}$n

$$E_{\text{tot}} = \frac{1}{2} kx^2$$

$$\lambda_{\text{ave}} = x = 6 \times 10^{-7}\ \text{m}$$

$$= \frac{1}{2} (3.95 \times 10^{2}\ \text{Nm}^{-1}) (6 \times 10^{-7}\ \text{m})^2$$

$$= 7.11 \times 10^{-11}\ \text{J}$$

The total number of increments $\Delta E$ required to reach even this tiny amplitude is thus of the order of $10^{22}$. Obviously, these steps are so finely spaced from a macroscopic point of view that they can never be noticed.

## 17.4 THE PHOTOELECTRIC EFFECT

Already in 1887 when Hertz performed experiment to show the existence of electromagnetic waves, he observed that the intensity of the spark between the two high voltage electrodes was increased on illuminating the electrodes by ultravoilet (UV) light. Later on, in a number of attempts it was demonstrated that the increase in the intensity of spark was due to electrons ejected from the metal surface by the light, as shown in Fig. 17.8. This phenomenon of electron ejection by light is called the photoelectric effect. In order to find out the details let us understand the experiment in Fig. 17.8.

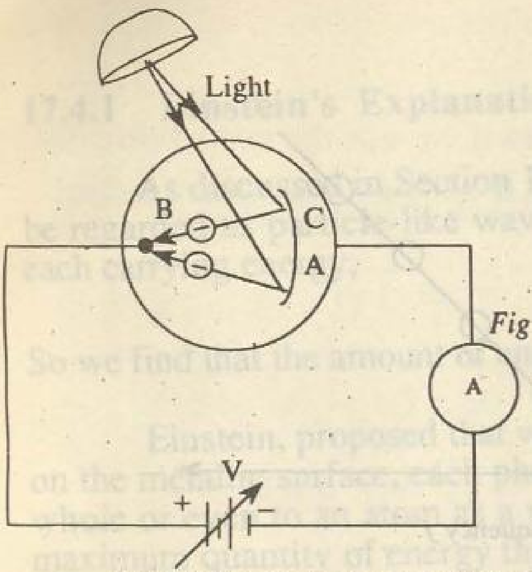


*Fig. 17.8 The photoelectric effect. UV light incident on the metal surface eject electron from the surface. These photoelectron as they are called, are drawn to the positive collector, thus producing a current.*

Light from some sources falls on one of the two plates, C and B, contained within an evacuated tube and connected to a source of variable voltage and a sensitive current measuring instrument (ammeter). The electrons are emitted from the photoemissive surface, say, of the plate C and are attracted by the collector plate B, connected to the positive terminal of the battery (the negative terminal of the variable high voltage source, battery, is connected to the plate C). The ammeter shows a current. When the light is switched off, the current stops.

The maximum energy of the ejected electrons can be determined by reversing the battery in the circuit and making the plate B more and more negative with respect to the plate C, till the current in the ammeter just disappears.

One observes that as the plate B is made negative with respect to the plate C, the current starts decreasing. Only those electrons which have energy greater than Ve (where V is the voltage across the electrodes of the tube and e is the electron charge) reach the plate B. At a certain retarding voltage $V_o$ (stopping potential) the current falls to zero. The stopping voltage corresponds to the maximum kinetic energy of the photoelectrons, $\frac{1}{2}\, mv^2_{max}$, which is equal to

$$\frac{1}{2}\, mv^2_{max} = V_o e$$

On plotting a graph of the maximum energy photoelectrons versus the frequency of the incident radiation, a straight line is obtained as shown in the Fig. 17.9. It intersects frequency axis at $f = f_o$, the threshold frequency or minimum frequency. Therefore, the equation that reprsents the relationship between $V_o$e and $f$ is

$$V_o e = af - af_o,$$

where $a$ is a constant and is equal to the slope of the straight line. It was found that photoemissive surfaces of different materials all produced straight-line relationship with the same slope but with different threshold frequency. The details of the observation of this experiment may be summarized below.

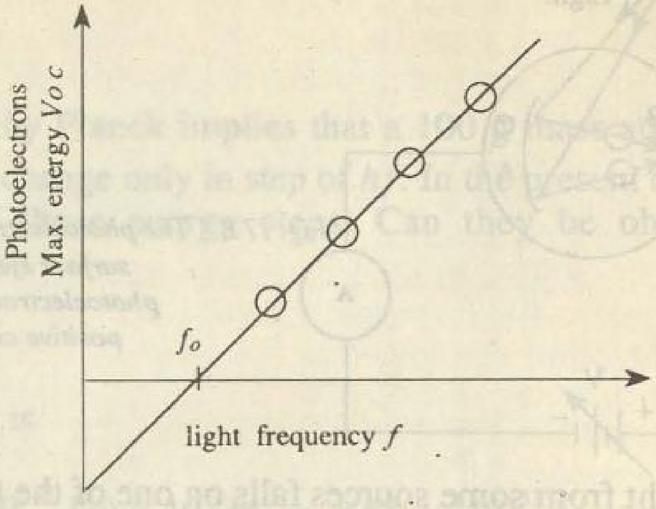


*Fig. 17.9 Shows the variation of maximum energy of photoelectrons with the frequency of incident light.*

(i) For each material the emission occurs only for certain definite frequency, $f_o$ of light, called the threshold frequency. Light with a lower frequency than the threshold frequency will not eject electrons (also called photoelectrons) no matter what the intensity of the light may be.

(ii) If the incident radiation is of a single frequency greater than the threshold frequency of the material, the number of electrons ejected per second is proportional to the intensity of the radiation.

(iii) The ejected electrons have various values of kinetic energy ranging from zero upto some maximum value depending upon frequency but independent of the intensity of incident radiation.

The observation of the photoelectric effect was not startling since it would be easily explained on the basis of classical theory of electromagnetism. According to this theory, light was known to consist of oscillating electric and magnetic fields that carry electromagnetic energy. Thus on striking a metallic surface light waves could transfer energy to electrons in the metal and when an electron had acquired sufficient energy from the light wave it would escape from the metal. But there were some other features of this experiment which could not be explained on the basis of classical electromagnetic theory. For example according to classical wave theory of light, an increase in intensity increases the energy of light falling on the surface and should have increased the kinetic energy of the ejected electrons. We see that this is not in agreement with observation. On the other hand, according to classical electromagnetic theory the electron would not have been emitted until and unless it had acquired sufficienet energy to escape. That is spontaneous emission of an electron for a weak intensity of light should not have been possible. This is again not in agreement with the experiments. Spontaneous emission of photoelectrons is recorded even for weak intensity of the radiation. Furthermore, wave theory of light can offer no explanation of the frequency dependence of the kinetic energies of the emitted electrons, nor why there should be a minimum frequency at which emission occurs.

### 17.4.1 Einstein's Explanation of the Photoelectric Effect

As discussed in Section 17.3, the nature of light, according to Einstein, can be regarded as particle-like wave packets of photons moving with speed of light each carrying energy,

$$E = hf$$

So we find that the amount of energy in a photon depends on the frequency.

Einstein, proposed that when a beam of photons in the incident light strikes on the metallic surface, each photon imparts all its energy, not to the material as a whole or even to an atom as a whole, but to one and only one electron. Thus the maximum quantity of energy that can be given to an electron in a particle-particle like collision between photon and electron is the full energy of one photon only. Unless this energy of the photon is sufficient, the electron in the metal would not break free from the metal surface. Increasing the intensity of the light does not increase the energy of the photons in it but merely supplies more photons. Thus, the frequency of the photon and hence its energy should be of some minimum value if it is to knock out an electron from the metal with some kinetic energy. This explains the occurence of the threshold frequency which is different in different materials.

The above discussion makes it clear that since one photon can strike out only one elctron from the metal, the number of electrons emitted, in a given time, by a surface is proportional to the number of incident photons i.e., to the intensity of the radiation. Furthermore, as soon as a photon strikes an electron it ejects it out from the material, explaining why photoemission begins instantaneously.

We conclude that the more energetic photons supply more energy to the electrons, and electrons are emitted with a higher maximum energy. It is because some electrons lose more energy than other on their way out of the metal, by colliding with atoms, therefore, come out with less kinetic energy than the maximum. Thus, the kinetic energy of an electron expelled out is the difference between the energy of the incident photon of light and the minimum energy necessary to break the free electron from the metal surface-called the work function of the material. That is,

$$(\text{electron } KE)_{\max} = (\text{photon energy}) - (\text{work function})$$

$$\frac{1}{2} mv^2_{\max} = \mathrm{h}f - \phi$$

where $\phi$ is the work function for the particular material and represents the minimum energy an electron must have in order to escape from the metal. The work function $\phi$ is of the order of a few electron volts $V_c$ for most of the metals.

The above equation is plotted in Fig. 17.9. The intercept at $f = f_0$ identifies the frequency at which the energy of an incident photon equals the work function $\phi$.

The threshold frequency immediately follows if we use the condition that the maximum kinetic energy of the electron is zero if $hf \leq \phi$. That is to say that when

$$(KE)_{max} = 0 = hf_o - \phi$$

or $$hf_o = \phi$$

**Example 17.5**

An electron changing from one atomic "orbit" to another causes a photon of frequency $1.5 \times 10^{15}$ vibrations per second to be emitted. What is the energy difference between the orbits?

**Solution**

The energy difference is equal to the energy of the photon emitted and is given as

$$E = hf$$

Here $f = 1.5 \times 10^{15}\ \text{Hz} = 1.5 \times 10^{15}\ \text{s}^{-1}$

$h = 6.625 \times 10^{-34}\ \text{J. s}$

$E = ?$

**Thus, on substituting and solving:**

$$\text{E} = 6.625 \times 10^{-34} \times 1.5 \times 10^{15}\ \text{J.s s}^{-1}$$

$$= 9.94 \times 10^{-19}\ \text{J} = 1.0 \times 10^{-18}\ \text{J}$$

**Example 17.6**

Find out the maximum kinetic energy of an electron expelled out from magnesium (for which $\phi = 3.7$ eV) when a light of frequency $1.5 \times 10^{15}$ Hz strikes it.

**Solution**

The energy of the photon is given as

$$E = hf$$

$$\text{Here, } f = 1.5 \times 10^{15} \text{ Hz} = 1.5 \times 10^{-15} \text{ s}^{-1}$$

$$h = 6.6 \times 10^{-34} \text{ J.s}$$

Therefore,

$$E = 6.6 \times 10^{-34} \times 1.5 \times 10^{15} \text{ J. s. s}^{-1}$$

$$= 9.9 \times 10^{-19} \text{ J} \times \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}}$$

$$= 6.2 \text{ eV}$$

Hence, the maximum kinetic energy is

$$(KE)_{\max} = \mathrm{h}f - \phi$$

$$= 6.2 \text{ eV} - 3.7 \text{ eV}$$

$$= 2.5 \text{ eV}$$

Careful and systematic measurements of the photoelectric effect for several elements were carried out by the group of American scientists under the guidence of Robert A. Milikan. He showed that the slope of the line $(\mathrm{Ve})_{\max}$ against $f$ was indeed consistent with Einstein's theory. The slope of the line furnished the values of $he^{-1}$ and combined with Milikan's earlier experiment on the determination of charge e, predicted the value of Planck's constant *h*.

It would be interesting to remark that Einstein was awarded Nobel Prize in 1921 for his explanation of the photoelectric effect (not for his relativity theory). Milikan also received Nobel Prize in 1923 for his experimental determination of e and *h*.

### 17.4.2 Photocell

A photoemissive cell based on photoelectric effect is shown in the Fig. 17.10. It consists of an evacuated glass envelope fitted with an anode wire and a cathode of an appropriate emitting surface. The material of the cathode can be chosen to respond to the frequency range over which the cell operates. The photoemissive cell is connected in a circuit to operate for a particular use of the cell as a source of photoemission.

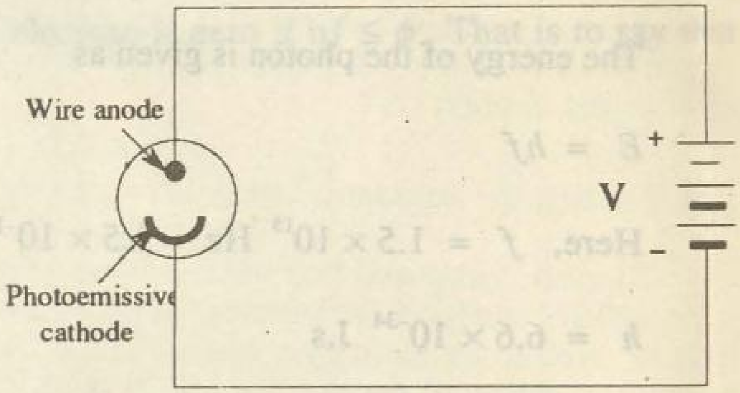


*Fig. 17.10 A photocell*

When light of frequency greater than the threshold frequency of the cathode is incident on cathode, it emits photoelectrons. These electrons are attracted by the positive anode, a current flows in the external circuit. The current increases with the intensity of light. The current would cease to flow if the light beam is interrupted.

A photoemissive cell can be used in any situation where a beam of light falling on a cell is broken. For example,

(i) to count vehicles or items on a conveyer belt;
(ii) to open door automatically;
(iii) to operate burgler alarm, etc.

### 17.4.3 Comptom Effect

Although Einstein presented his photon model for the photoelectric effect in 1905, it was not untill 1924, that A. H. Compton confirmed the existence of photon with the help of another more direct method. In the photoelectric effect it was assumed that a photon strikes an electron, gives away all its energy (kinetic energy) to the electron and vanishes. Compton argues that if a photon is really a particle-like entity, it should be scattered on collidng with an electron in much the same manner as do two colliding balls. The only requirement for this to demonstrate is that a photon should have sufficient energy, so that after it had imparted energy, it has energy for itself to escape. Compton knew that the binding energy of the outer electrons in an atom is comparable to the kinetic energy of the photons of ultravoilet light. Thus, instead of using ultravoilet light, whose photons have energy slightly greater than the electron binding energy (a few electron volts), Compton used x-ray photons of energy 17.5 keV, to make the impact over poweringly strong. x-ray are high frequency electromagnetic waves and like light, they are composed of photons. Because this energy is so much greater than the binding energy of the electrons, therefore, a bound electron in the material can be treated almost free.

Now if the particle-like entity or a photon with a certain frequency strikes whith an electron it is scattered with a frequency different from the frequency of the original wave and the frequency of the scattered beam depends only on the direction of the scattering.

If the particle-like nature of the photon and the electron is assumed, and if a photon of energy h$f$ incident on an electron at rest (Fig. 17.11) then the scattering process of these two particles can be treated in a strictly classical way. Fig. 17.11 illustrates what happens when an x-ray photon strikes an electron in a piece of graphite. Like two billiard balls colliding on a pool table, the x-ray photon scatters in one direction and the electron recoils in another direction after the collision. Compton observed that the frequency $f'$ of the scattered photon is less than the frequency of the incident photon. Furthermore, he observed that the difference between the two frequencies depend on the angle $\theta$ at which free scattered photon leaves the collison. The phenomenon in which an x-ray photon is scattered from an electron, the scattered photon having the smaller frequency than the incident photon, is called Compton effect.

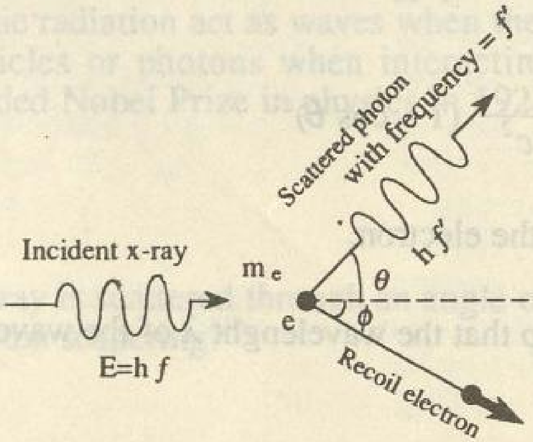


*Fig. 7.11 Schematic diagram of the Compton scattering of a photon of energy hf by an electron.*

We have two conservation laws which must be satisfied by this scattering experiment.

(i) Conservation of energy: The kinetic energy before collision (the photon kinetic energy $hf$ plus the electron rest energy) must be equal to the energy after collision (the energy $hf'$ of the scattered photon plus the total energy of the recoiling electron).

(ii) Conservation of linear momentum in the direction of the incident photon, the momentum for the incident photon $P = \frac{E}{c} = \frac{hf}{c}$ must be equal to the sum of the components of electron momentum and the momentum of the scattered photon in the direction of the incident photon.

(iii) Conservation of linear momentum in the direction transverse to the incident photon.

On writing down the above two conservation laws in a mathematical language and performing some lengthy calculations, one can determine the relation between the original frequency $f$, the frequency of the scattered photon $f'$ and the scattering angle $\theta$ as

$$\frac{1}{f'} = \frac{1}{f} + \frac{h}{m_o c^2}(1 - \cos\theta)$$

where $m_o$ is the rest mass of the electron.

Using the relationship that the wavelenght $\lambda$ of the wave is given as

$$\lambda = \frac{c}{f}$$

we have

$$\lambda' = \lambda + \frac{h}{m_o c}(1 - \cos\theta)$$

Thus, the change in the wavelength $\Delta\lambda$ of the photon is given

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_o c}(1 - \cos\theta)$$

In this equation $m_o$ is the rest mass of the electron. The quantity $h/(m_o c)$ is referred to as the Compton wavelength of the electron, and has the value $(h/ m_o c) = 2.43 \times 10^{-12}$ m. Since $\cos\theta$ varies between +1 and –1 the shift $\Delta\lambda$ in the wavelenth can vary between zero and $2h.(m_o c)$

This shows that the change in wavelength depends only on the scattering angle $\theta$. For $\theta = 90°$, the change in the wavelength will be

$$\Delta\lambda = \frac{h}{m_o c} = 2.42 \times 10^{-12}\ \text{m}$$

This is known as the Compton wavelength of the electron.

Compton carried out experiments to measure $\Delta\lambda$ as a function of $\theta$ for several different initial wavelengths $\lambda$ and determined the change in wavelength $\Delta\lambda$. He found that the experimental value of $\Delta\lambda$ for a given angle $\theta$ was in excellent agreement with the theoretical predictions. The theory–experiment agreement confirmed that the photon is a particle–like wave not only in the photoelectric effect but in the scattering process, as well. The photoelectric effect and the Compton effect provided compelling evidence that the electromagnetic waves can exhibit particle-like characteristics attributed to energy packets called photons. Light and other electromagnetic radiation act as waves when they move from place to place but behave as particles or photons when interacting with material substance. Compton was awarded Nobel Prize in physics in 1927 for these experiments and their interpretations.

**Example 17.7**

A 50 keV x-ray is scattered through an angle of 90°. What is the energy of the x-ray after Compton scattering?

**Solution**

According to the Compton effect, we have

$$\frac{1}{f'} = \frac{1}{f} + \frac{h}{m_o c^2}(1-\cos\theta)$$

Dividing this equation by $h$, we get

$$\frac{1}{hf'} = \frac{1}{hf} + \frac{1}{m_o c^2}(1-\cos\theta)$$

Using the values $m_o c^2 = 511$ keV and $\cos 90° = 0$, we find,

$$\frac{1}{hf'} = \frac{1}{50\text{ keV}} + \frac{1}{511\text{ keV}}$$

or $$hf' = \frac{(50 \times 511)\,(\text{keV})^2}{(50+511)\,(\text{keV})}$$

$$= \frac{25550}{561}\text{ keV} = 45.5\text{ keV}$$

The electron, of course, carries off the remainder of the incident energy

$$KE = 50 \text{ keV} - 45.5 \text{ keV} = 4.46 \text{ keV}$$

### 17.4.4 Pair Production

In the previous section we have seen that a low energy photon (visible light) on striking with an electron loses entire energy (the photoelectric effect), whereas, a high energy photon (x-ray) loses a part of its energy (Compton scattering). In the first situation the photon vanishes after imparting its energy to the photoelectron and in the second case it is scattered with the remaining energy with a energy smaller than the initial energy. A photon may also lose its energy in another way, too, when it interacts with the nucleus of an atom. In this process it disintegrates and an electron-positron pair is created (Fig. 17.12).

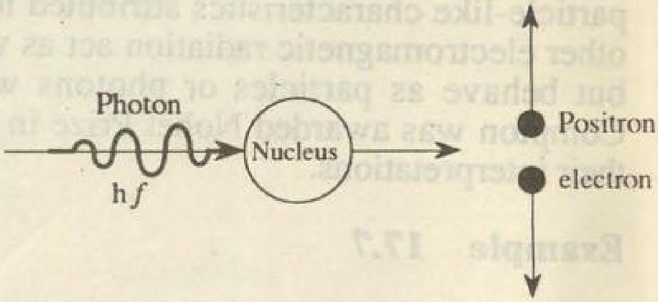


*Fig. 17.12 A schematic diagram of pair production.*

A positron is an antiparticle of an electron. It is also called as a positive electron. The phenomenon of disintegration of photon on striking a nucleus into an electron and a positron is known as pair-production. The role of the nucleus in this process is just to share some energy and momentum in order to conserve the two quantities.

The process of pair production is also called as materialization of energy. According to Einstein relation, $E = mc^2$, mass and energy are interconvertible. In order to create an electron–positron pair, an energy $2m_oc^2$ is needed. Thus, a photon with energy h$f$ greater than $2m_oc^2$ is required for the pair–production to take place. In fact, the photon energy in this process is equal to the sum of the $2m_oc^2$ and the kinetic energies of electron plus positron. Mathematically,

$$hf = 2m_oc^2 + KE_{e-} + KE_{e+}$$

### 17.4.5 Annihilation of Matter

We have said in the preceding paragraph that energy and mass are interconvertible. In the process of pair-production we have seen that energy (the photon) is converted into mass (the electron plus the positron). The reverse of this process is also a reality. That is an electron and positron when combine together, they annihliate each other and the mass of these particles appears as energy of two photons (Fig. 17.13). Each photon will have an energy equal to the rest mass of an electron.

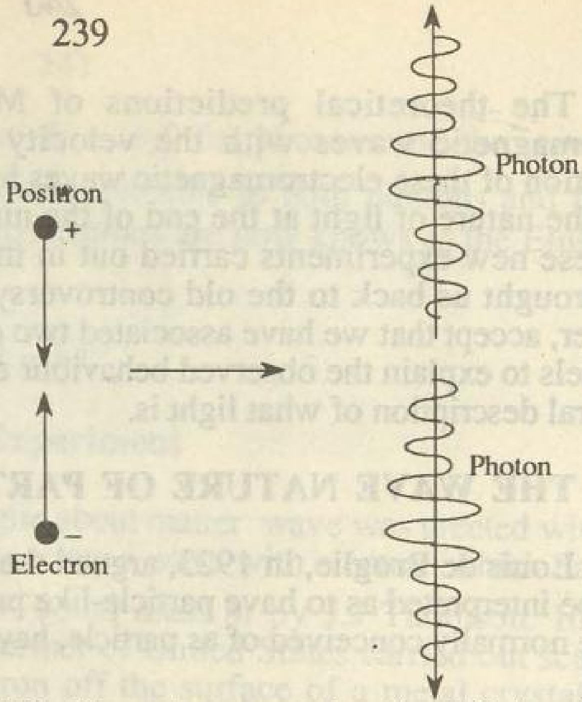


*Fig. 17.13 Schematic representation of annihiliation of positron and electron and the appearance of two photon moving into the opposite direction.*

The photons produced will move in the opposite directions and will thus conserve the total momentum of the creation–annihilation process. Charge and energy are also conserved in this process. We know that the rest mass energy of the electron and positron is equal to 1.02 MeV. Therefore, each photon will have an energy equal to 0.51 MeV.

It is interesting to remark that the exsitence of positron-a positive electron, shows the presence of antimatter in the universe. The prediction of antimatter (the positron) was made theoretically by P. A. M. Dirac in 1928 onwards, while studying the electron behaviour in terms of relativity and quantum physics. In 1932 and early 1933 Anderson provided the experimental observation of the positive electron, while, studying the energy spectrum of the secondary electron produced in the atmosphere and other materials by the incoming cosmic radiation. Now it is believed that for each ordinary particle that we know of there should exist a corresponding antiparticle having the same mass and spin as the particle but opposite charge and magnetic moment. Following the discovery of the positron, evidence for the antiproton was sought in cosmic ray studies for some twenty years, without success. In 1955 the elusive particle was finally tracked down in an accelerator experiment at the University of California's Lawrence Radiation Laboratory and for this Segre and Chamberlain were awarded Nobel prize in physics in 1959. Evidence of antineutron followed rapidly, in 1956 completing the basic electron, proton and neutron "antitrio".

Experiments discussed in the preceding paragraphs confirmed the particle aspect associated with the electromagnetic waves, such as light. We conclude that the electromagnetic waves when strike with matter behave as if they were particles (particle-like waves or photons) with definite energy and momentum. Yet light unquestionably behaves like a wave in such phenomena as reflection, refraction, diffractions and interference. The diffraction and interference are two very convincing evidences to accept the wave nature of the electromagnetic radiation.

The theoretical predictions of Maxwell about the propagation of electromagnetic waves with the velocity of light and Hertz's experimental production of these electromagnetic waves had settled the long discussed problem about the nature of light at the end of the nineteenth century. Therefore, it seems that these new experiments carried out in the beginning of the twentieth century have brought us back to the old controversy about the nature of light. We must, however, accept that we have associated two different attributes (wave and particle) or models to explain the observed behaviour of light; neither may not necessarily be the literal description of what light is.

## 17.5 THE WAVE NATURE OF PARTICLES

Louis de Broglie, in 1923, argued that if the behaviour of wavelike objects could be interpreted as to have particle-like properties, why not, then, those objects, that are normally conceived of as particle, have wave properties? Indeed, why not?

Louis de Broglie argued the following.

The photon had been observed to be emitted and absorbed as discrete bundles of energy. The energy of these packets was related to the frequency by

$$E = hf$$

Also, the momentum of the photon, which moves with a velocity c is related to the energy as

$$E = pc$$

Therefore, these give

$$hf = pc$$

But $\lambda f = c$

With this a relation between the wavelength, $\lambda$ (a parameter associated with a wave motion) and momentum, $p$, (a parameter for the motion of the particle) is obtained as

$$\lambda = \frac{h}{p}$$

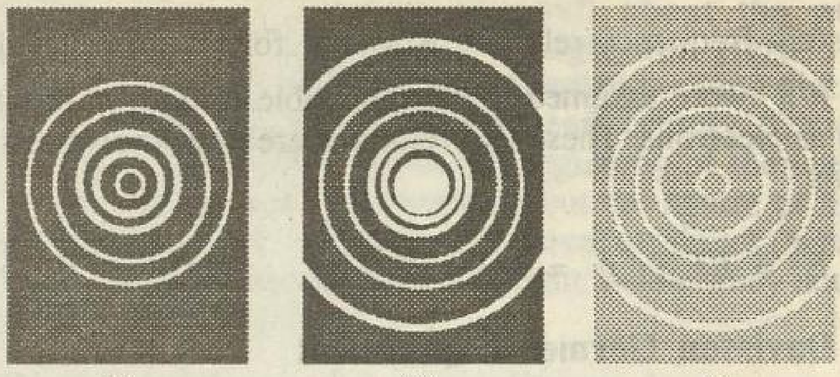
Thus, the Einstein frequency relation $E = hf$ for photons and the wavelength relation $p = h/\lambda$ were assumed to be applicable to both photons and material particles, such as electrons. These two relations are now knows as the Einstein-de Broglie relations:

$$E = hf \ ; \ p = h/\lambda = h\lambda^{-1}$$

### 17.5.1 The Davisson Germer Experiment

The new idea of Louis de Broglie about matter wave was greeted with great astonishement. It looked strange to associate a wave with electron which had been identified as particle having charge $^-e$ and mass $m$ by J.J Thomson. In 1926, Clinton. J. Davisson and Laster H. Germer of United States carried out scattering experiments of the low-energy electron off the surface of a metal crystal. They observed that the scattered electrons came off in rather peculiar peaks showing many electrons coming off at certain angles and very few at others analogous to that seen when x-ray are diffracted by a crystal. The wavelength of the electrons revealed by the diffraction pattern matched that predicted by de Broglie. hypothesis, $\lambda = \frac{h}{mv}$. They had no idea why such peaks were appearing. In one of the conferences at Oxford, Davisson reported the unexplained data. There it was suggested to him that these peaks might be interpereted as diffraction of associated electron waves postulated by de Broglie. The later investigations confirmed that the electrons are diffracted from crystals in exactly the same manner as x-rays or any other wave.

A more direct analogy of electron diffraction with x-ray diffraction as manifestations of the wave interference effects was soon demonstrated by G. P. Thomson (son of J. J. Thomson). He obtained diffraction by passing x-rays and electrons of approximately the same wavelength through a thin aluminium foil (see Fig. 17.14). The similarity for the two diffraction pattern alongwith that obtained with neutron is strikingly illustrated in this Fig. 17.14. These experimental observations of the interference effect were a direct proof of the fact that the electrons and neutrons have wave-like properties. Thus a beam of electrons or neutrons can be used to study the crystal structure in a similar fashion as is determined with the help of x-rays. Davisson and Thomson shared the 1937 Nobel prize in physics for these experiments that established the wave nature of electron.



(a) (b) (c)

*Fig. 17.14 Diffraction rings produced by (a) x-ray with A= 07.12 A°; (b) 6000 V electrons (0.50A°) and (c) 0.0568 V neutrons.*

It is interesting to note that the waves assoicated with the motion of macroscopic particle has wavelength which is small as compared to the width of the atoms in a crystal. Therefore, the diffraction produced by such particles is too small to be observable. For example, the de Broglie wavelength of a tennis ball having mass 20 g moving at a speed of 50 cm $s^{-1}$ will be

$$h = 6.6 \times 10^{-34} \text{ J.s}$$

$$m = 20 \times 10^{-3} \text{ kg}$$

$$v = 50 \times 10^{-2} \text{ ms}^{-1}$$

$$\lambda = \frac{6.6 \times 10^{-34} \text{ Js.}}{50.10^{-2} \text{ ms}^{-1} \times 20 \times 10^{-3} \text{ kg}}$$

But $J = \dfrac{\text{kg m}^2}{s^2}$

So $\lambda = \dfrac{6.6 \times 10^{-34} \,.\, \text{kg m}^2 \text{s}^{-2} \,.\, \text{s}}{50 \times 10^{-2} \times 20 \times 10^{-3} \text{ kg m. s}^{-1}}$

$$\lambda = 6.6 \times 10^{-32} \text{ m}$$

Since this length is many orders of magnitude smaller than atomic dimenstions, it is obviously impossible to measure such a small wavelength.

n electron moving with a velocity $10 \times 10^6$ $ms^{-1}$ has a

$$= \frac{6.6 \times 10^{-34}\ \mathrm{J.s}}{9.1 \times 10^{-31}\ \mathrm{kg} \times 10^{7}\ \mathrm{m.s^{-1}}}$$

$$= \frac{6.6 \times 10^{-34}\ .\ \mathrm{kg\ m^2\ s^{-2}\ .s.s}}{9.1 \times 10^{-31} \times 10^{7}\ \mathrm{kg\ m}}$$

$$= 0.72 \times 10^{-10}\ \mathrm{m}$$

This wavelength is in the short x-rays range and can be measured experimentally. Since the above value of the wave length of electron is comparable to the distance between two adjacent atoms on the lattice sites in a crystal, the electron beam can pass through the crystal and produce an inteference effect which may be seen on a photographic plate.

## 17.6 WAVE PARTICLE DUALITY

· The assumption about the photonic or particle nature of electromagnetic waves, although introduced discontinuity in the continuous, does not pose much problem as far as interpretation of its nature is concerned. Of course, there is no paradox at all. The energy of a quantum depends on the frequency. Even for short radiowaves the product of frequency and Planck's constant *h* gives an exceedingly small amount of energy of the quanta in the radiowaves emitted from a transmitter. Thus the radiowaves emitted from a transmitter have stupendous quantities of quanta. The motion of these particles in certain pattern censtitutes a wave. The wind blowing in one's face is an analogy. Here huge quantities of molecules give the impression of a soft pressure of air. Thus, it is only the overall averaged picture of the movement of the molecules that creates what is perceived as a wave or wind. So the scheme of things seems quite logical.

The problem arises when a whole wave is associated with a single particle like electron, proton, or neutron as was postulated by de Broglie. Especially, the association of a wave with an electron which was for a long time pictured as a minute particle - a Newtonian classical material point, was entirely a new concept. The experiments of Davisson and Germer and G. P. Thomson had confirmed that both corpuscular and wave properties could be associated with matter in all its manifestation. This new concept was known as wave-particle duality. In other words, both light and electrons manifest what could seem to be mutually exclusive properties of particles (corpuscles) and waves.

Now the question arises how an electron can be a particle and a wave at the same time? Surely it is impossible. When we say that the electron is a wave and a particle, we are only recognising the fact that the electron strictly speaking is neither the one nor the other. To put it in the words of Einstein, "It is the theory which decides what can be observed". Thus, when one speaks that the electron is a particle, it means that it can be approximately described as a particle.

As long as the concept of a wave had not been associated with microparticles, or material point, the particles were treated as Newtonian classical particles. A material object was considered to be located at a specific place at a definite instant of time and moving with a definite veloicty. Furthermore, it was considered possible to predict the course of all future events by following trajectories of the particles in space and time. In other words, the coordinates and velocity of the particle could be specified with absoulte precision in classical physics.

## 17.7 THE UNCERTAINTY PRINCIPLE

In the previous paragraph we said that the electron could only approximately be regarded as a material particle. It meant that the coordinates and momenta of the particle could only be specified approximately, with a certain error. This problem was solved mathematically by W. Heisenberg and is known as Heisenberg uncertainty principle.

According to the uncertainty principle, one can determine either the position or the momentum of a particle exactly, but both can not be known exactly at the same time. The more we know one, the less we can know of the other. No exception to this principle have been found.

As an example, let us see how do we determine the position and speed of a particle in space and time. This can be accomplished in two ways; either by touching it or by looking at it through a beam of light. smaller is the size of the particle the more difficult would it be to locate the position of the particle. Thus the position of a particle is located through the photons which are scattered from the particle. The light which consists of photons having momentum $p = h/\lambda$ used for determining the position is called as the probe.

Now we find that the accuracy with which we can locate the position of an object depends upon the wavelength of the light used for this purpose. The shorter the wavelength of the probe the more fine details of an object can be observed with it. In other words, the deviation $\Delta x$, in the precise determination of an object will be less if $\lambda$ is small and greater if $\lambda$ is greater. This may be expressed mathematically as

$$\Delta x \text{ (electron)} \simeq \lambda \text{ (photon)}$$

Thus the accurate determination of position of some micro-object requires the use of a probe of a short wavelength. This means that the beam of light to be used for this purpose should have high frequency ($f = 1/\lambda$, the small $\lambda$ means large value of $f$) and hence of large momentum.

Now obviously for large momentum of the probe particle, the change in the original momentum of the electron will be greater because larger the momentum of the photon (the probe particle) striking the electron the greater amount of its momntum it will transfer to the momnetum of the electron. Putting it in mathematical form.

$$\Delta p\,(\text{electron}) \simeq p\,(\text{photon}) = \frac{h}{\lambda}\ (\text{photon})$$

On multiplying the expressions for Δx and Δp we get the uncertainty relation as

$$(\Delta x)\ (\Delta p) \simeq \lambda . \frac{h}{\lambda}$$

$$\Delta x\ \Delta p \simeq h$$

This means that the product of deviation in the precise determination of position and momentum at the same time will be approximately equal to Planck's constant $h$.

Similar to the position-momentum uncertainty relation there is another relation connecting the imprecision of th energy and indeterminancy of the time interval during which the process occurs:

$$(\Delta E)\ (\Delta t) \simeq h$$

where $\Delta E$ is the uncertainity in our knowledge of the energy and $\Delta t$ is the time interral during which the particle is in a particular state.

Since the value of the Planck's constant is very small, this explains why the uncertainty does not bother us when working with large–scale (macroscopic) particles. The uncertainty or the deviation in both the position and momentum of an object of the size of a tennis ball is negligible in comparison with the size and momentum of the ball would have under normal conditions. In dealing with small particles (microscopic) such as electron etc. Planck's constant is comparatively large and the uncertainty may therefore be very important.

**Example 17.8**

The position of a free electron is determined within an uncertainty of $10^{-7}$ m. What is the uncertainty in its velocity? What would be the position of the electron after this ?

**Solution**

According to the uncertainty relation

$$\Delta x\ \Delta p \simeq h$$

Here $\Delta p_x = m_e\ \Delta v_x$

$m_e$ is the mass of the electron and $\Delta v_x$ is the uncertainty in its velocity. So that

$$\Delta v_x \simeq \frac{h}{m_e \, \Delta_x}$$

Here $m_e = 9.1 \times 10^{-31}$ kg

$$h = 6.6 \times 10^{-34} \text{ J.s}$$

$$\Delta_x = 10^{-7} \text{ m}$$

Substituting and solving, we get

$$\Delta v_x = \frac{6.6 \times 10^{-34} \text{ J.s}}{(9.1 \times 10^{-31} \text{ kg})\,(10^{-7} \text{ m})}$$

$$\simeq 7 \times 10^{-3} \text{ ms}^{-1}$$

Therefore, after 1 second the electron could be anywhere within a distance of $7 \times 10^{-3}$ m .

**Example 17.9**

Find the uncertainty in the energy of a photon which is emitted from an atom radiating for approximately $10^{-8}$ s.

**Solution**

According to uncertainty relation

$$\Delta E \simeq \frac{h}{\Delta t}$$

Here

$$\Delta t = 10^{-8} \text{ s}$$

$$h = 6.6 \times 10^{-34} \text{ J.s}$$

$$\therefore \quad \Delta E = \frac{6.6 \times 10^{-34} \text{ J.s}}{10^{-8} \text{ s}} \times \frac{1}{1.6 \times 10^{-19} \text{ J/eV}}$$

$$= 4 \times 10^{-7} \text{ eV}$$

# SUMMARY

***Frame of Reference:*** A frame of reference is required to describe the relative position of the body in space. The simplest frame is a system of Cartesian coordiantes consisting of three mutually perpendicular axes originating from a common origin. An inertial reference frame is unaccelerated and non-rotating with Newton's laws of motion obeyed in it.

***Principle of Relativity:*** The simple assumption that all possible reference frames moving with uniform velocity relative to one another are equivalent for the statement of laws of physics is called the principle of relativity. All inertial reference frames are completely equivalent for all laws of physics.

***Postulates of relatevity :*** The special theory of relativity is based on two postulates.

*(i)* All laws of physics are the same in all inertial refrence frames.

*(ii)* The speed of light in vacuum is the same for all observers in uniform translational relative motion and is independent of the motion of the source i.e., the speed of light is a universal constant "*c*"

***Black Body Radiation:*** A perfect black body may be defined as a body of some material capable of absorbing all heat radiation incident on it and can emit in turn all the radiation at constant temperature after it is in equilibrium with it. The radiation emitted by a block body are called blackbody radiation.

***Photon:*** Light (electromagnetic radiation) consists of particle like wave called photon, which are packets of energy. The rest mass of a photon is zero and it moves with the speed of light "*c*".

***Photoelectric Effect:*** The process of ejection of loosely bound electrons from a metallic surface (photosensitive surface) by the absorption of photons is called photoelectric effect. This effect is a quantum phenomenon and is governed by the Einstein's equation

$$\frac{1}{2} mv_{max}^2 = hf - \phi$$

where the symbols have their ususal meanings.

***Photo Cell:*** It is a device which makes use of the photoelectric effect and consists of an evacuated glass bulb fitted with an anode and a cathode of appropriate surface which is sensitive to photons.

***Compton Effect:*** Compton effect is a phenomenon in which electromagnetic waves exhibit corpuscular properties. It is a process of scattering of an x-ray photon by an electron. The Compton effect results in the shift in wave length of the scattered photon and the incident photon by an amount $\Delta\lambda = \lambda' - \lambda = \frac{h}{m_o c}(1 - \cos\theta)$. Thus the scattered photon has a frequency less than that of the incident photon.

***Pair Production:*** When we try to stop a photon it loses its identity and disintegrates into an electron and a positron. This is called pair production.

***Annihilation of Matter:*** A process reverse to the pair production is known as annihilation of matter. Such a process takes place when an electron and a positron combine together in such a way that both of them disappear and their combined mass reappear as two gamma ray photons.

***Wave Particle Duality:*** Radiation and matter may exhibit particle, as well as, wave like characteristics. This is known as wave-particle duality.

***Uncertainty Principle:*** The Heisenberg uncertainty principle states that the product of the uncertainties in momentum ($\Delta p$) and position ($\Delta x$) of particle at some instant is approximately of the order of Planck's constant "$h$" i.e., $\Delta p . \Delta x \simeq h$. In other words this means that it is impossible to measure the position and momentum of a particle (say electron ) simultaneously with perfect accuracy. There is always an uncertainty in the measurements of momentum and position if we try to determine their values simultaneously.

# QUESTIONS

**17.1** Suppose you are standing at the plateform and a train passes by you. A passenger sitting in the train and you are looking at a clock on the train. Who measures the proper time interval? You or the passenger? Justify your answer.

**17.2** The speed limit on our highways is 65km per hour. If the speed of hight was 65km per hour would you be able to drive at the speed limit?

**17.3** Usually the masses of various elementry particls in physics are given in units of energy, such as in MeV (million electron volts) rather than in kilograms. Why is this possible?

**17.4** Suppose a spaceship is approaching you with speed of light. A person on board this spaceship throws a ball towards you with a speed of light relative to the speed of the space ship. The person on the space ship sees the ball move away at a very high speed. Yet you see the ball move away from the spaceship very slowly. why? Use the velocity addition formula to guide your thinking.

**17.5** Radiation with a given wavelength causes electron to be emitted from the surface of one metal but not from the surface of another. Explain why this could be.

**17.6** The photographic films are exposed when light photons strike the film. Although x-ray photon are more energetic than the light photon, yet the x ray machines used at the airport security checkpoints do not expose the photographic films. Explain.

**17.7** In a Compton scattering experiment, an electron is accelerated straight ahead in the direction of the incident x-ray photon. Which way does the scattered photon move?

**17.8** A basket ball is dropped from rest from the top of a building. Explain what happens to the de Broglei wavelength of the basket ball as it falls.

**17.9** If we keep on applying force on material object, can it gain the speed of light?

**17.10** Does the fact that an atom can emit a photon, violate the law of conservation of energy? Explain.

**17.11** Can pair production takes place is vacuum? Explain.

**17.12** Is it ever posible to see a star moving away from us at a uniform velocity equal to the velocity of light?

**17.13** Why must the rest mass of a photon be zero?

# PROBLEMS

**17.1** An electron changing from one atomic orbital to another causes a photon of frequency $1.5 \times 10^{15}$ vibration per second to be emitted. What is the energy difference between the orbits?

[Ans. $E = 9.94 \times 10^{-19}$ J]

**17.2** What is the wavelength of a proton with a speed of $1000\text{m s}^{-1}$ ?

[Ans. $3.96 \times 10^{-10}$ m]

**17.3** What is the wavelength of a photon with an energy of $16 \times 10^{-19}$ J?

[Ans. $1.24 \times 10^{-7}$ J]

**17.4** According to the theory of electricity and magnetism, the equation $E = cp$ gives the relation between the amount of energy and the amount of momentum in a series of electromagnetic waves. From this equation and Planck's equation $E = hf$ obtain de Broglie's equation for photons.

**17.5** A Certain atom has a separation in energy of $3 \times 10^{-18}$ J between two of its orbital levels. What frequency photon is required to make the atom change from the lower state to the higher one?

**17.6** An astronaut, using a metre stick that is at rest relative to a cylindrical spacecraft, measures the length and diameter of the spaceraft to be 82 and 21 m, respertively. The spacecraft moves with a constant speed of $v = 0.95c$ relative to the earth. What are the dimemsions of the spacecraft, as measured by an observer on earth.

[Ans. L = 26m , diameter = 21m ]

**17.7** An electron is accelerated from rest to a speed of $v = 0.9995c$ in a particle accelerator. (a) Determine the rest energy $E_o$ of an electron (rest mass = $9.10 \times 10^{-31}$ kg), (b) find the total energy of the high speed electron (c), what is the kinetic energy of the electron?

[Ans.$E_o = 8.187 \times 10^{-14}$ J

$E_o = 2.59 \times 10^{-12}$ J

$E_o = 2.51 \times 10^{-12}$ J]

**17.8** In converting electrical energy into light energy a sixty - watt incandescent light bulb operates at about 2.1% efficiency. Asuming that the light is green light (vacuum wavelength = 555 nm) determine the number of photon given off per second by the bulb.

[Ans. $3.6 \times 10^{-18}$ photons / second ]

**17.9** The work function for a silver surface is $\phi = 4.73$eV. Find the minimum frequency that light must have in order to eject electrons from this surface.

[Ans. $f_o = 1.14 \times 10^{15}$ Hz ]

# 18

# THE ATOMIC SPECTRA

On the basis of alpha particle scattering experiments, Rutherford in 1911, concluded that the atom consisted of a tiny but massive positively charged central part, called the nucleus, surrounded by orbiting electrons at some distance away. Now an electron in an orbit is an accelerating charge and according to electromagnetic theory it should give off energy in the form of radiation continuously. An electron moving in a circular orbit as proposed by Rutherford, should continuously lose energy, and thus spiral inward until it is swallowed up by the nucleus. Contrary to this fact, the atoms are known to remain stable and nothing of the sort happens. A brilliantly successful answer to this difficulty was proposed by Niels Bohr. In the Bohr theory an electron in an atom could not move in an orbit of arbitrary energy but could only be found in certain discrete set of energy levels, which would imply that only discrete set of radii were physically permissible. When moving in one of these allowed orbits, the energy of the electron would be constant, so emission of energy would be impossible. The second important feature of the Bohr's atomic model was that an electron could make instantaneous jumps from one allowed orbit to another, the change in energy of the atom being balanced by the emission or absorption of radiation. Another successful aspect of this theory was that Bohr's formula for the allowed frequencies of emitted radiation exactly fitted the experimentally observed atomic spectrum of hydrogen atom.

## 18.1 SPECTRUM OF HYDROGEN ATOM

The radiation emitted from hydrogen filled discharge tube shown in Fig. 18.1, when viewed by dispersing devices such as prism, gratings etc, shows a line spectrum.

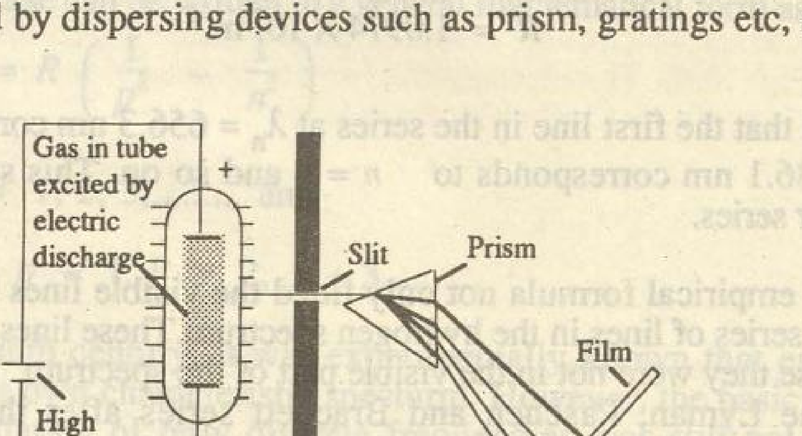


*Fig. 18.1 A schematic representation of the experimental arrangement for observing spectra of different atoms.*

One finds in Fig. 18.2 that the line spectrum of hydrogen atoms consists of a series of spectral lines with wavelengths 656.3 nm, 486.1 nm, 434.0 nm and 410.1 nm.

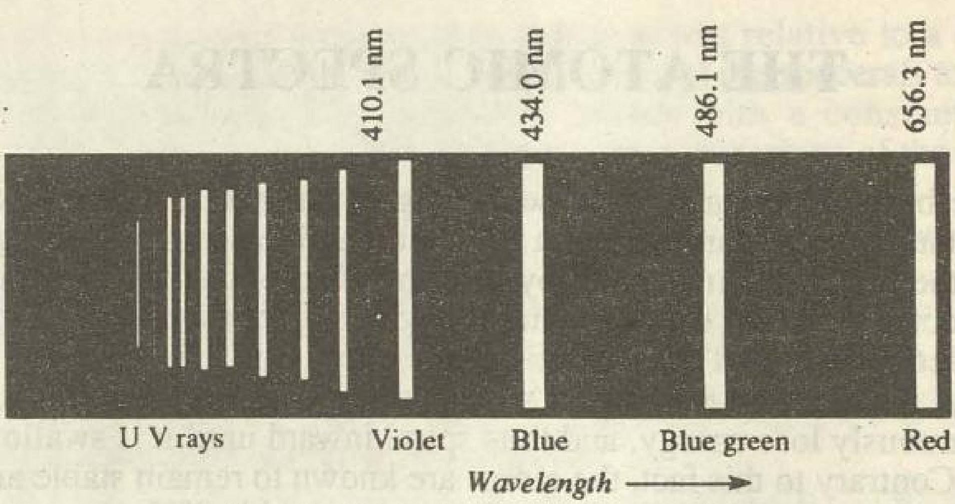


Fig. 18.2 *The Balmer series of spectral lines for hydrogen*

Johann Balmer explained the wavelengths of these lines with the help of an empirical formula

$$\frac{1}{\lambda_n} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \; n = 3,4,5,\ldots\ldots\ldots \qquad (18.1)$$

where $\lambda_n$ is the wavelength corresponding to $n$, which may have the integral values 3,4,5,...., and $R$ is a constant. The value of $R$ was determined by Rydberg and hence it is known as Rydberg constant. If $\lambda_n$ is in metres, then

$$R = 1.0974 \times 10^7 \text{ m}^{-1}$$

Thus we find that the first line in the series at $\lambda_n = 656.3$ nm corresponds to $n = 3$, the line at 486.1 nm corresponds to $n = 4$ and so on. This series of lines was called Balmer series.

Balmer empirical formula not only fitted the visible lines but also predicted several other series of lines in the hydrogen spectrum.These lines had not been seen before because they were not in the visible part of the spectrum. The series are now known as the Lyman, Paschen and Brackett series after the names of their discoverers. These are described by the formulae as under

Lyman series

(Ultra violet region) $$\frac{1}{\lambda_n} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), \; n = 2,3,4,\ldots\ldots\ldots \qquad (18.2)$$

Paschen series

(Infra red region) $\frac{1}{\lambda_n} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$, $n = 4,5,6,.....$ (18.3)

Brackett series

(Infra red region) $\frac{1}{\lambda_n} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$, $n = 5,6,7,.....$ (18.4)

The Balmer series evidently fits into the scheme between the Lyman and the Paschen series (Fig. 18.3).

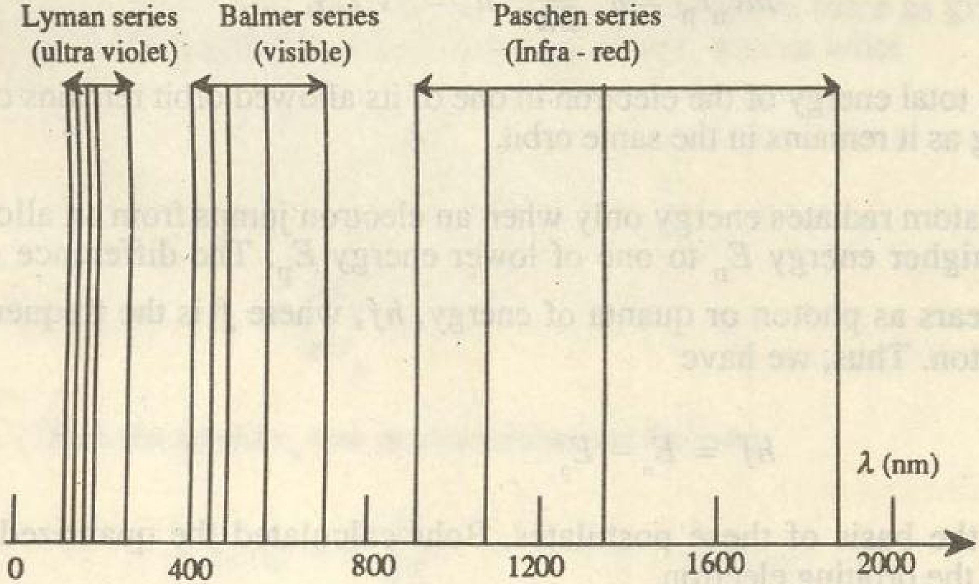


*Fig. 18.3: The three series of spectral lines given off by hydrogen atoms.*

All the above formulae can be written in a general mathematical form as

$$\frac{1}{\lambda_n} = R\left(\frac{1}{p^2} - \frac{1}{n^2}\right) \quad (18.5)$$

where $p = 1, 2, 3,.........$ and

$n = p + 1, P + 2, p + 3 .........$

In the nineteenth century, it was experimentally known that each element appeared to have its own characteristic specturm. However, the basic mechanism responsible for radiation of only discrete frequencies was still not understood before the Bohr's atomic theory.

## 18.2 THE BOHR MODEL OF HYDROGEN ATOM

Bohr in 1913, proposed a model of an atom according to which an electron moves in a circular orbit about the nucleus under the influence of the Coulomb force of attraction between the electron and positively charged nucleus. This model is based upon the following postulates:

1. An electron can not revolve in an arbitrary orbit. Only those orbits are possible for which the angular momentum of the electron about the nucleus is an integral multiple of $\frac{h}{2\pi}$, where $h$ is Planck's constant. These orbits are called allowed orbits. The angular momentum of an electron of mass $m$, moving with velocity $v_n$ in an orbit of radius $r_n$ is given by $mv_nr_n$. Thus, according to this postulate, we have

$$mv_nr_n = n\ \frac{h}{2\pi},\ n\ =\ 1,2,3,\ ...... \tag{18.6}$$

2. The total energy of the electron in one of its allowed orbit remains constant as long as it remains in the same orbit.

3. An atom radiates energy only when an electron jumps from an allowed orbit of higher energy $E_n$ to one of lower energy $E_p$. The difference of energy appears as photon or quanta of energy, $hf$, where $f$ is the frequency of the photon. Thus, we have

$$hf\ =\ E_n - E_p \tag{18.7}$$

On the basis of these postulates, Bohr calculated the quantized radii and energy of the orbiting electron.

### 18.2.1 The Radius Of the Quantized Orbit

According to Bohr's picture of an atom the hydrogen atom consists of an electron of charge $-e$ circling around a proton (nucleus) of an equal but opposite charge $+e$ (Fig 18.4). The electrostatic force $F$ of attraction between the proton and the electron is given by Coulomb's force law:

$$F\ =\ k\frac{e^2}{r_n^2} \tag{18.8}$$

where $k$ is a constant. Its value is $9 \times 10^9\ \mathrm{Nm^2\ C^{-2}}$. If $v_n$ is the speed of the electron in the nth orbit of radius $r_n$, the centripetal force $F_c$ required for the electron to move along a circular orbit will be

$$F_c\ =\ \frac{mv_n^2}{r_n}$$

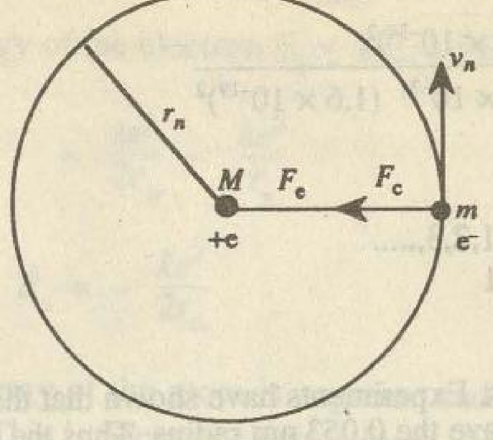


*Fig. 18.4: Bohr's model of hydrogen atom*

This centripetal force is provided by the Coulomb attractive force as given by Eq. (18.8). Assuming the massive proton to remain at rest, we can write

$$\frac{m\,v^2_{\rm n}}{r_{\rm n}} = \frac{ke^2}{r^2_{\rm n}} \quad (18.9)$$

$$r_{\rm n} = \frac{ke^2}{mv^2_{\rm n}} \quad (18.10)$$

From Eq. (18.6) the speed $v_n$ can be determined as follows

$$mv_{\rm n}\,r_{\rm n} = n\,\frac{h}{2\pi}, \qquad n = 1,2,3 \;.....$$

$$v_{\rm n} = n\,\frac{h}{2\pi\, mr_{\rm n}} \quad (18.11)$$

Putting the value of $v_{\rm n}$ in Eq. (18.10) we get

$$r_n = \frac{ke^2}{m} \times \frac{4\,\pi^2\,m^2 r^2_{\rm n}}{n^2\,h^2}$$

Thus on simplifying, we get

$$r_{\rm n} = n^2h^2/4\,\pi^2me^2k, \quad n = 1,2.3,.... \quad (18.12)$$

Substituting the values of the constants *e, h, m* and *k*, in SI units, in Eq. (18.12) we obtain

$$r_{\rm n} = \frac{n^2\,(6.63 \times 10^{-34})^2}{4\pi^2 \times 9 \times 10^9 \times 9.1 \times 10^{-31}\;\;(1.6 \times 10^{-19})^2}$$

$$r_n = \frac{n^2 (6.63 \times 10^{-34})^2}{4\pi^2 \times 9 \times 10^9 \times 9.1 \times 10^{-31} \ (1.6 \times 10^{-19})^2}$$

$$= n^2 \times 0.53 \times 10^{-10} \text{ m}$$

or $r_n = n^2 \times 0.053$ nm, $n = 1,2,3,.....$ (18.13)

For the first orbit i.e for $n = 1$

$$r_1 = 0.053 \text{ nm}$$

which is called the Bohr radius. Experiments have shown that the ground state of the hydrogen atom do indeed have the 0.053 nm radius. Thus the theoretical value of $r_1$ is of the correct order of magnitude for the size of an atom. Using Eq. (18.13), the radius of the electron orbits in a hydrogen atom for any value of $n$ can be determined. One finds from Eq. (18.13) that the radii of the allowed orbits of the electron around the nucleus are quantized.

### 18.2.2 Energy Of Electron In An Orbit

As already discussed, the electron revolving around the nucleus, according to the Bohr's postulate, does not radiate energy as expected in the classical electromagnetic theory. Instead, it is assumed that the total energy of the electron in an orbit remains constant.The total energy of the electron in an atom is the sum of its kinetic energy (KE) and electric potential energy (PE). The KE of the electron in the nth orbit is

$$\text{KE} = \frac{1}{2} m v_n^2$$

Now on rearranging Eq. (18.9) we get

$$\frac{1}{2} m v_n^2 = \frac{ke^2}{2r_n} \qquad (18.14)$$

The absolute potential at a distance $r$ from a charge $q$ is given by $\frac{kq}{r}$, where $k$ is constant and its value is $9 \times 10^9 \text{ N m}^2\text{C}^{-2}$. As potential is work done per unit charge, the electric potential energy of a charge $q'$ at a distance $r$ from the charge $q$ is $\frac{kqq'}{r}$. In the present case $q' = {}^-e$, the charge on the revolving electron and $q = +e$, the charge on the nucleus and $r = r_n$. Thus, the electric potential energy of the electron at a distance $r_n$ from the positive charge is

$$\text{PE} = -\frac{ke^2}{r_n} \qquad (18.15)$$

The negative sign of the PE means that this amount of energy must be given to the electron in order to take it to infinity, where the value of PE is taken as zero.

Thus the total energy of the electron $E_n$ = KE + PE

$$= \frac{ke^2}{2r_n} - \frac{ke^2}{r_n}$$

$$E_n = -\frac{ke^2}{2r_n} \tag{18.16}$$

The total energy of the atom is negative and it becomes more negative as $r_n$ becomes smaller. This is because electron loses PE as Coulomb force pulls it closer to the nucleus. Substituting the value of $r_n$ from Eq. (18.12) in Eq. (18.16) we find that

$$E_n = -\left(\frac{1}{n^2}\right)\left(\frac{2\pi^2 e^4 k^2 m}{h^2}\right), \quad n = 1,2,3,...$$

Putting the constant factor $\frac{2\pi^2 e^4 k^2 m}{h^2} = E_o$

$$\text{we get } E_n = -\frac{E_o}{n^2}, \quad n = 1,2,3,..... \tag{18.17}$$

Substituting the values of various constants, we find

$$E_o = \frac{2\pi^2 m k^2 e^4}{h^2}$$

$$= \frac{2\pi^2 (9.11 \times 10^{-31} \text{ kg}) (9.0 \times 10^9 \text{ Nm}^2/\text{C}^{-2})^2 \times (1.6 \times 10^{-19} \text{ C})^4}{(6.63 \times 10^{-34} \text{ Js})^2} = 2.17 \times 10^{-18}\text{J}$$

$$= 13.6 \text{ eV} \qquad (1.6 \times 10^{-19} \text{ J} = 1\text{eV})$$

Therefore,

$$E_n = \frac{-13.6}{n^2} \text{ eV}, \quad n = 1, 2, 3, ........ \tag{18.18}$$

Each value of $n$ corresponds to a given stationary state of the atom. The lowest stationary state called the normal or ground state corresponds to $n = 1$ and has an energy

$$E_1 = -13.6 \text{ eV}$$

This is in agreement with the result previously found for the ground state energy of the hydrogen atom using Balmer empirical relation. The next state, corresponding to $n = 2$, has an energy

$$E_2 = -3.4 \text{ eV}$$

### 18.2.3 Emission Of Spectral Lines (Energy Level Diagram)

In Bohr atomic theory, the spectral lines emitted from hydrogen atoms are calculated assuming that if the electron jumps from the nth orbit to the pth orbit, the difference of energy is emitted as a photon of energy $hf$ as given by Eq. (18.7). that is

$$hf = E_n - E_p$$

Using Eq. (18.17), we get

$$hf = E_o \left( \frac{1}{p^2} - \frac{1}{n^2} \right)$$

The frequency of this radiation is

$$f = \frac{c}{\lambda}$$

where $c$ is the velocity of light. Thus,

$$\frac{hc}{\lambda} = E_o \left( \frac{1}{p^2} - \frac{1}{n^2} \right)$$

or

$$\frac{1}{\lambda} = \frac{E_o}{hc} \left( \frac{1}{p^2} - \frac{1}{n^2} \right) \tag{18.19}$$

Putting $\frac{E_o}{hc} = R_H$ in Eq. (18.19) we get

$$\frac{1}{\lambda} = R_H \left( \frac{1}{p^2} - \frac{1}{n^2} \right) \tag{18.20}$$

where $p = 1, 2, 3$ ...... and $n = p + 1, p + 2, p + 3$ ....

Compare the theoretical equation (18.20) of the Bohr theory with the empirically formulated equation (18.5) that fit in the observed spectrum of hydrogen atom. The form of equation is the same. The value of $R_H$ in Bohr model was also found in excellent agreement with the value of the Rydberg's constant determined empirically in the Balmer formula, i.e.,

$$R_H = R = 1.0974 \times 10^7 \text{ m}^{-1}$$

The single electron in hydrogen atom normally occupies the lowest level or the ground state. If the atom absorbs energy in some way, its electron is raised to one of the higher energy levels (Fig. 18.5). The atom is now unstable and is said to be in excited state. It returns to the ground state by emitting energy which was originally absorbed. The atom emits one spectral line if the transition occurs direct from some higher energy state to the ground state or a number of lines of different wavelengths, if the electron stops temporarily at a number of lower states during transition.

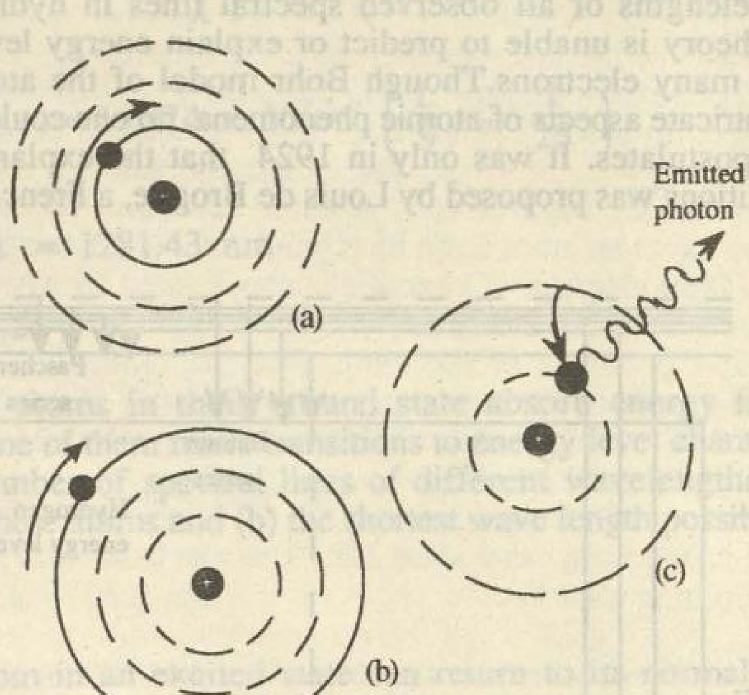


*Fig. 18.5 A model of the hydrogen atom in (a) ground and (b) in excited states (c) return to ground state*

When there are large number of atoms, different transitions take place simultaneously. Each transition involves the emissions of a photon whose energy depends on the energy difference of the two levels. (Eq. 18.7). Thus, radiation of different wavelengths are emitted. This is exhibited by the line spectrum of hydrogen which consists of light of many wavelengths as shown in Fig. 18.2 and Fig. 18.3.

It is convenient to represent the energy of the quantized states of the atoms on an energy level diagram. The energy level diagram for hydrogen atom is shown in Fig. 18.6. The energy levels of the atom are represented as a series of horizontal lines and the transitions between the levels are represented by vertical arrows.

The transitions from various energy levels terminating on the lowest level ($n = 1$) give rise to Lyman series. Balmer series occur for transitions ending at second energy level ($n = 2$). The Paschen, Brackett and Pfund series occur for transitions from various energy levels to third ($n = 3$), fourth ($n = 4$) and fifth ($n = 5$) energy levels, respectively. Only the Balmer series is in the visible part of the spectrum. The Lyman series is in the ultraviolet and the others are in the infrared region. Two of the possible transitions are also shown in the Fig. 18.6 that result in spectral lines in the hydrogen spectrum. The transition from $n = 3$ state to $n = 2$ state produce a photon of wavelength 656.3 nm, the resulting spectral line occurs in the Balmer series. Similarly the spectral line with wavelength 121.6 nm is given off due to transition from $n = 2$ state to $n = 1$ state. The wavelength calculated in this way are in excellent agreement with those observed experimentally in the line spectrum of hydrogen. Bohr theory successfully explains Balmer formula and predicts the wavelengths of all observed spectral lines in hydrogen spectrum. However, Bohr theory is unable to predict or explain energy levels in complex atom, which has many electrons.Though Bohr model of the atom successfully explained some intricate aspects of atomic phenomena, no one could understand the origin of Bohr's postulates. It was only in 1924 that the explanation of Bohr's quantization conditions was proposed by Louis de Broglie, a French physcist.

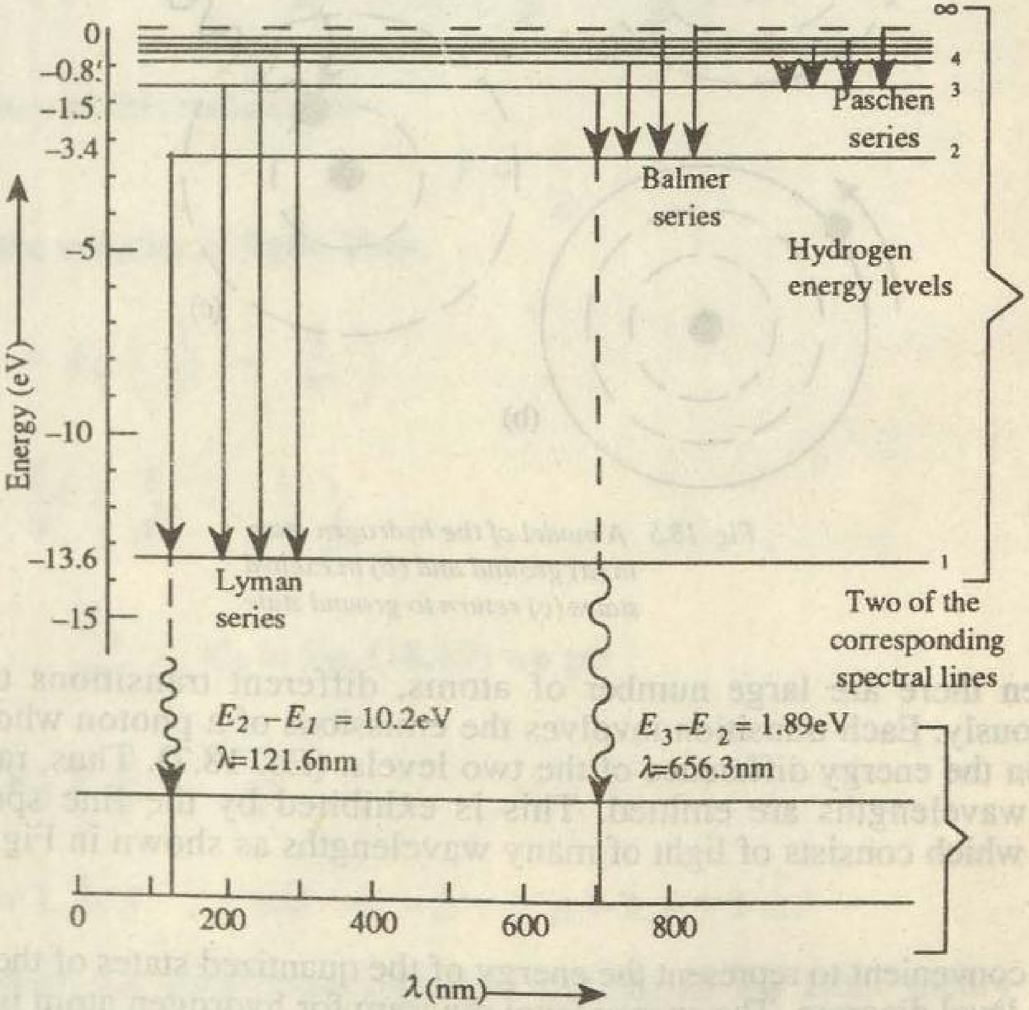

0
–0.8
–1.5
–3.4
–5
–10
–13.6
–15
Energy (eV)
∞
4
3
2
1
Paschen series
Balmer series
Hydrogen energy levels
Lyman series
Two of the corresponding spectral lines
$E_2 - E_1 = 10.2$eV
λ=121.6nm
$E_3 - E_2 = 1.89$eV
λ=656.3nm
0
200
400
600
800
λ(nm)

*Fig. 18.6: The energy level diagram for hydrogen atom also shown are two of the possible corresponding spectral lines.*

## Example 18.1

What is the wavelength of the second line of the Paschen series?

## Solution

We know that the Paschen series results from transition to the $n = 3$ state. The second line occurs when an electron falls from the $n = 5$ state to the $n = 3$ state in an atom. Therefore, from Eq. (18.20), we get

$$\frac{1}{\lambda} = R_H \left( \frac{1}{3^2} - \frac{1}{5^2} \right)$$

or $$\frac{1}{\lambda} = 1.0974 \times 10^7 \text{m}^{-1} \left( \frac{1}{9} - \frac{1}{25} \right)$$

or $$\lambda = 1281.43 \text{ nm}$$

**Example 18.2**

Hydrogen atoms in their ground state absorb energy from the incident photons and some of them make transitions to energy level characterized by $n = 4$. Find (a) the number of spectral lines of different wavelengths observed in the radiation from these atoms and (b) the shortest wave length possible.

**Solution**

(a) An atom in an excited state can return to its normal state by making transitions to any lower level, and each of these transitions can be followed by other transitions to lower levels until the ground level is reached. The possible transitions from level $n$ to level $p$ can be expressed as

| $n$ | $\rightarrow$ $p$ | Number of transitions |
|---|---|---|
| 4 | 3, 2, 1 | 3 |
| 3 | 2, 1 | 2 |
| 2 | 1 | 1 |
| Total | | 6 |

Thus the total number of transitions is six. There is a different wavelength for each transition, therefore, the number of different wavelengths emitted is also 6.

(b) The shortest wavelength results from the highest energy level transition, which is from $n = 4$ state to $p = 1$ state. Hence from Eq. (18.20) we obtain

$$\frac{1}{\lambda} = R_H \left( \frac{1}{p^2} - \frac{1}{n^2} \right)$$

or $$\frac{1}{\lambda} = 1.0974 \times 10^7 \left( 1 - \frac{1}{16} \right) \text{m}^{-1}$$

or $$\lambda = 97.2 \text{ nm}$$

## 18.3 EXCITATION AND IONIZATION POTENTIAL

The Bohr model of the atom predicts that the total energy of electron in the atom is quantized. For example, Eq. (18.18) gives the allowed energy values of the electron in one electron atom. The state for n = 1 is said to be the ground state, whereas states with $n = 2, 3, 4, .....$ are called the excited states. If an atom absorbs energy so that the electron in it is raised to one of its excited states, the energy absorbed to move an atom from its ground state to an excited state is called excitation energy. If an electron of charge $e$ is accelerated by applying a potential difference of $V$ volts to achieve that amount of energy, the applied potential $V$ is known as excitation potential of the atom. For example, the first and second excitation energies (written in electron volt (eV) unit) of hydrogen atom are approximately 10.2eV and 12.1eV, respectively. The corresponding excitation potentials are 10.2 volts and 12.1 volts.

The excited states have very short life. The electron soon returns to lower energy levels by emitting photons.

If an atom absorbs sufficient amount of energy, an electron may be raised to a level $n = \infty$. The electron then becomes free from the attractive influence of the nucleus. In other words, the electron is removed completely from the atom. An atom which has lost one or more electrons is said to be ionized. The minimum energy required to ionize an atom, which is in its ground state, is called ionization energy of the atom. This means, the supplied energy to the electron should be equal to its energy in the ground state minus its energy at $n = \infty$ state. Since the energy at $n = \infty$ state is zero, therefore, the energy to be supplied should be simply equal to the ground state energy of the electron. Thus the ionization energy is the ground state energy of the particular electron which is removed from the atom. If the energy is written in electron volt (eV), the corresponding potential $V$ is called the ionization potential.

There is only one value of the ionization potential equal to 13.6 V for hydrogen atom which corresponds to 13.6 eV as the ground state energy of its single electron. In the case of atoms with several electrons there can be more than one value of ionization potentials. The potential $V$ which corresponds to the energy required to remove the outermost or most loosely bound electron is called the first ionization potential. The energy to remove next most loosely bound electron corresponds to the second ionization potential and so on. The second ionization potential is obviously higher than the first, as more energy is required to remove the second electron which is held more firmly by the nucleus.

### Example 18.3

Find longest wavelength of light capable of ionizing a hydrogen atom. How much energy is needed to ionize a hydrogen atom?

**Solution**

The wavelength of light capable of ionizing the hydrogen atom will make the electron to raise from $p = 1$ to $n = \infty$

Using relation

$$\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

one gets $\frac{1}{\lambda} = 1.097 \times 10^7 \left(1 - \frac{1}{\infty}\right) \mathrm{m}^{-1}$

or $\lambda = 0.912 \times 10^{-7}$ m

$\lambda = 91.2$ nm

The energy required is the energy of the photon and is given as

$$E = hf = \frac{hc}{\lambda}$$

$$= \frac{6.63 \times 10^{-34} \text{ Js } 3 \times 10^8 \text{ m s}^{-1}}{91.2 \times 10^{-9} \text{ m}}$$

$$= 2.18 \times 10^{-18} \text{ J}$$

Thus $E = 13.6$ eV

## 18.4 INNER-SHELL TRANSITIONS AND X-RAYS

The possible energy levels for hydrogen atoms are of the order of a few electron volts. Thus the hydrogen spectrum consists of lines in ultraviolet, visible and infra red regions.

In the heavy atoms, the electrons are assumed to be arranged in concentric shells at increasing distance from the nucleus. These shells are labelled as K, L, M, N, etc. the K-shell being closest to the nucleus, the L-shell next, and so on. The outer shell electrons are loosely bound and relatively small amount of energy is sufficient to raise them to excited state or even for ionization. On their return to normal states, spectral lines of wavelength in or near the visible region are emitted. However, the situation is quite different for transitions involving inner shell electrons. They are closer to the nucleus and hence more tightly bound. Large amount of energy is required for their displacement from their normal energy levels.

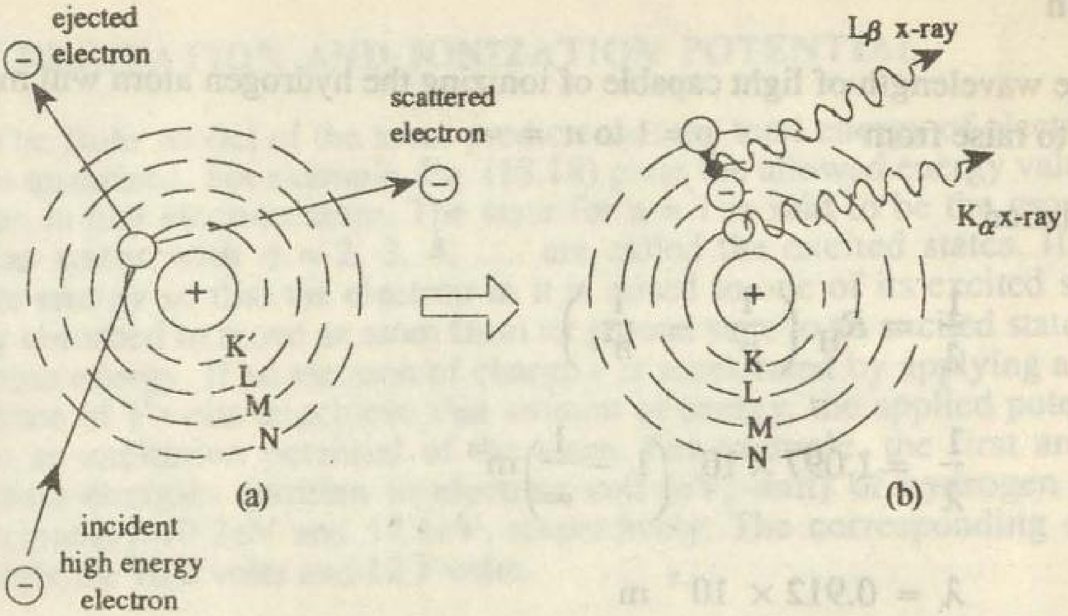


*Fig. 18.7 (a) Creation of vacancy in K-shell (b) Emission of x-ray*

Consequently, photons of larger energy are emitted when the atoms return to their normal states. Thus the transitions of inner shell electrons in heavy atoms give rise to the emission of high energy photons or x–rays. For that purpose, the sample usually consists of material of high atomic number and is bombarded with a beam of electrons that has been accelerated through several thousand volts. Some of these electrons will collide with inner-shell electrons of the sample and knock them out of their respective atoms. Let a K-shell electron is removed from an atom creating a vacancy in the K-shell (Fig. 18.7 a), then an electron from either the L, M, or N-shell will quickly jump down to fill the vacancy in K-shell emitting the excess energy as x-ray photon (Fig. 18.7 b). An x-ray photon due to transition from L–shell to the vacancy in the K - shell is called $K_\alpha$ characteristic x-ray. The transitions from M and N-shells to the K-shell give rise to $K_\beta$ and $K_\gamma$ x-rays, respectively. Similarly L, M and N series x-rays, relatively of lower energy, are produced due to the ejection of electrons from L, M and N-shells, respectively.

In addition to characteristic x-rays spectrum as described above, there has been observed a continuous back-ground radiation from the sample target. This has nothing to do with inner-shell transitions and is caused due to the sudden slowing down of electrons by the target material. The energy lost by decelerating electrons appears in the form of photons in the x-ray range. This is an effect similar to inverse photo-electric effect.

### 18.4.1 Production of x-rays

A modern x-ray unit essentially consists of

(i) A source of electron – a filament which also serves as cathode,

(ii) An evacuated metal or glass tube to accelerate the electrons,

(iii) A system to supply high potential difference to accelerate the electrons,

(iv) A target, usually of Tungsten, to serve as anode on which the electrons strike to produce x-rays.

A schematic diagram of an x-ray unit is shown in Fig. 18.8

The tube is evacuated so that the focussed beam of electrons from the filament is pulled on to the anode at very high speed because of very high potential difference (25 to 100 kV) applied between the filament and the target. The high potential difference is provided by a transformer from a.c. supply. The accelerated electrons are suddenly decelerated on impact with the target and some of the kinetic energy is converted into electromagnetic energy, as x-rays. The x-rays spread out from the target and pass easily through the glass tube walls. x-rays are only produced on the half cycle of a.c. when target is positive with respect to the filament. Most of the kinetic energy of the electron goes into producing heat. The heat is conducted away from the target by a copper rod in which the target material is embedded. The rod is cooled by circulating oil/water through it or by the use of cooling fins to dissipate the thermal energy.

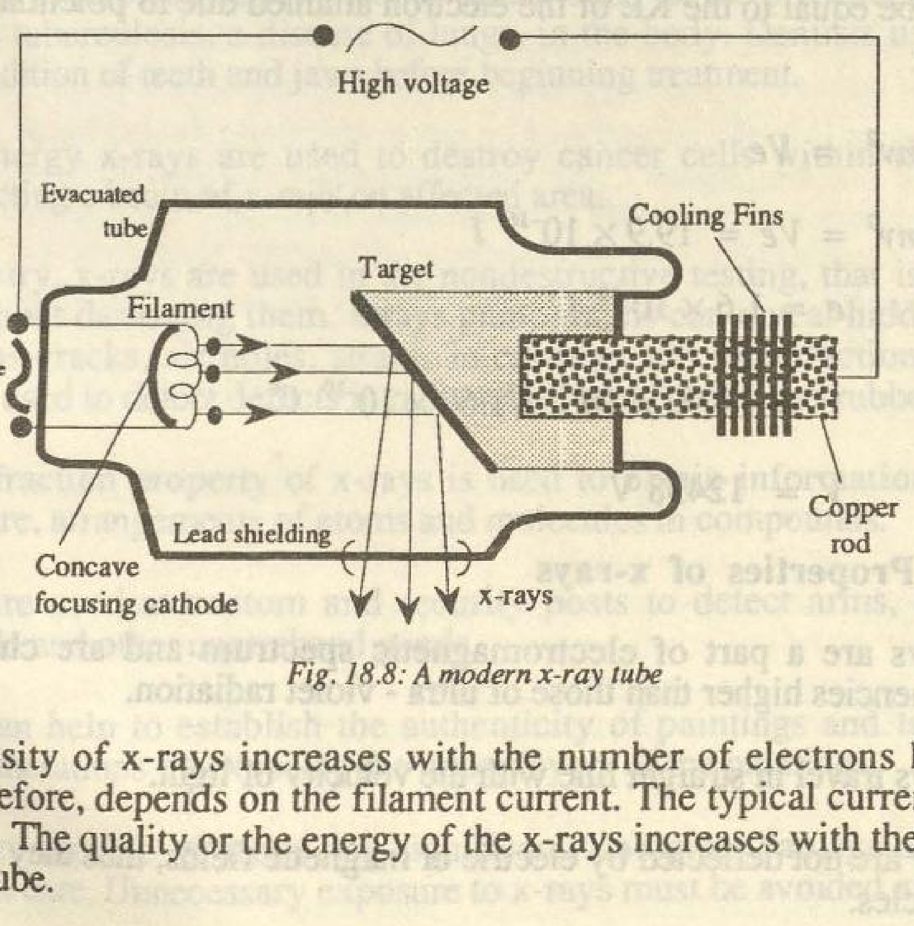


*Fig. 18.8: A modern x-ray tube*

The intensity of x-rays increases with the number of electrons hitting the target and, therefore, depends on the filament current. The typical current range is 100 to 500 mA. The quality or the energy of the x-rays increases with the operating voltage of the tube.

**Example 18.4**

Calculate the potential difference through which an electron must be accelerated in order that short wave limit of the continuous x-ray spectrum shall the exactly $1 \times 10^{-10}$ m.

**Solution**

The frequency corresponding $1 \times 10^{-10}$ m is given by

$$f = \frac{c}{\lambda} = \frac{3 \times 10^{8} \text{ ms}^{-1}}{10^{-10} \text{ m}}$$

$$f = 3 \times 10^{18} \text{ Hz}$$

The energy of the photon is

$$\text{h}f = 6.63 \times 10^{-34} \text{ J.s} \times 3 \times 10^{18} \text{ s}^{-1}$$

$$= 19.9 \times 10^{-16} \text{ J}$$

This must be equal to the KE of the electron attained due to potential difference *V*. Therefore,

$$\frac{1}{2}\, mv^2 = Ve$$

$$\frac{1}{2}\, mv^2 = Ve = 19.9 \times 10^{-16} \text{ J}$$

since $e = 1.6 \times 10^{-19}$ C

$$V = 19.90 \times 10^{-16} \text{ J}/1.60 \times 10^{-19} \text{ C}$$

$$V = 12438 \text{ V}$$

### 18.4.2 Properties of x-rays

i) x-rays are a part of electromagnetic spectrum and are characterized by frequencies higher than those of ultra - violet radiation.

ii) x-rays travel in straight line with the velocity of light.

iii) They are not deflected by electric or magnetic fields, thus they are not charged particles.

iv) They can be diffracted by a crystal lattice.

v) They can pass through many solids which are opaque to ordinary light such as wood or flesh etc. The penetration is the least with material of high electron density.

vi) The photographic film is blackened by x-rays depending upon the intensity of x-rays striking it.

vii) They can cause fluorescence in materials such as zinc sulphide, barium platinocyanide or cadmium tungstate by causing the excitations of their atoms.

viii) They cause ionization in gases through which they pass.

ix) x-rays liberate photoelectrons on striking some metals.

x) x-rays can produce damage in living tissues.

### 18.4.3 Uses of x-rays

1. The major use of x-rays is in medical diagnostics and treatment. The absorption of x-rays by dense material is made use of in x-ray imaging of the human body. Bones are composed of much denser matter than flesh, hence, x-rays passed through the body on to a photographic film produce a shadow photograph of bones in flesh. Physicians use them to see fractures in bones, location of foreign bodies such as swallowed safety pins inside the body or the presence of diseased areas such as tuberculosis, a disease of lungs, in the body. Dentists use x-rays to check the condition of teeth and jaws before beginning treatment.

High energy x-rays are used to destroy cancer cells within the body by carefully directing a beam of x-rays on affected area.

2. In industry, x-rays are used in the nondestructive testing, that is, testing of materials without damaging them. x-rays photographs can reveal hidden flaws in metals such as cracks, air holes, strains in castings and imperfections in welds. They are also used to detect defects in radio tubes, tennis balls and rubber tyres etc.

3. The diffraction property of x-rays is used to obtain information about the crystal structure, arrangements of atoms and molecules in compounds.

4. x-rays are used at custom and security posts to detect arms, explosives, precious metals and other contraband goods.

5. x-rays can help to establish the authenticity of paintings and to detect re-touching and alterations by exposing the under layers of the paintings.

x-rays are capable of producing significant biological damage and must be used with great care. Unnecessary exposure to x-rays must be avoided at all times.

## 18.5 THE LASER PRINCIPLE

The name LASER stands for Light Amplification by Stimulated Emission of Radiation, which is a concise description of how a laser works. The device can produce a very narrow, intense beam of coherent light by stimulated emission. This is a process in which de-excitation of an atom is caused by an incident photon with the emission of a second photon of the same energy which is coherent with the original photon as shown in Fig. 18.9.

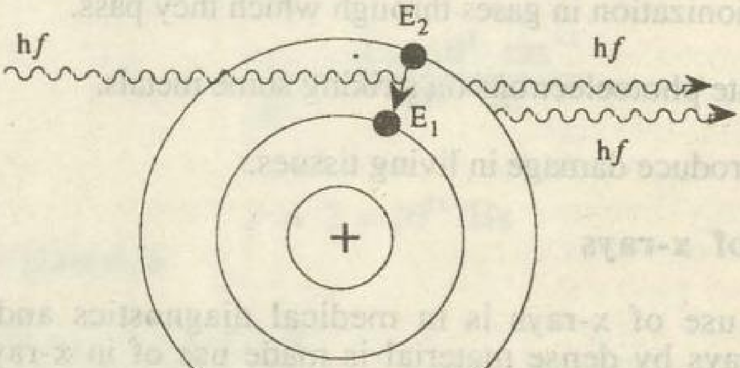
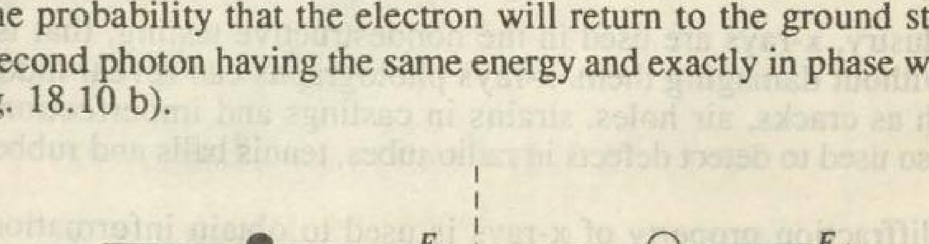


*Fig. 18.9 Stimulated emission of radiation*

Suppose an atom is in an excited state $E_2$ and a photon of energy exactly equal to the difference of the two states ($hf = E_2 - E_1$) is incident on it (Fig. 18.10 a). When the excited atom is in the presence of an electromagnatic field (Photon) that oscillates exactly with the frequency $f$, the field will stimulate the atom to de-excite by emitting a photon in phase with external field (Photon). The incident photon thus increases the probability that the electron will return to the ground state ($E_1$) by emitting a second photon having the same energy and exactly in phase with incident photon (Fig. 18.10 b).



(a) (b)

*Fig. 18.10. (a) A photon is incident on an atom in excited state.*
*(b) Stimulated emission .*

These photons can, in turn, stimulate other atoms to emit photons in a chain of similar events as shown in Fig. 18.11.

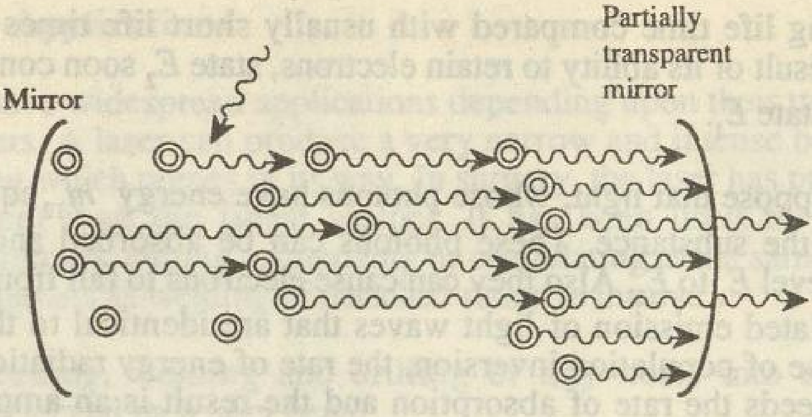

Partially
transparent
mirror
Mirror

*Fig. 18.11 Stimulated emission builds up a strong coherent radiation in a laser tube.*

### 18.5.1 Laser Operation

The operation of laser depends upon the existence of metastable (temporarily stable) state in the atoms of some substances. The metastable states are excited atomic states that can persist for unusually long periods of time. For example, the life time of most excited states is only about $10^{-8}$ second, but there exist in some substances such excited states (metastable) in which the life time is about $10^{-3}$ second or even more.

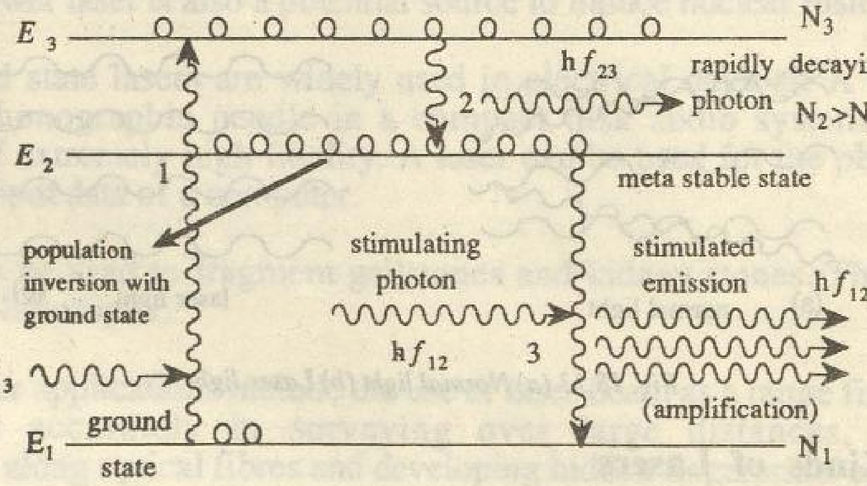

$E_3$
$N_3$
$hf_{23}$
rapidly decaying
2
photon
$N_2>N_3$
$E_2$
1
meta stable state
population
inversion with
ground state
stimulating
photon
stimulated
emission
$hf_{12}$
$hf_{12}$
3
$hf_{13}$
(amplification)
ground
$E_1$
state
$N_1$

*Fig. 18.12 The transitions in laser action*

When light is made incident on such a substance usually called the active medium, there is a net absorption of photons due to more atoms in the ground state than in the excited state. It is only if the situation can be inverted, that is, there should be more number of atoms in the excited state than in the ground state, a net emission of photons can result. Such a situation is called "population inversion".

Let the incident photons of energy $hf_{13}$ equal to $E_3 - E_1$ pump up the atoms from ground state $E_1$ to excited state $E_3$ (Fig. 18.12) but the excited atoms do not decay back directly to state $E_1$. Instead, they decay spontaneously emitting photon of energy $hf_{23}$ to the lower energy state $E_2$ which is a metastable state. This state has the special property of having a large stimulated emission probability because of relatively long life time compared with usually short life times of other excited states. As a result of its ability to retain electrons, state $E_2$ soon contains more atoms than ground state $E_1$.

Now suppose that light, whose photons have energy $hf_{12}$ equal to $E_2 - E_1$, is incident on the substance. These photons can be absorbed and excite the few electrons in level $E_1$ to $E_2$. Also they can cause electrons to fall from $E_2$ to $E_1$ giving rise to stimulated emission of light waves that are identical to the incident light wave. Because of population inversion, the rate of energy radiation by stimulated emission exceeds the rate of absorption and the result is an amplification of the ordinary light.

The emitted photons must be confined in the assembly long enough to stimulate further emission from other excited atoms. This is achieved by using mirrors at the two ends of the assembly. One end is made totally reflecting, and the other end is partially transparent to allow the laser beam to escape. As the photons move back and forth between the reflecting mirrors they continue to stimulate other excited atoms to emit photons. As the process continues the number of photons multiplies, and the resulting radiation is, therefore, much more intense and coherent than light from ordinary sources in which the emission of individual atoms are not coordinated as shown in the Fig. 18.13 (a) and (b).

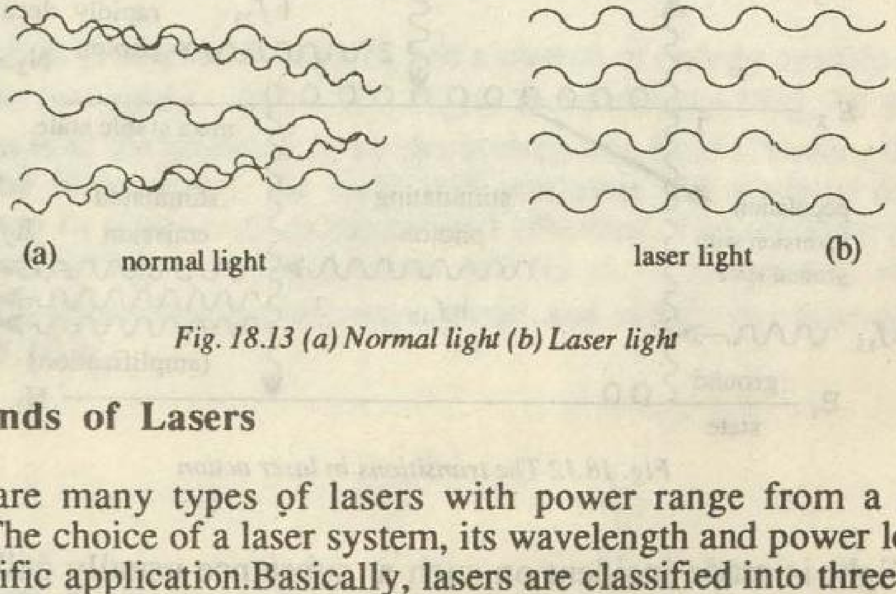


*Fig. 18.13 (a) Normal light (b) Laser light*

### 18.5.2 Kinds of Lasers

There are many types of lasers with power range from a milliwatt to megawatts. The choice of a laser system, its wavelength and power level depends upon its specific application.Basically, lasers are classified into three major kinds depending upon their light amplifying substance: 1. solid lasers 2. Liquid lasers and 3. gas lasers.

In solid lasers, a fluorescent crystal, such as that of a ruby, glass or a semiconductor is used as light amplifying substance. The liquid lasers usually make use of a dye dissolved in methanol or a similar liquid. Gas lasers have a gas or mixture of gases as their light amplifying substance. Helium – neon, argon ion, and carbondioxide gas lasers are the most widely used types of gas lasers.

### 18.5.3 Laser Applications

1. The lasers have widespread applications depending upon their type and other relevant parameters. A laser can produce a very narrow and intense beam that can vapourize anything which comes in its way. In surgery, the laser has proved a more delicate instrument than the finest scalpel. It has been used for bloodlessly removing small tumours, cutting, cauterizing or coagulating at the same time and in such delicate operations as welding of detached retina to the eye.

2. Precision cutting, welding and drilling of tiny holes into even hardest materials can be achieved by a laser beam.

3. A very interesting application is the production of three dimensional images called holograms. A hologram records not only the intensity of the light but also the phase difference between two different image beams that causes the interference pattern to reconstruct an image suspended in mid air as if the object is there in reality.

4. The laser is a potential environmental monitoring instrument. Some types of lasers can be tuned to a desired frequency. These lasers can be used as very sensitive detectors of pollutants in the atmosphere.

5. A high power laser is also a potential source to induce nuclear fusion reaction.

6. Tiny solid state lasers are widely used in electrical devices. A laser beam replaces the phonographic needle in a compact disk audio system for music reproduction of extremely high fidelity. A laser can be used for the photographic recording of output data of a computer.

7. Laser can be used to fragment gallstones and kidney stones. The device is known as laser lithotripter.

8. Some other applications include the use of laser beam as a range finder, lining up equipment accurately in surveying over large distances, telephone communication along optical fibres and developing hidden fingerprints etc.

The use of laser beam need special care. Even a beam of moderate power can cause skin burns and a very low power beam can damage the eye.

# SUMMARY

**Balmer Formula:**

Balmer relation to explain the wavelengths of the lines in the hydrogen spectrum is

$$\frac{1}{\lambda_n} = R\left(\frac{1}{2^2} - \frac{1}{n}\right) \qquad n = 3, 4, 5, \ldots\ldots\ldots$$

where $\lambda_n$ is the wavelength corresponding to $n$ which may have integral values 3, 4, 5, ... and $R$ is Rydberg constant.

**Bohr's Postulates :**

To explain the line spectra emitted by atoms as well as their stability, Bohr postulated that the electron can exist only in certain orbits such that angular momentum, $m v_n r_n$, is an integral multiple of $\frac{h}{2\pi}$, where $h$ is Planck's constant.

An electron in a quantized state does not emit radiation. When an electron jumps to a lower state it emits a photon whose energy equals the difference in energy between the two states.

**Orbital radii and energies :**

Assuming the circular orbits and a Coulomb force of attraction between electron and proton, the quantized radii ($r_n$) and energies ($E_n$) of the orbiting electron for hydrogen atom are;

$$r_n = n^2 \frac{h^2}{4\pi^2 m e^2 k} \text{ where } n = 1, 2, 3, \ldots\ldots$$

$$\text{and} \quad E_n = -\left(\frac{1}{n^2}\right)\left(\frac{2\pi^2 e^4 k^2 m}{h^2}\right), \quad n = 1, 2, 3, \ldots\ldots$$

where $k$ is the Coulomb constant, $e$ is the charge on the electron, $m$ its mass and $n$ is an integer called quantum number.

**Emission of spectral lines :**

If the electron in the hydrogen atom jumps from an orbit whose quantum number is $n$ to an orbit whose quantum number is $p$, it emits a photon of wavelenght $\lambda$, given by

$$\frac{1}{\lambda_n} = \frac{2\pi^2 me^4 k^2}{h^3 c} \left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

where *c* is speed of light.

**Emission of x-rays :**

Characteristic x-rays are emitted due to transitions of inner shell electrons in heavy atoms. In an x-ray tube, x-rays are produced when high speed electrons are decelerated upon striking a metal target.

**Laser :**

A laser is a device that produces an intense, monochromatic and coherent beam of light based on stimulated emission of photons from atoms.

**A laser application :**

An interesting application of laser is the production of three dimensional images called holograms.

## QUESTIONS

**18.1** What are the main characteristics of a line spectrum? Explain how line spectra can be used for the identification of elements?

**18.2** Suppose you are given a glass tube having two electrodoes sealed on both ends. The gas inside it is either hydrogen or helium. How can you tell which one is it without breaking the tube?

**18.3** The hydrogen atom contains only a single electron and yet the hydrogen spectrum contains many lines. Why is this so?

**18.4** The electron in a hydrogen atom requires an energy of 10.2 eV for excitation to a higher energy level. A photon and an electron, each of energy 10.5 eV, are incident on the atom. Which of these can excite the atom? Give explanation in support of your answer.

**18.5** Describe the atomic processes in the target of an x-ray tube where by x-ray line spectra are produced.

**18.6** Explain clearly why x-ray emission in the range of 0.1 nm are not observed from an x-ray tube when a low atomic number metal is used as the target in the tube?

**18.7** Why does the frequencies of characteristic x-rays depend on the type of material used for the target?

**18.8** Explain the difference between laser light and light from an incandescent lamp.

**18.9** Give a brief account of the laser principle? Write down some applications of lasers.

**18.10** Why does laser usually emit only one particular colour of light rather than several colours?

## PROBLEMS

**18.1** The quantized orbits of the hydrogen atom are given by the expression $r_n = 0.053 \times n^2$ nm, where $n$ is an integer. Calculate radii of first, second and third Bohr orbits of hydrogen.

[ Ans: 0.053 nm, 0.212 nm, 0.477 nm ]

**18.2** The first line of the Lyman series corresponds to an electron transition from $n = 2$ to $n = 1$ state. Calculate the wavelength and in which part of the spectrum does this line occur?

[ Ans: 121.5 nm, ultraviolet ]

**18.3** What is the shortest wavelength radiation in the Balmer series? What value of $n$ must be used?

[ Ans: 364.5 nm, $n = \infty$ ]

**18.4** (a) Calculate the longest and shortest wavelengths for the Paschen series, (b) determine the photon energies corresponding to these wavelengths.

[ Ans: (a) $\lambda_{max} = 1875$ nm, $\lambda_{min} = 820.0$ nm, (b) 0.661 eV, 1.51 eV ]

**18.5** A sample of hydrogen atoms in their ground state absorbs energy from the incident photons, and some of the atoms make transitions to energy level characterized by $n = 5$. Find the number of spectral lines of different wavelengths observed in the radiation from these atoms.

[ Ans: 10 ]

**18.6** The energy levels of the hydrogen atom given by the expression

$$E_n = -\frac{2.18 \times 10^{-18}}{n^2} \text{ J, where } n \text{ is an integer.}$$

(a) What is the ionization energy of the atom?

(b) What is the wavelength of the spectral line which arises from transition between $n = 3$ and $n = 2$ levels?

[ Ans: (a) $2.18 \times 10^{-18}$ J (b) 656.2 nm ]

**18.7** What is the longest wavelength of light that will remove the electron from a hydrogen atom in its ground state?

[ Ans: 91.2 nm ]

# 19

# THE ATOMIC NUCLEUS

In the previous chapters we have mainly focussed our attention on the bulk behaviour of matter as it appears in solid, liquid and gaseous forms. However, a deeper insight into the mechanisms of these variety of forms and behaviours require the understanding of the basic building blocks of matter. This was realized even by the learned men of ancient times. The early Greeks believed that the bulk phenomenon associated with matter was just a complicated manifestation of a much simpler behaviour of some fundamental consituents of matter. They believed that these fundamental species of a matter were too small entities to be seen and could not be divided further. They called these fundamental constituents of matter as atoms. Interestingly enough, the idea that matter is made up of atoms is the one that is nearer to our present concept of matter.

The ideas of Greek philosphers did not base on investigations. The experiments in this direction were mostly carried out by chemists. All these efforts culminated into an atomic theory enunciated by Dalton in 1808. According to this theory matter was made up of tiny, indivisible particles which Dalton also called atoms. Dalton's atom could neither be divided into smaller parts nor be destroyed. The idea that the matter is composed of atoms is well accepted today.

By the turn of the 19th century, the concept of indivisibility, however, started losing ground mostly due to work of physicists. The passage of electricity through electrolytic solutions, conduction of electric current through gases — a phenomenon which is used now-a-days in familiar neon lighting. Natural and artificial radioactivity and production of atomic and molecular spectra showed definite signs of the presence of subatomic particles.

## 19.1 DISCOVERY OF THE ELECTRONS

In 1891, J. J Thomson performed famous experiment relating to the passage of electric current through a gas. At ordinary pressure little or no current will pass through a gas. But when the pressure is reduced to about one hundreth of normal atmospheric pressure, an electric current can be made to flow fairly readily.

The typical apparatus used in Thomson experiment consisted of two metal plates A and C, each with a pinhole through its centre, which were sealed into a closed tube evacuated by a vacuum pump as shown in Fig. 19.1. When a high-voltage was connected across the plates, a current flowed between them. It was noticed that two streams passed through the pinholes in the plates and flashed on zincblende. In order to know the nature of these streams a magnetic field was applied across the tube. The two thin streams were found to be bending by the magnet. It was sure test to believe that the stream consists of electrically charged particles. Thomson and his collaborators and other carried out a series of experiments of different kind and were successful to discover a number of properties of these new particles. It was found that the current consisted of negatively charged particles which are now called electrons. These experiments led scientists to conclude the following:

The atoms of the gas, far from being indivisible, are readily broken up into fragments, which carry an electrical charge.

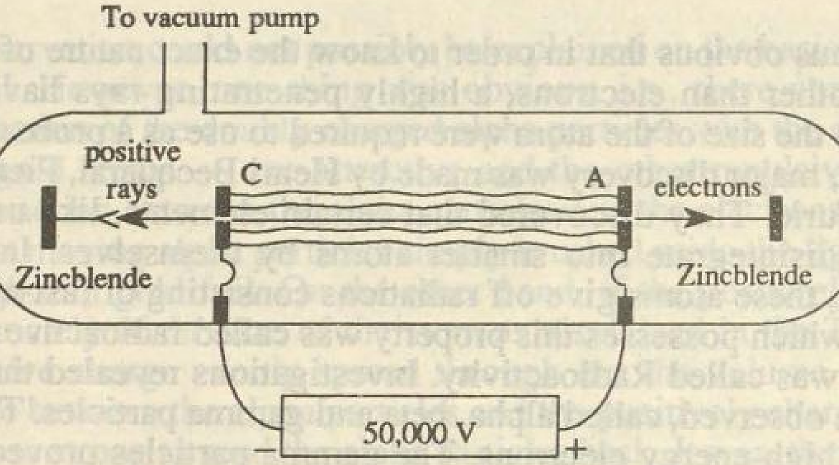


*Fig. 19.1 Shown two metal plates, each with a pinhole through its centre sealed into a closed tube, and attached to vacuum pump. High voltage is connected across the plates. Two thin streams of charged particles pass through the pinhole and flashes on the zincblende.*

The negatively charged particles were attracted by positively charged plate (A), where a few of them escaped through the pinhole. Similarly, the postively charged particles were attracted towards the negatively charged plate, and some again escaped.

In 1909, R. A. Milikan successfully determined the charge on the electron. Knowing the charge it became possible to determine mass of the electron using Thomson's value for the charge to mass ratio of electron. This showed that an electron was much smaller than an atom. Thus this series of experiments proved that atoms contain electrons and that the atom can be subdivided.

## 19.2 NUCLEUS DISCOVERED

Note that, before the discovery of the electron and possibility of subdivision of the atom the chemists had determined the atomic weights of the elements in terms of the weight of hydrogen atoms. No one, however, at that time knew how big the atom was and what was its actual weight. By the end of the 19th century, efforts were made to get an idea of the size of atom. One of these methods based on the idea of finite dilution of small drop of oil on the surface of water to get one molecule. In this way the length of a long molecule of oil was measured. Knowing the number of atoms it contained, a rough figure for the size of the atom could be calculated. Assuming the atom to be of round solid lump, a diameter of $10^{-10}$m was calculated for the atom. This crude estimate turned out to be quite accurate as confirmed later on by some very sophisticated experimental techniques.

On the basis of experimental data available at that time, Thomson tried to visulize a model for an atom. Since the whole atom is electrically neutral, the removal of electron in Thomson experiment left the lump of the atom with an equal but opposite charge. These postively charged fragments were attracted to the negative plate in his experiments. Thus, this led Thomson to propose that an atom had a central core like a pudding with one or more electrons like plums embeded in the surface.

It was thus obvious that in order to know the exact nature of an atom and its constituents other than electrons, a highly penetrating rays having wavelength comparable to the size of the atom were required to use as a probe. Luckily, around the same time, major discovery was made by Henri Becqueral, Pierre Curie and his wife Maire Curie. They discovered that certain elements, like uranium, radium, thorium etc, disintegrate into smaller atoms by themselves. In the process of disintegration, these atoms give off radiations consisting of fast-moving particles. The element which possesses this property was called radioactive element and the phenomenon was called Radioactivity. Investigations revealed that three types of particles were observed, called alpha, beta and gamma particles. The beta particles proved to be high-energy electrons. The gamma particles proved to be photons, much like light, but of much shorter wavelength and much greater penetrating power. The alpha particles were shown to be positively charged helium atoms that had lost two electrons, with a mass about four times that of hydrogen.

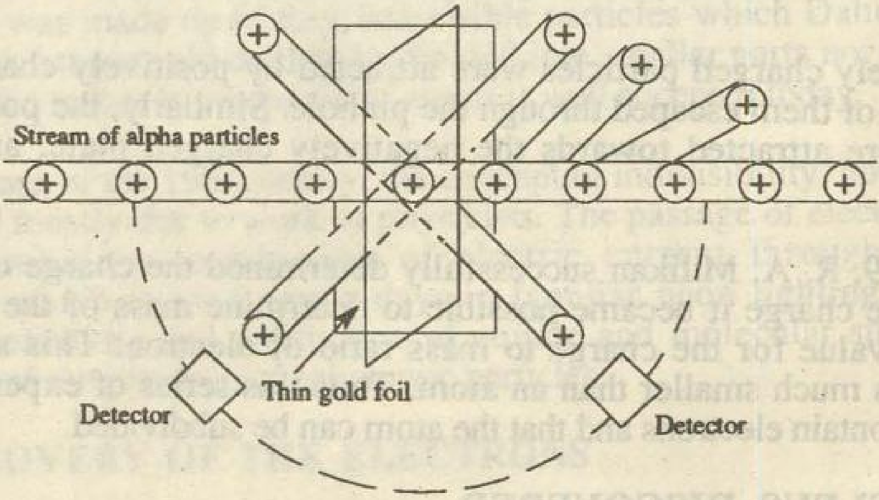


*Fig. 19.2 A thin gold foil is shown bombarded with a stream of alpha particles to study their scattering*

The alpha paticles can pass through opaque matter, therefore, Ernest Rutherford, a New-Zealand Physicist working at the Manchester University on radioactive elements, thought to explore the Thomson atom by using alpha particles as a probe. Around 1911, he bombarded a thin sheet of gold with a stream of fast moving alpha particles and studied deflection produced (Fig. 19.2).

The mineral coated screen was used as a detector to detect the scattering of the alpha particles from the thin gold foil. On striking the screen of the detector the particles made tiny flashes, observable by the human eye. To his surprise, it was observed that most of the alpha particles passed right through the gold foil, but a few of them were deflected through large angles, some of them even coming straight back. Rutherford expresses :

> "It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inches shell at a piece of tissue paper and it came back and hit you."

This observation could not possibly be explained on the basis of the Thomson atomic model. However, one thing was obvious, i.e., there could only be two types of interaction of the doubly charged alpha particles with the electrons and the cores of the gold atoms — one attractive and the other repulsive. As far as the attractive interaction between the alpha particles and the electrons is concerned, it seems that fast moving alpha particles simply brushed aside the light electrons and passed through thin gold foil. On the other hand, if the positively charged cores were approximately of the size of the atom as visulaized in the Thomson model, then the positive charge evenly spread through out the volume of the foil. The repulsive force between the alpha particles and the positively charged cores would not have been so strong to send the alpha particles back. It was, therefore, logical to conclude that the positive charge in atoms of the gold was not uniformly spread in each atom, rather it is concentrated into a smaller core, called Nucleus. Now, as electrical interaction force is inversely proportional to the distance between the charged particles, close encounters of an alpha particle with a nucleus produced large deflections, whereas less close encounters produced small deflection. The fluffy pudding atomic model of Thomson had to be replaced by a model consisting of hard scattering centres. Rutherford even determined the size of these nuclei to be approximately equal to $10^{-14}$m; i.e., about 1/10,000 th of the size of the atom.

Experiments were performed using other elements as a target material and it was found that the atoms of all other elements too had nuclei.

Rutherford proposed a new model of the atom after the disocvery of the nucleus containing only positive charge. He imagined it almost like a small solar system. The electrons were now assumed to be circling around nucleus like planets around the sun. The electrical attraction between the opposite charges on the electrons and the nucleus was assumed to play the same role as played by gravitational attraction between the planets and the sun. This model had to be abandoned, because of the reasons discussed in the introductory part of chapter 18.

### 19.2.1 Structure of the Nucleus

As stated in the preceding section Rutherford from his experiments with alpha particles and thin gold foil had concluded that the atom contained its positive charge in a small, massive centre having diameter of the order of $10^{-13}$ m. He also determined the total charge on the nucleus but could not know whether this charge in the nucleus is of a single positively charged particle or an aggregate of many positively charged particles. Since the total charge determined by Rutherford for the central part of the atom of gold turned out to be ~ 100e, he believed that nucleus is made up of 49 atoms of helium, each carrying a charge of 2e. It was natural for him to believe this in view of the expulsion of helium atoms (the alpha particles) carrying two unit charges from the radioactive matter.

In 1919, Rutherford bombarded nitrogen with alpha particles and to his surprise, he detected new radiation emitted from it. The nitrogen turned into oxygen. The new radiation was identified to be hydrogen nuclei. Rutherford, therefore, concluded that the nitrogen atom (nucleus) was broken up under the intense forces developed in a collision with fast alpha particles. The hydrogen atom which was liberated from a consituent part of the nitrogen nucleus. In 1920, Rutherford called hydrogen nuclei as protons.

Thus the nucleus of a particular atom, with a given number of orbitting electrons, must contain the same number of protons to balance the charge. The mass of the actual nucleus was found to be roughly double the value attained on the basis of the number of protons in the nucleus. This difficulty was resolved with James Chadwick's new particle, the Neutron.

Chadwick studied the scattering of alpha particles from a beryllium target. He found that when beryllium nuclei were bombarded with alpha particles, some of them turned into carbon nuclei with the emission of heavy neutral particles of almost the same size as a proton. The newly discovered particle was called the neutron.

We now believe that a model of the atom consists of a central region called nucleus with a nuclear diameter of the order of $10^{-12}$ m surrounded by a cloud of electrons, giving an atomic radius of order of $10^{-10}$ m. The nucleus contains proton and neutrons — protons to have a positive charge equal in magnitude to the charge of electrons, and then roughly an equal number of neutrons to give the observed total mass. Neutrons and protons together are called as nucleons.

Nuclei of different elements are now identified by their atomic number Z, an integer equal to the number of protons in the nucleus, which is also equal to the number of electrons in the outer cloud. The atomic mass number of a nucleus, designated by the symbol A, is the total number of nucleons in the nucleus. A nuclide is a particular nucleus with a specified number of protons and neutrons. Any nuclide may be represented by its chemical symbol X, together with its atomic and mass number: ${}_zX^A$ , for example:

$${}_1H^2\,,\ {}_6C^{12}\,,\ {}_7N^{14}\,,\ {}_{92}U^{235}$$

The nucleus of a radioactive element is called as radio-nuclide.

### 19.2.2 Nuclear Mass

It is known that a gram mole of any substance should contain $6.022 \times 10^{23}$ number of atomic nuclei. Thus the mass of a nucleus is of the order of $10^{-24}$ kg. Since it is very small number, therefore, the nuclear mass is expressed in terms of unified (U) mass scale. The unified mass which is based on the rest mass of an atom of carbon $C^{12}$ taken exactly equal to 12 units. Thus one atomic mass unit (amu) is equal to 1/12 of the mass of the carbon atom. Its relationship with gram can be found using the equation

$$12 \text{ atomic mass units of carbon } C^{12} = \frac{12}{6.022 \times 10^{23}} \text{ g}$$

$$\therefore \quad 1 \text{ amu} = 1.660 \times 10^{-27} \text{ kg}$$

The mass of proton on a unified atomic mass scale is 1.007276 and that of neutron is 1.008665. The electron mass has only 0.000548 amu. The atomic masses of some light elements ($Z \leq 4$) is given in Table 19.1. (a)

Table 19.1(a) : Atomic Masses of Some Light Elements

| Nucleus | Z | Mass (amu) |
|---|---|---|
| n | 0 | 1.008665 |
| $H^1$ | 1 | 1.007276 (stable) |
| $H^2$ or $D^2$ (deuterium) | 1 | 2.014102 (stable) |
| $H^3$ or $T^3$ (tritium) | 1 | 3.016050 |
| $He^3$ | 2 | 3.016030 (stable) |
| $He^4$ | 2 | 4.002603 (stable) |
| $He^5$ | 2 | 5.012297 |
| $He^6$ | 2 | 6.018893 |
| $He^7$ | 2 | 7.028031 |
| $Li^5$ | 3 | 5.012538 |
| $Li^6$ | 3 | 6.015125 (stable) |
| $Li^7$ | 3 | 7.016004 (stable) |
| $Li^8$ | 3 | 8.022487 |
| $Be^6$ | 4 | 6.019717 |
| $Be^7$ | 4 | 7.016929 |
| $Be^8$ | 4 | 8.005308 |
| $Be^9$ | 4 | 9.012186 (stable) |
| $Be^{10}$ | 4 | 10.013534 |

The properties of particles in the atom are summarized in table 19.1 (b).

Table 19.1 (b). The Properties of Electron, Proton and Neutron.

| | | **Rest mass** | |
|---|---|---|---|
| **Particle** | **Electric Charge (C)** | **Kilograms (kg)** | **Atomic Mass Units (amu)** |
| Electron | $-1.60 \times 10^{-19}$ | $9.109390 \times 10^{-31}$ | $5.485799 \times 10^{-4}$ |
| Proton | $+1.60 \times 10^{-19}$ | $1.672623 \times 10^{-27}$ | 1.007276 |
| Neutron | 0 | $1.674929 \times 10^{-27}$ | 1.008665 |

### 19.2.3 Isotopes

F.W. Aston developed an instrument, called mass spectrometer, which uses electric and magnetic fields to sort out atoms according to their masses. The schematic diagram of this instrument is as shown in Fig. 19.3. The source is a device to obtain positively charged portion of the atom, called +ve ion. The positive terminal of a battery is connected to the slit $S_1$, while the negative to the slit $S_2$. Thus the positive ion escaping the slit $S_1$ are accelerated to the negative slit $S_2$ due to the potential difference $V$ through which the ion falls. Let it be a single charged ion having mass $m$ and positive charge $e$. Then the kinetic energy attained by it upon reaching the slit $S_2$ will be $Ve$. If $v$ is the velocity of this ion at $S_2$, then we have

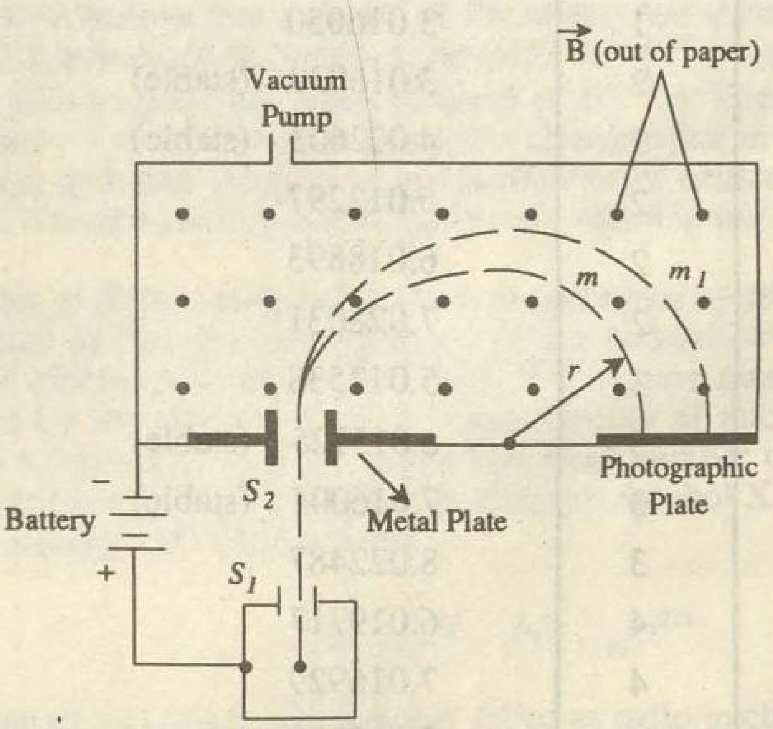


*Fig. 19.3. A schematic representation of Aston mass spectrometer. After being accelerated through a potential difference V, positive ions enter a constant magnetic field* **B** *that is directed out of page, toward the reader. The dashed lines are the paths travelled by two ions of different masses.*

$$\frac{1}{2}mv^2 = Ve \tag{19.1}$$

The ion on entering the evacuated chamber through the slit $S_2$ with a speed $v$ is deflected in a circular path by a magnetic field perpendicular to the plane of the paper (pointing outward, in the Fig. 19.3). The ions with different masses are deflected differently and thus strike at different places of the photographic plate. The magnetic force on the ion is *Bev* and is equal to the centripetal force supplied to the ion. Mathematically,

$$Bev = \frac{mv^2}{r} \tag{19.2}$$

where $r$ is the radius of the circular path as shown in the figure. Substituting the value of $v$ from Eq. (19.1), one gets the following expression for $r$ :

$$r = \sqrt{\frac{2Vm}{B^2 e}} \tag{19.3}$$

This relation shows that for *V*, *B* and *e* to be constant, *r* depends upon the mass of the ion. Thus ions with different masses will strike the photographic plate at different places.

Aston observed that there are certain elements which give more than one beam of ions and hence more than one spot on the photographic plate. The masses calculated on the basis of the radii were found out to be different for atoms belonging to the same element. This led to the discovery that many of the common elements do not have entirely identical atoms in them. That is, all the atoms in a given element although have the same number of protons in their nuclei, yet some with the same chemical properties (the same number of electrons) but different atomic masses. Such atoms in an element were called Isotopes (iso-equal, tope-place). Thus "Isotopes are two or more nuclides of the same element, that is with the same chemical symbol X and atomic number Z, but with different number of neturons hence different mass number A". For example, ${}_6C^{12}$ and ${}_6C^{14}$ both have six protons but six and eight neutrons, respectively. Chlorine is a mixture of isotopes of amu. 35 and amu 37 each having the same number of protons inside the nucleus. Natural uranium contains two isotopes ${}_{92}U^{238}$ (99%) and ${}_{92}U^{235}$ (little less than 1%). ${}_{92}U^{235}$ is used as a fuel for nuclear reactors, therefore, some very sophisticated facilities have been developed to separate ${}_{92}U^{235}$ from natural uranium. The process is known as enrichment. Hydrogen is found in three isotopic forms, ${}_1H^1$ , ${}_1H^2$ and ${}_1H^3$. The nucleus ${}_1H^1$ is just a proton, ${}_1H^2$ is a proton and a neutron bound together and is called a Deuteron — the nucleus of heavy hydrogen. The isotope ${}_1H^3$ consists of a proton and two neutrons, and is called a Triton.

It may be remarked that except for the lightest atoms, different isotopes have similar properties. However, different isotopes may behave differently specifically in nuclear reactions.

### 19.2.4 Stability Of Nucleus

Having said that a nuclues is composed of protons (positively charged particles) and neutrons (electrically neutral particles) packed together at a distance of less than $10^{-15}$m in some way, we must answer the basic question of what holds these nucleons together. Classically, we are aware that two types of forces — gravitational and electromagnetic — should be operative amongst the individual particles of the nucleus. But it is not difficult to show that these two cannot account for the stability of nuclear matter. For example, the forces of repulsion due to the positive charge on each proton is

$$F_{\text{coulomb}} = \frac{Ke^2}{r^2} \tag{19.4}$$

the gravitational force of attraction is

$$F_{\text{grav}} = \frac{Gm_p m_p}{r^2} \tag{19.5}$$

and the ratio of the two is

$$\frac{F_{\text{grav}}}{F_{\text{coulomb}}} = \frac{Gm_p^2}{Ke^2} \cong 10^{-36} \tag{19.6}$$

This shows that the electrostatic repulsive force between positively charged protons is extremely stronger than the attractive gravitational force, which is found to be negligibly small. It may be remarked here that electromagnetic repulsive forces between protons in the nucleus are of the order of million electron volts. Say, for example, for a nucleus of 50 protons in the nucleus the addition of one more proton to form Z = 51 requries 7.2 MeV

$$\left[ = \frac{K50e^2}{r} = \frac{9 \times 10^9 \times 50 \times (1.6 \times 10^{-19})^2}{10^{-14}} = 1.15 \times 10^{-12}\ \text{J} = 7.2\text{MeV} \right]$$

It is worth comparing this number with the energy released in atomic transitions which is just tens of electron volts. Thus, in view of such a powerful coulomb repulsive force it can be imagined that the state of the nucleus should be like a compressed spring ready to straighten out. Since the nucleus holds together, completely new and specifically different type of force is exerted between protons and neutrons. This force is the strong nuclear force and is one of only four fundamental forces that have been discovered. This force should be attractive and stronger than the repulsive electromagnetic force between positive charges.

Another problem which makes the question regarding the stability of nucleus all the more important is that of disintegration of neutron itself. It is known that a free neutron disintegrates just in about 12 minutes. On the otherhand, it is stable inside the nucleus.

Perhaps one way to look at the stability of the nucleus is by associating a motion with the nucleons. The nature of this motion is such that it restores the system. In other words the stability of the nucleus is invested in the dynamic equilibrium. This situation is somewhat similar to the case of still water surface of a pond. One finds that here the apparent immobility and stability hides intense motion, too. Molecules of water are constantly evaporating from the surface and conversely, molecules of vapour are constantly being captured by the water. If these two counter streams of molecules are the same, the water level does not change and we have equilibrium.

In order to understand the role of dynamic equilibrium in maintaining the stability of the nucleus, let us imagine, two men carrying a load. Let this load is too heavy for a single person to carry it far without dropping it. This is obvious that in the absence of the second person, the first person would carry the load for sometime and then sooner or later bound to drop it. On the other hand, if the two persons constantly hand it over to one another once in a while they can carry the same load easily for a much longer period of time without dropping it. But to do this they would have to be close together. How close depends on the mass exchanged between the two. The larger the mass the closer will have to be the two men.

Thus the exchange of the load serves as a link between the two men and is responsible for the successful continuation of carrying the load.

Using the above explained analogy, it seems reasonable to assume that the presence of proton alongside the neutron stabilizes the latter. This is achieved through some exchange between neutron and proton in motion and thus bind them together inside the nucleus.

We discussed in the previous chapters that the charge particles interact with each other through the "fields" created by the charged particles around them. Similarly, we can say that a charge creates or emits quanta (intermediary particle) which is then absorbed by the other charge. The exchange of intermediary particles are responsible for interaction between charged particles.

Now the range and the strength of interaction between the nucleons, of course, should depend upon the type of the mediating particle which in turn depends upon its mass. It can be understood again by the analogy of the two men and the heavy load. The only way they could carry the load was by constantly handing it over to one another. But to do this they had to be close together. How close together? It would depend upon the mass of the load. The bigger the mass of the mediating particle the shorter should be the distance between them. That is, the range of interaction and its strength between the nuclear particle by way of exchange is given as

$$\phi(r) \propto m^{-1}$$

where $r$ is distance between the particles exchanging mediatory particle of mass $m$. The necessity of exchange binds the particle in the nucleus. To an outside observer, it will appear as if attractive forces are operating amongst nucleons. Thus along with the gravitational and electromagnetic forces, there exist weak and strong nuclear forces inside the nucleus depending upon the type of exchange interaction between the nuclear particles.

It has been established theoretically, as well as, experimentally that strong nuclear forces emerge because of exchange of mesons between the nucleons. The mesons are either charged (positive or negative) or neutral particles. The interaction between neutron - neutron or proton-proton is mediated through the exchange of neutral mesons ($\pi^\circ$), whereas, neutron-proton interaction is caused by the exchange of charged mesons ($\pi^{\pm}$). Recently, some very heavy particles like $W^+$, $W^-$ and $Z_0$ bosons have been observed to play the role of interaction between the nucleons.

It would be interesting to note that the emission of alpha, beta and gamma radiation from the nucleus, mentioned earlier in this chapter, is the result of nuclear interaction, which is very weak as compared to nuclear interaction occuring, i.e., in the sun or in the explosion of a nuclear bomb. It has recently been shown that the weak nuclear force and the electromagnetic force (which includes the Coulomb force) are aspects of the same phenomenon and can be combined at very high energies. Pioneering work in this direction was carried out by Dr. Abdus Salam of Pakistan for which he was awarded Nobel Prize in Physics in 1979. The combined or unified force is called the electroweak force, the particles $W^+$, $W^-$ and the $Z^+$ bosons are the carriers of electroweak interaction at very high energies.

The strong forces between protons and protons, protons and neutrons, and neutrons and neutrons are all approximately equal in strength. Unlike the Coulomb and gravitational forces, the nuclear forces fall off much more rapidly with increasing distance between the nucleons. Hence, the nuclear forces are short-range forces.

### 19.2.5 Mass-Defect : Nuclear Binding Energy

Another paramerter with the help of which the concept of the stability of the nucleus may be appreciated is the nuclear binding energy. Because of the strong nuclear force, the nucleons in a stable nucleus are held tightly together. Therefore, energy is required to separate the nucleus into its constituent protons and neutrons. The most stable the nucleus is, the greater the amount of energy needed to tear it apart. The required energy is called the nuclear binding energy. The binding energy of the nucleus is usually calculated using mass-energy equivalence relationship of Einstein. According to this relationship

$$E = mc^2 \qquad (19.7)$$

where $m$ is the mass of the particle moving with velocity of light. If $m_o$ is the rest mass of a particle, then $m$ and $m_o$ are related with each other through an equation

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{19.8}$$

Eq. (19.8) is called the relativistic mass variation relationship.

In view of the mass variation formula, Einstein proposed that the kinetic energy of an object in relativistic limits should be given as

$$\text{Kinetic energy} = KE = (m - m_o)\, c^2 = \Delta\, mc^2 \tag{19.9}$$

where $\Delta m$ is the change in mass of the particle. Einstein showed that the Eq. (19.7) is applicable to any form of energy and that the mass and energy are inter changeable. That is, the change in mass $\Delta m$ of a particle corresponds to an energy difference equal to $(\Delta m)c^2$ and vice-versa. Since the velocity of light ($c$) is a constant quantity, the difference in energy is proportional to the mass difference. In other words, the greater the mass difference the greater will be the difference of energy it has.

Now consider the case of the simple nucleus that depends on the nuclear force for its existence - the deuteron. We have discussed that a deuteron consists of one proton and one neutron. It has been found out that the mass of the deuteron ($m_d$) is slightly less than the combined masses of a free proton ($m_p$) and a free neutron ($m_n$). Using the atomic masses of hydrogen ($m_H$) deuterium nucleus ($m_D$) and the neutron, we have

| | | |
|---|---|---|
| Mass of hydrogen | $m_{\mathrm{H}}$ | = 1.007825 amu |
| Mass of neutron | $m_{\mathrm{n}}$ | = 1.008665 amu |
| (1) Total | | = 2.016490 amu |
| (2) Observed mass of deuterium | $m_{\mathrm{D}}$ | = 2.014102 amu |

Mass difference [(1) – (2)]

$$\Delta m = (m_{\mathrm{H}} + m_{\mathrm{n}}) - m_{\mathrm{D}} = 0.002388 \text{ amu}$$

**This result in fact is quite general: all nuclii have masses that are less than the masses of the constituent protons and neutrons together in the free state. The mass of a heavy nucleus, like that of uranium is about 1% lighter than the sum of the**

masses of its constituent nuclear particles. This apparent loss of mass is known as mass deficit or mass defect.

The magnitude of this mass deficit for a particular nucleus is indicative of the degree of binding of the protons and neutrons in that nucleus. According to the Einstein's mass energy relation [Eq. (19.7)], the mass difference $\Delta m$ corresponds to an energy difference $E_B$ which for deuteron is given as

$$E_B = (\Delta m) \times c^2$$

$$= (0.002388 \text{ amu}) \times c^2$$

The energy equivalent of one amu is 931.481 Mev ≅ 931.5 Mev

Thus

$$E_B = (0.002388 \text{ amu}) \times \left(931.5 \frac{\text{MeV}}{\text{amu}}\right) = 2.22 \text{ MeV}$$

This result means that 2.22 MeV of energy should be emitted when the neutron and proton combine to form a deuteron. Conversely, this is the energy required to change the deuteron into a free proton and a free neutron.

In general, the energy that must be supplied in order to separate particles held together by a force, is called the binding energy. Therefore, the binding energy of the nucleus (the ground state energy of the nuclear system, if the energy of the nucleons separated from each other is taken as zero) can be measured either as the energy required to break the nucleus into its constituents and to separate them or as the mass deficit between the bound nuclear system and that of proton and neutron out of which it is made. Thus the stronger the forces holding the nucleons together, the greater is the mass deficit.

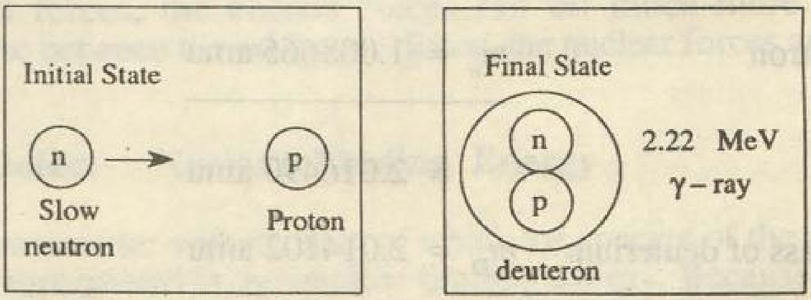


*Fig. 19.4. Schematic representation of the capture of a slow neutron by a proton to form a deuteron. The deuteron energy is released in the form of a γ-ray*

**The nuclear binding energy can also be interpreted in the following way. If, for example, a slow moving neutron is captured by a proton to form a deuteron, the**

initial mass-energy of the system, $(m_H + m_n)c^2$, is greater than the final mass-energy $m_D c^2$ as discussed above. And so the energy difference $E_B$ is radiated in the form of a γ ray, as shown in Fig. 19.4.

Large nuclei contain more protons. These protons rèpel one another because of their electric charge, and thus act against the attractive strong force discussed in the preceding section. The electric force is much weaker than the strong force, but it acts at a long range. Therefore, each proton is bound by strong force to its nearest neighbours, but it is repelled by every other proton in the nucleus. Despite greater strength of the strong force, there is a maximum number of protons that can be lodged together in a stable nucleus. These protons, because of their charge, act against the strong force; for large nuclei, the mass deficit per particle actually decreases somewhat as the nucelus increases in mass number rather than becoming greater. Therefore, there is a size of a nucleus for which the mass per particle is greatest. The simplified argument we have given here suggests a graph similar to that shown in Fig. 19.5. The Fig. 19.5 shows that the binding energy of the element with Z > 83 decreases with increasing mass number.

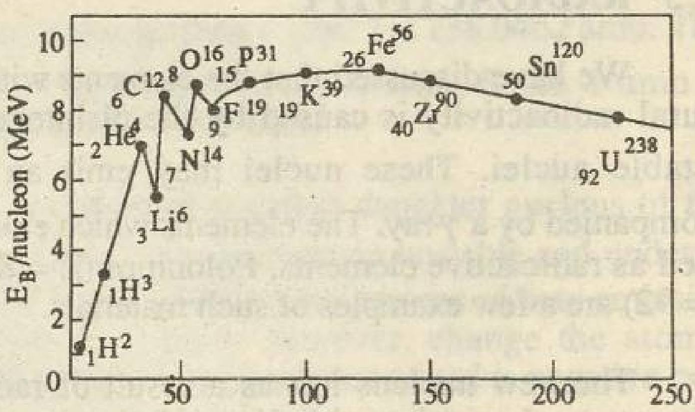


*Fig. 19.5 The binding energy per nucleon as a function of mass number*

The termination of the nuclear stability near A = 210 is the consequence of the long-range electric and short-range nuclear forces discussed above. Thus the Coulomb repulsion dominates the attractive effect of the nuclear force of heavy element. In fact, there are no stable nuclei with Z > 83. These instabilities on either side of the most stable region have some important consequences: we shall discuss in turn, where as beginning with radioactivity in the following.

**Example 19.1**

How much energy is required to break up a $C^{12}$ nucleus into three alpha particles ?

**Solution**

By definition, the atomic mass of $C^{12}$ is 12 amu (exactly). And,

$$3 \times \text{mass of alpha particles} = 3 \times (4.002603 \text{ amu}) = 12.007809 \text{ amu}$$

Therefore, the energy required is

$$E = \left[3 \times m\left(He^4\right) - m\left(C^{12}\right)\right] \times c^2$$

$$= (12.007809 \text{ amu} - 12 \text{ amu}) \times c^2$$

$$= (0.007809 \text{ amu}) \times \left(931.481 \ \frac{\text{MeV}}{\text{amu}}\right)$$

$$= 7.274 \text{ MeV}$$

Conversely, we can conclude that when $3\alpha$ particles combine to form a $C^{12}$ nucleus 7.274 MeV of energy is released.

## 19.3 RADIOACTIVITY

We have discussed that the elements with $Z > 83$ are unstable and that the natural radioactivity is caused by the disintegration (decay) or break up of the unstable nuclei. These nuclei may emit an $\alpha$-radiation, $\beta$-radiation, and accompanied by a $\gamma$-ray. The elements which emit these three types of radiation are called as radioactive elements. Polonium ($Z = 84$), radium ($Z = 88$), and uranium ($Z = 92$) are a few examples of such materials.

The new nucleus left as a result of radioactive decay may now have a different atomic number with the result that a new element is formed. This is called the disintegration of the atom and the process is called radioactivity.

When a nucleus emits an alpha radiation/particles its atomic number drops by 2 i.e., $(Z - 2)$ and nucleon number drops by 4 i.e., $(A - 4)$. That is,

$$\underset{\text{Parent nucleus}}{{}_{Z}X^{A}} \xrightarrow[\text{emission}]{\alpha \text{ radiation}} \underset{\text{Daughter nucleus}}{{}_{Z-2}X^{A-4}} + Q$$

where $Q$ is the disintegration energy. $Q$ is always positive as the process is spontaneous. In fact, the energy released by radioactive decay is responsible, in part, for keeping the interior of the earth hot and, in some places, molten.

**Example 19.2**

Determine the energy released when α decay converts ${}_{92}U^{238}$ into ${}_{90}Th^{234}$

**Solution**

The disintegration of ${}_{92}U^{238}$ into ${}_{90}Th^{234}$ with the emission of $\alpha$ particle is given as

$${}_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4} \text{ ( alpha particle)}$$

The mass of ${}_{92}U^{238}$ is $M_U = 238.0508$ amu

The mass of ${}_{90}Th^{234}$ is $M_{Th} = 234.0436$ amu

The mass of ${}_{2}He^{4}$ is $M_{He} = 4.0026$ amu

The total mass of thorium and alpha particle ( ${}_{2}He^{4}$ ) is 238.0462 amu. The decrease in mass is 238.0508 amu −238.0462 = 0.0046 amu. Since 1 amu is equivalent to 931.5 Mev, the energy released is 4.3 Mev.

The decay product ${}_{Z-2}X^{A-4}$ in this case is called daughter nucleus of the parent nucleus ${}_{Z}X^{A}$. The daughter nucleus may also remain unstable and undergo further disintegration till it attains stability. Similarly, the process of beta emission involves no change in mass number A. It does, however, change the atomic number Z by −1 or +1 depending upon whether the particle emitted is negative beta particle (electron) or a positive beta particle (positron). Thus the $\beta$-radioactivity may lead to either of the following disintegrations :

$$\underset{\text{Parent nucleus}}{{}_{Z}X^{A}} \xrightarrow[\text{emission}]{\beta^-} \underset{\text{Daughter nucleus}}{{}_{Z+1}X^{A}} + \text{Antineutrino} + Q_1$$

$$\underset{\text{Parent nucleus}}{{}_{Z}X^{A}} \xrightarrow[\text{emission}]{\beta^+} \underset{\text{Daughter nucleus}}{{}_{Z-1}X^{A}} + \text{neutrino} + Q_2$$

The prototype for beta decay is the decay of the neutron itself. The neutron, in free state, is unstable, decaying to a proton and electron with a half-life of about 12 minutes

$${}_{0}n^{1} \longrightarrow {}_{1}p^{1} + {}_{1}e^{0} + \text{(antineutrino)}$$

One of the best known beta decay is from the naturally occuring isotope, $C^{14}$.

As an illustration of $\beta^-$ decay, consider the thorium ${}_{90}Th^{234}$ nucleus, which decays by emitting a $\beta^-$ particle

$$\underset{\substack{\text{Parent nucleus}\\(\text{Thorium})}}{{}_{90}\mathrm{Th}^{234}} \longrightarrow \underset{\substack{\text{Daughter nucleus}\\(\text{Protactinum})}}{{}_{91}\mathrm{Pa}^{234}} + \underset{\beta\ \text{particle}}{{}_{-1}e^{\circ}}$$

Thus $\beta^{-1}$ decay, like $\alpha$ decay, causes the disintegration of one element into another. This is known as transmutation.

The electron emitted in $\beta^{-1}$ decay does not actually exist within the parent nucleus and is not one of the orbital electrons. Instead, the electron is created when a neutron decays into a proton and an electron; when this occurs the proton number of the parent nucleus increases from Z to Z + 1 and the nucleon number remains unchanged.

Most frequently the alpha or beta emission leaves the daughter nuclide in an excited state. Such a nuclide may go back to its ground state (unexcited state) by emitting one or more gamma ($\gamma$) - rays. Since $\gamma$-rays are massless photon, their emission will cause no change either of A or of Z of the parent nuclide.

The $\gamma$-decay process is written as follows:

$$\underset{\substack{\text{Parent nucleus}\\\text{in excited energy}\\\text{state}}}{{}_{Z}\mathrm{P}^{A}} \longrightarrow \underset{\substack{\text{Daughter Nucleus}\\\text{in lower energy}\\\text{state}}}{{}_{Z}\mathrm{P}^{A}} + \underset{\gamma\text{-rays}}{\gamma}$$

$\gamma$ decay does not cause a transmutation of one element into another.

### 19.3.1 Half-life of a Radioactive Element

Another very important concept associated with radioactive substances and radioactivity is the half-life of the these materials. It is observed that not all the radioactive atoms of a material disintegrate at the same time, (i.e., spontaneously), to give rise to the radioactivity. Contrary, different radioactive atoms undergo nuclear disintegration at different time. This process of disintegration takes place randomly, i.e., we can never say which atom decays at a particular instant.

It has been observed that, on the average, the actual number of atoms which decay at any instant is proportional to the number of atoms present. As time goes on there are fewer and fewer undecayed atoms. Moreover, the number of radioactive atoms in a material that decay to one half of the original number in a time is characteristic of the material. Thus, the time taken for the atoms or a radioactive material to decay to half of original number is called the half-life of the material. The half-life of a radioactive material is a nuclear property and is not influenced in any way by the physical conditions of the bulk material (such as the temperature, or the pressure or the crystalline form).

Suppose, the half-life of a particular substance is 4 hours. Then if there are N atoms at one stage, $\frac{1}{2}$ N atoms are left undecayed after 4 hours. In the next 4 hours i.e., total of 8 hours or two half-lives the number of atoms remaining $= \frac{1}{2} \times \frac{1}{2}\text{ N} = \frac{1}{4}\text{ N}$. Thus $\frac{3}{4}$ N atoms have decayed in 8 hours. In the next 4 hours, half of the remaining $\frac{1}{4}$ N atoms decay, that is, after 12 hours or three half-lives, the number of atoms left $= \frac{1}{2}\left(\frac{1}{4}\text{ N}\right) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\text{ N} = \frac{1}{8}\text{ N}$ Thus $\frac{7}{8}$ atoms have decayed in 12 hours.

The variation in the number of atoms decaying in a particular time can be graphically represented as shown in the Fig. 19.6. The half-life is represented by $T_{1/2}$ and is determined from the graph as the time taken for the radioactivity to fall to half of the initial value $N_o$.

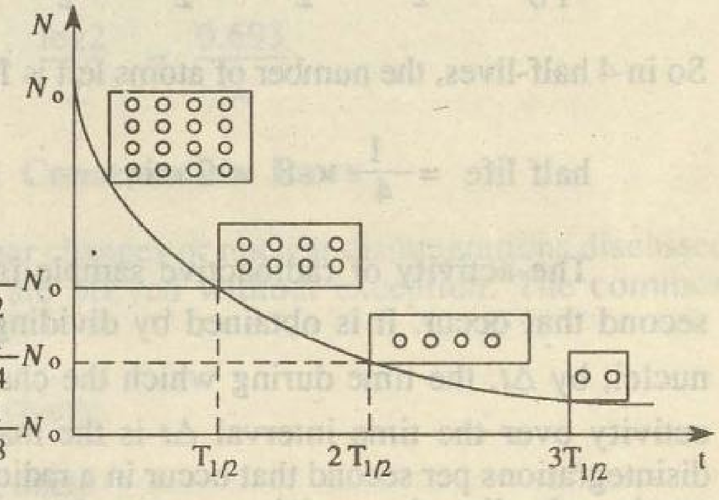


*Fig. 19.6 Decay curve for a radioactive material*

Different radioactive materials have different half-lives. So the materials can be identified by measuring their half-lives. Radium has a half-life of about 1600 years, radon has a half-life of about four days and polonium a hlaf-life of 8 minutes.

**Example 19.3**

A radioactive element has a half-life of 6 minutes. If the initial count rate is 824 per min, how long does it take to reach a count rate of 206 per minute? What fraction of the original number of atoms is then left?

**Solution**

After successive 6 min intervals, the count rate reduces from 824 to 412 (half) to 206 (half). Since there are two half-lives

time taken $= 2 \times 6\text{ min} = 12\text{ min}$

Also, fraction of atoms left $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

**Example 19.4**

If 15/16 of the radioactive atoms of an element decay in 8 min, find the half-life of the element.

**Solution**

since 15/16 decay, fraction of atoms left is 1/16

Now, $\frac{1}{16} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

So in 4 half-lives, the number of atoms left is 1/16 of the original

half life $= \frac{1}{4} \times 8 = 2$ min.

The activity of radioactive sample is the number of disintegrations per second that occur. It is obtained by dividing ΔN, the change in the number of nuclei, by $\Delta t$, the time during which the change takes place. Then, the average activity over the time interval $\Delta t$ is the magnitude of $\Delta N/\Delta t$. The number of disintegrations per second that occur in a radioactive element is proportional to the number of radioactive nuclei present, so that

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

where $\lambda$ is the proportionality constant referred as the decay constant. The minus sign is present in this equation because each disintegration decreases N.

The SI unit for activity is the becquerel (Bq); one becquerel equals one disintegeration per second. Activity is also measured in terms of a unit called curie (Ci), in honour of Marie and Pierre Curie, the discoverers of radium and polonium.

$$1 \text{ Ci} = 3.70 \times 1010 \text{ Bq}.$$

The activity of radium put into the dial of a watch to make it glow in the dark is about $4 \times 10^4$ Bq, and the activity used in radiation therapy for cancer treatment is approximately $4 \times 10^{13}$ Bq.

The graph shown in Fig. 19.6 of N versus $t$ may he expressed in a mathematical form using calculus. The result is

$$N = N_o\ e^{-\lambda t}$$

where $N_o$ is the initial number of radioactive nuclei at $t = 0$; and N is the number of radioactive nuclei present at time $t$. The exponential has the value e = 2.718 .... By substituting $N = \frac{1}{2} N_o$ and $t = T_{1/2}$ into above equation, we find that

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

Taking the natural logarithm of both side of this equation we get

$$T_{1/2} = \frac{\log 2}{\lambda} = \frac{0.693}{\lambda}$$

### 19.3.2 Nuclear Changes And Conservation Laws

In all the processes of nuclear changes or nuclear disintegrations discussed above various conservation laws are obeyed without exception. The common conservation laws are:

(i) The conservation of electric charge,

(ii) The conservation of energy, mass

(iii) The conservation of linear momentum,

(iv) The conservation of angular momentum.

(v) The conservation of nucleon number

According to these laws, the total charge, energy, linear momentum, angular momentum and nucleon number of a system should remain constant before and after a certain change or transformation.

These conservation laws can be traced back to certain invariance (non alteration) and symmetry properties associated with physical systems. For example, an isolated molecule, composed of many electrons, and nuclei atoms must be described in exactly identical terms no matter where it is located relative to the observer. But we know that the total momentum of an isolated system is constant. Hence, conservation of linear momentum in a given direction is a result of translational invariance of physical conditions in that direction. Similarly, the conservation of angular momentum is a consequence of rotational invariance of the laws describing the system. The energy conservation will therefore be a direct consequence of the invariance of physical laws describing the system related to the chosen origin of time translation (linear passage of time). In the same manner we can attribute the charge conservation to be the result of invariance of the laws of electromagnetic field with respect to some transformation or change which will not affect the electric field or the form of the field equations.

## 19.4 INDUCED NUCLEAR REACTIONS

In the preceding section we have discussed that a nucleus of an atom can be bombarded by nucleons (that is, proton or neutron), by groups of nucleons (for example, deutrons or alpha particles) or by gamma radiation. We have observed that as a result of the bombardment the nucleons are added to, removed from or rearranged within a target nucleus. Such processes of nuclear transformations are known as Nuclear Reactions.

### 19.4.1 Neutron Capture

The capture of a neutron by a proton to form a deuteron is given as ${}_1H^2$.

$${}_1H^1 + {}_0n^1 \xrightarrow{\qquad} {}_1H^2 + \gamma$$

Neutrons are the ideal projectile for such nuclear bombardments because they carry no electrical charge and thus suffer no electrical repulsion in their approach to positively charged atomic nuclei. Neutron when incident on $C^{12}$ can eject another neutron from the $C^{12}$ nucleus without being captured itself:

$${}_6C^{12} + {}_0n^1 \xrightarrow{\qquad} {}_6C^{11} + 2\,{}_0n^1$$

If the incident neutron is a slow neutron (that is, neutron having negligibly small kinetic energy), it can be captured by the nucleus with the release of an amount of energy equal to the separation energy of a neutron in the final nucleus. For the case of the deuteron, we found in section 19.2.5, that this energy release is 2.224 MeV.

### 19.4.2 Charged-Particle Reactions

In 1919, Rutherford, who for the first time identified a nuclear reaction, used alpha particles from a radioactive source to bombard nitrogen gas. He found that protons were produced in the process of disintegration of nitrogen nuclei by energetic alpha particles. The nuclear reaction that Rutherford had observed was

$$\underset{\substack{\text{Nitrogen}\\ \text{(Target)}}}{{}_7N^{14}} + \underset{\substack{\text{Incident}\\ \alpha\text{ Particle}}}{{}_2He^4} \xrightarrow{\qquad} \underset{\text{Oxygen}}{{}_8O^{17}} + \underset{\text{Proton}}{{}_1H^1}$$

(we have used atomic notation i.e., ${}_2He^4$ for $\alpha$-particles and ${}_1H^1$ for proton). This reaction was reproduced in 1925 by P.M.S. Blackett in a cloud-chamber and the tracks of particles in a nuclear reaction were photographed. This photograph is reproduced in Fig. 19.7. Becauses the incident $\alpha$–particle induces the transmutation of nitrogen into oxygen, this reaction is an example of an induced nuclear transmutation.

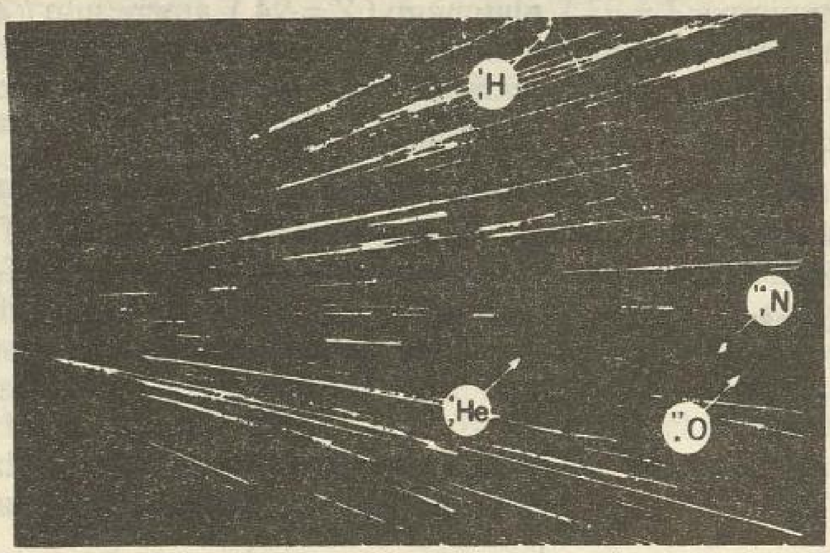


*Fig. 19.7 Nuclear Callion. by (i) α-particle, which transmutes nitrogen atom to an oxygen atom. The streaks shows α-particles moving through nitrogen gas.*

Just like the balancing of chemical equation, all nuclear reaction equations must be balanced with respect to the number of protons and neutrons before and after the reaction. For the above reaction, we have

$${}_7N^{14} + {}_2He^4 \xrightarrow{\qquad} {}_8O^{17} + {}_1H^1$$

No of protons : $7 + 2 = 8 + 1$;

No of neutrons : $7 + 2 = 9 + 0$

One can always identify the unknown newly created nuclei in a nuclear reaction by balancing a nuclear reaction equation.

**Example 19.5**

${}_5B^{10}$ is bombarded with neutrons and $\alpha$-particles are observed to be emitted. What is the residual nucleus?

**Solution**

$${}_5B^{10} + {}_0n^1 \xrightarrow{\qquad} (?) + {}_2He^4$$

No of protons : $5 + 0 = Z + 2$;

No of neutrons : $5 + 1 = N + 2$

clearly, $Z = 3$ and $N = 4$; therefore, the residual nucleus is ${}_3Li^7$.

Induced nuclear transmutation can be used to produce isotopes that are not found naturally. In 1934 Enrico Fermi, an Italian scientist, suggested a method for producing elements with a higher atomic number than uranium ( Z = 92 ). These element - neptunium ( Z = 93 ), plutonium ( Z = 94 ), americium ( Z = 95 ) and so on are known as transuranium elements. These do not occur naturally. They are created in a nuclear reaction. For exmple, when a neutron is captured by a uranium $_{92}U^{238}$ nucleus, $_{92}U^{239}$ is produced along with $\gamma$-ray. The $_{92}U^{238}$ nucleus is radioactive and decays with a half-life of 2.3 min, into neptunium $_{93}Np^{239}$ Neptunium is also radioactive and disintegrates with a half-life of 2.4 days into plutonium $_{94}Pu^{239}$ . Plutonium is the final product and has a half-life of 24100 Years. This nuclear reaction is shown as

$$
\begin{array}{l}
{}_0n^1 + {}_{92}U^{238} \longrightarrow {}_{92}U^{239} + \gamma \\
\qquad\qquad\qquad\quad \downarrow 23.5 \text{ min} \\
\qquad\qquad\qquad\quad {}_{93}Np^{239} + {}_{-1}e^{0} + \gamma \\
\qquad\qquad\qquad\quad \downarrow 2.4 \text{ days.} \\
\qquad\qquad\qquad\quad {}_{94}Pu^{239} + {}_{-1}e^{0} + \gamma
\end{array}
$$

## 19.5 NUCLEAR FISSION

In 1939, the German radio-chemist, Otto Hahn and his collaborator Fritz Strassman bombarded uranium with neutrons with the expectation to produce a uranium isotope with atomic mass higher than that of ordinary uranium. They made an important discovery that gave birth to the atomic age. On studying the resulting radioactive material they found to their surprise that among the products of neutrons absorption by uranium there were radioactive atoms of a much lighter element, barium (Z = 56). Shortly later, two German Physcists, Lise Meitner and Otto Frisch, working then as refugees in Sweeden, showed that in Hahn's and Strassman's experiments the nuclei of $U^{235}$ were split by incident neutrons into two massive parts (fragments) of nearly equal masses, almost as if it were a drop of liquid breaking up:

$$
{}_{92}U^{235} + {}_0n^1 \longrightarrow \underset{\substack{\text{compound unstable} \\ \text{nucleus}}}{{}_{92}U^{236}} \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1
$$

It was indeed a new type of nuclear reaction not similar to the one discussed above. Instead of exchanging a few nucleon between the incident particle and the target nucleus, as was the case in the nuclear reaction discussed earlier, this discovery showed the possiblility of a real breakup, called fission, of a nucleus into two smaller sized fragments called fission fragments or fission products.

The splitting of a heavy nucleus into two smaller sized nuclei was indeed a great discovery because the energy liberated in such nuclear reactions was many order greater than in ordinary nuclear reactions. If the energy released in artificial nuclear transformations were a few million electron volt, the uranium liberated 200 MeV per atom as compared to explosion of TNT molecule which releases about 30 eV.

We have discussed in section 19.2.4 that the stability of a nucleus depends upon the dominent role of strong nuclear forces over the repulsive Coulomb forces. Furthermore, the nuclear forces are short-range as compared to long-range Coulomb force. It was also pointed out in section 19.2.5 that the binding energy of nuclei decreases with the increase of mass number A. Thus, the long range repulsive forces in the heavy elements become almost comparable to the nuclear forces. The addition of one more particle in the nucleus easily disturbs the equilibrium and the nucleus becomes unstable. Accoridng to Bohr and Wheeler, the breakup of the heavy nuclei on capturing a slow neutron is a resolution of the conflict between the attractive nuclear and repulsive Coulomb forces acting in the nucleus. This can best be explained if we imagine an atomic nucleus to be an electrically charged droplet (Fig.19.8). It is obvious that an addition of another particle would transform its initial spherical shape into a more or less dumbell shape (Fig. 19.8) causes two kinds of forces to act on it:

1. The forces of nuclear surface tension attempting to restore the nucleus to its original spherical shape.
2. Coulomb repulsive forces between the electric charges on the opposite ends of the dumbell attempting to break up the nucleus into two halves.

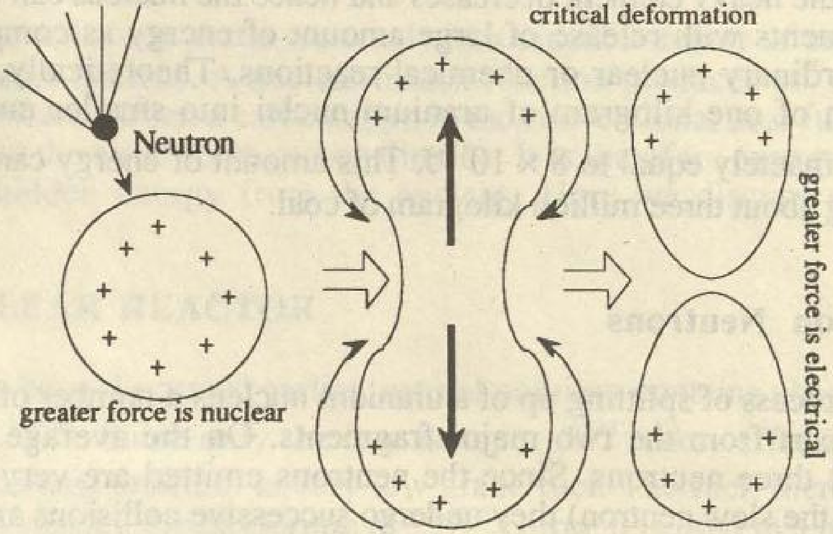


*Fig 19.8 A schematic representation of a massive nucleus break up in fission fragments. Arrows inside indicate electric repulsion between charges and arrows outside indicate surface tension forces.*

Fission reaction given above is only one of the many possible reactions that can occur when uranium fissions. For example, another reaction is

$$ {}_{92}U^{235} + {}_{0}n^{1} \longrightarrow \underset{\substack{\text{compound}\\ \text{nucleus}\\ \text{(unstable)}}}{{}_{92}U^{236}} \longrightarrow \underset{\text{Xenon}}{{}_{54}Xe^{140}} + \underset{\text{Strontium}}{{}_{38}Sr^{94}} + 2\,{}_{0}n^{1} $$

In some other reactions as many as 5 nentrons are produced. Howere, the average number produced per fission is 2.5.

We have already discussed that naturally occuring uranium is composed of two isotopes: ${}_{92}U^{238}$ and ${}_{92}U^{235}$. Although the $U^{238}$ is the most abundently found (92.275%) isotope, the probability that it will capture a neutron and fission is very small. That is why $U^{238}$ is not used for generating nuclear energy. Contrary to this ${}_{92}U^{235}$ undergoes fission readily when a neutron of kinetic energy about 0.04 eV or less strikes it. A neutron with such an energy is called a thermal neutron since the energy is comparable to the average thermal energy of a molecule at room temperature.

### 19.5.1 Nuclear Energy

The discovery of fission was a great landmark in the field of science and history of mankind because the process of fission suggested a new way of releasing tremendous amount of energy for a power-hungry world. For, if a nucleus of a heavy element is broken into half to make two small-sized ones, the binding energy per nucleon of each would increase as can be seen in Fig. 19.5 of binding energy. Thus according to this figure with the increase of mass number the binding energy per nucleon of the heavy element decreases and hence the nucleus can break up into two light fragments with release of large amount of energy as compared to that produced by ordinary nuclear or chemical reactions. Theoretically, the neutron induced fission of one kilogram of uranium nuclei into smaller nuclei releases energy approximaetely equal to $8 \times 10^{13}$ J. This amount of energy can be obtained only by burning about three million kilogram of coal.

### 19.5.2 Fission Neutrons

In the process of splitting up of a uranium nucleus a number of neutrons are also emitted apart from the two major fragments. On the average each fission produces about three neutrons. Since the neutrons emitted are very fast (10,000 time more than the slow neutron) they undergo successive collisions and are slowed down. This gives the obvious possiblility of a chain reaction in the lump of uranium, in which the slow neutrons emitted from the fission, trigger the fission of other nuclei. In this way a "family tree" (see Fig. 19.9) for fission can build up and the energy releasing process does not fizzle out, but may grow rapidly and will constitute a Chain Reaction. An uncontrolled chain reaction accompanies an enormous amount of energy in the nuclear bomb. Since the number of fissions increase a thousand fold within a few millionths of a second, with an average energy of about 200 MeV being released per fission, an uncontrolled chain reaction can generate an incredible amount of energy in a very short time.

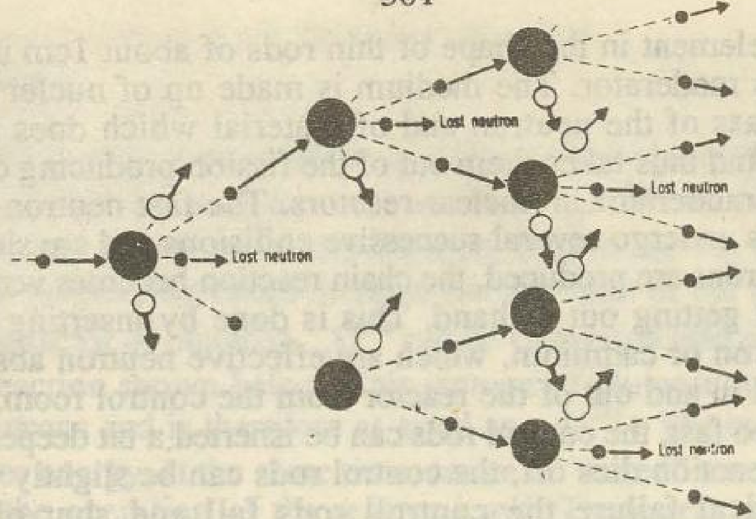


*Fig. 19.10 An uncontrolled series of fission event. Here each fission is shown to generate 2 neutrons*

The first useful (Fig. 19.10) application of the fission reaction appeared by controlling the chain reaction in a nuclear reactor. In a controlled fission, the number fo neutrons in the environment of the fissioning nuclei are limited to on the average to only one neutron that fission another nucleus. Man made release of nuclear energy was carried out successfully in a squash court in an obscure corner of Chicago University, in 1942, under the direction of Enrico Fermi.

We have discussed above that a seemingly inexhaustible source of energy is hidden inside the atomic nucleus. A nuclear fission reaction produces $10^8$ times as much energy as is released when a carbon atom in the conventional coal furnace combine with oxygen in the air to form carbon dioxide. It is therefore expected that we can extract this hidden energy from the nucleus. Here we discuss some of these methods.

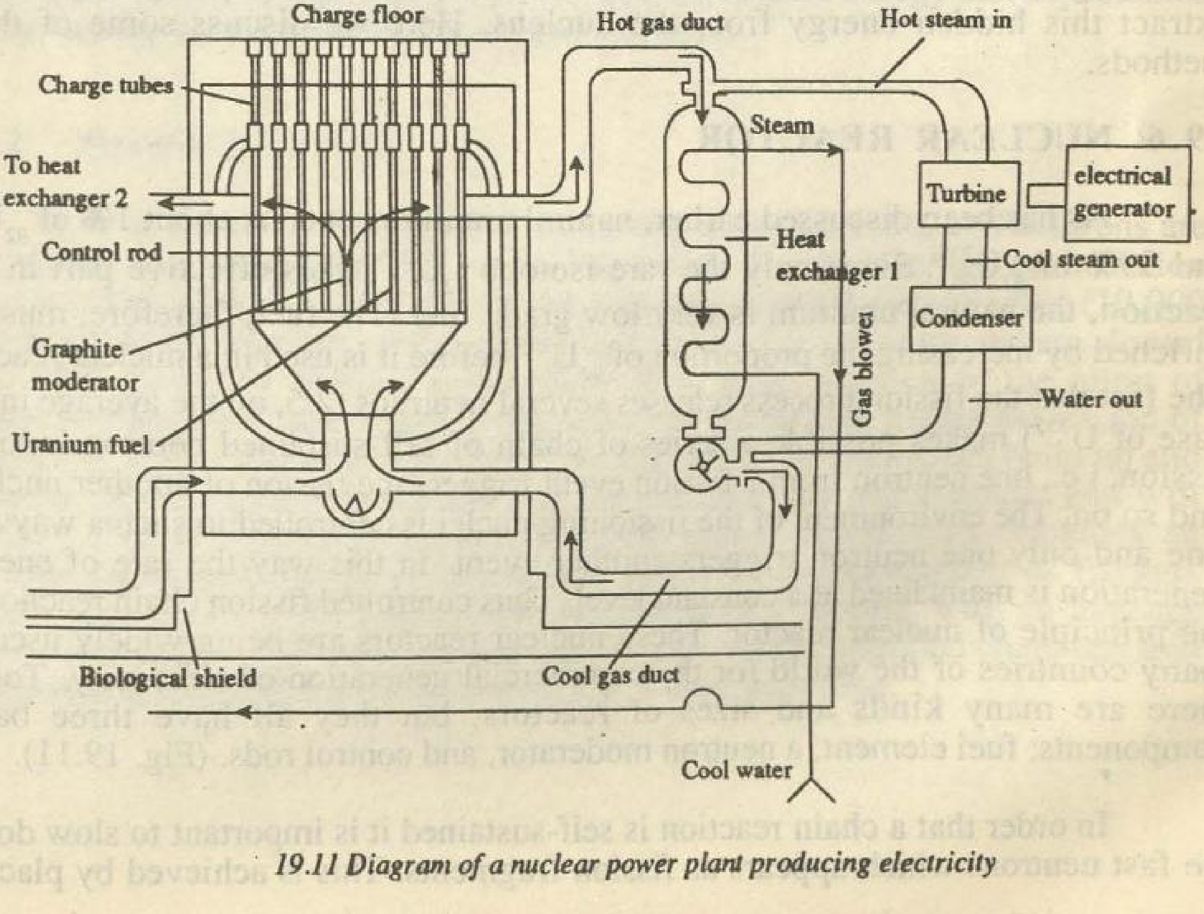
## 19.6 NUCLEAR REACTOR

As has been discussed earlier, natural uranium contains about 1% of ${}_{92}U^{235}$ and 99% of ${}_{92}U^{238}$. Since only the rare isotope ${}_{92}U^{235}$ takes effective part in the reaction, the natural uranium is very low grade fuel. The fuel, therefore, must be enriched by increasing the proportion of ${}_{92}U^{235}$ before it is used in a nuclear reactor. The fact that the fission process releases several neutrons (2.5, on the average in the case of $U^{235}$) makes possible a series of chain of self-sustained neutron induced fission, i.e., one neutron from a fission event triggers the fission of another nucleus and so on. The environment of the fissioning nuclei is controlled in such a way that one and only one neutron triggers another event. In this way the rate of energy generation is maintained at a constant level. Thus controlled fission chain reaction is the principle of nuclear reactor. These nuclear reactors are being widely used in many countries of the world for the commercial generation of electricity. Today there are many kinds and sizes of reactors, but they all have three basic components; fuel element, a neutron moderator, and control rods. (Fig. 19.11).

In order that a chain reaction is self-sustained it is important to slow down the fast neutrons which appears as fission fragments. This is achieved by placing uranium, the fuel element in the shape of thin rods of about 1cm in diameter in a medium known as moderator. The medium is made up of nuclei of comparable mass to that of mass of the neutron and of material which does not absorb the precious neutrons and thus takes them out of the fission producing cycle. Graphite is usually used as moderator in nuclear reactors. The fast neutron produced in a fission process thus undergo several successive collisions and are slowed down. In case too many neutrons are produced, the chain reaction becomes very fast and thus must be prevented getting out of hand. This is done by inserting in the reactor control rods of boron or cadmium, which are effective neutron absorbers. These rods can be moved in and out of the reactor from the control room. If the fission chain starts to go too fast, the control rods can be isnerted a bit deeper and neturons absorbed. As the reaction dies off, the control rods can be slightly withdrawn. In the case of electrical failure the controll rods fall and shut off the reactor automatically. In a large power reactor there may be thousand of fuel element placed closed together, the entire reaction of fuel elements being known as the reactor core.

A charge machine is used to start the reactor. The uranium rods, each of about 1 m long and 2 cm in diameter are lowered into about 1700 fuel channels in the graphite core (Fig. 19.11). The boron or boron-steel rods are then raised slowly and placed at certain position as soon as the chain reaction proceed at the required rate. The reactor is now said to have "gone critical" and heat is produced steadily.

The hot gas is led into heat exchanger outside the reactor. It heats up water and gets converted into high-pressure and low pressure steam. The steam is used to drive the turbines which turn electrical generators.


Charge floor
Hot gas duct
Hot steam in
Charge tubes
Steam
To heat exchanger 2
Turbine
electrical generator
Control rod
Heat exchanger 1
Cool steam out
Condenser
Graphite moderator
Gas blower
Water out
Uranium fuel
Biological shield
Cool gas duct
Cool water

*19.11 Diagram of a nuclear power plant producing electricity*

### 19.6.1 The Liquid Metal Fast Breeder Reactor (LMFBR)

In a nuclear reactor discussed above, only the less abundant $_{92}U^{235}$ isotopes produces nuclear energy, because the more abundant $_{92}U^{238}$ isotope does not fission with thermal neutrons. If we could find some method to use the abundant isotope $_{92}U^{238}$ as fuel in a reactor, we could increase the life span of our existing uranium fuel supplies by a factor of hundred. $_{92}U^{238}$ can be converted into plutonium $_{94}Pu^{239}$ by the nuclear reaction shown below. This isotope of plutonium can be fissioned with thermal neutrons and is therefore as good as $_{92}U^{235}$ . The reactors have been built that produces energy, at the same time, create or "breed" more plutonium than is consumed in the operation of the reactor itself. These reactors are known as breeder reactor. For this reason there is much interest in breeder reactors, because they will allow us to make use of the plenty supply of $_{92}U^{238}$ rather than $_{92}U^{235}$ as a reactor fuel. For this aim we can produce $_{94}Pu^{239}$ from $_{92}U^{238}$ by neutron bombardment according to the following schemes of nuclear reactions:

$$_{92}U^{238} + {_0n^1} \longrightarrow {_{92}U^{239}}$$

$$_{92}U^{239} \longrightarrow {_{-1}\beta^0} + {_{93}Np^{239}} \quad \left(T_{1/2} = 23.5 \text{ min}\right)$$

$$_{93}Np^{239} \longrightarrow {_{-1}\beta^0} + {_{94}Pu^{239}} \quad \left(T_{1/2} = 2.4 \text{ days}\right)$$

The isotope $_{94}Pu^{239}$ produced is also radioactive and decays into $_{92}U^{235}$ and an alpha particle, but the half-life of this isotope is so long $\left(T_{1/2} = 2.44 \times 10^4 \text{ year}\right)$ that large quantities can be collected and used in a power reactor, where it fissions under neutron bombardment with the release of energy in greater amounts i.e.,

$$_0n^1 + {_{94}Pu^{239}} \longrightarrow 2\,{_0n^1} + X + Y + 200 \text{ MeV}$$

where, X and Y are fission fragments consisting of a variety of isotopes near the middle of the periodic table. Thus we see that starting with an abundant nonfissile nucleus, $_{92}U^{238}$, we can breed a fissile nucleus $_{94}Pu^{239}$, which can then be utilized in a reactor to produce roughly the same amount of energy as in the case of fission of $_{92}U^{235}$. Since two neutrons are released in each fission the isotope $_{94}Pu^{239}$ can also be used in a nuclear bomb or in a nuclear reactor to produce tremendous amount of energy. The $_{94}Pu^{239}$ breeder reactor is called a liquid-metal fast breeder reactor (LMFBR), because a liquid metal like sodium is used as a coolant instead of water which slows down the neutrons and hinder the process of breeding. Sodium is a solid at room temperature but becomes liquid at 98°Ç. Hence there is no need to pressurise the reactor to keep the sodium from vapourizing. It is also a high specific heat liquid and is therefore a good heat transfer fluid. A schematic diagram of LMFBR as shown in Fig 19.12. From the diagram it is seen that this reactor is just like a conventional reactor except that the reactor core is made up of 15 to 30% ${}_{92}U^{235}$, surrounded by a blanket of ${}_{92}U^{238}$. Since fast neutrons are more efficient in converting ${}_{92}U^{238}$ to ${}_{94}Pu^{239}$, there is no need for a moderator to slow down the neutrons produced in fission.

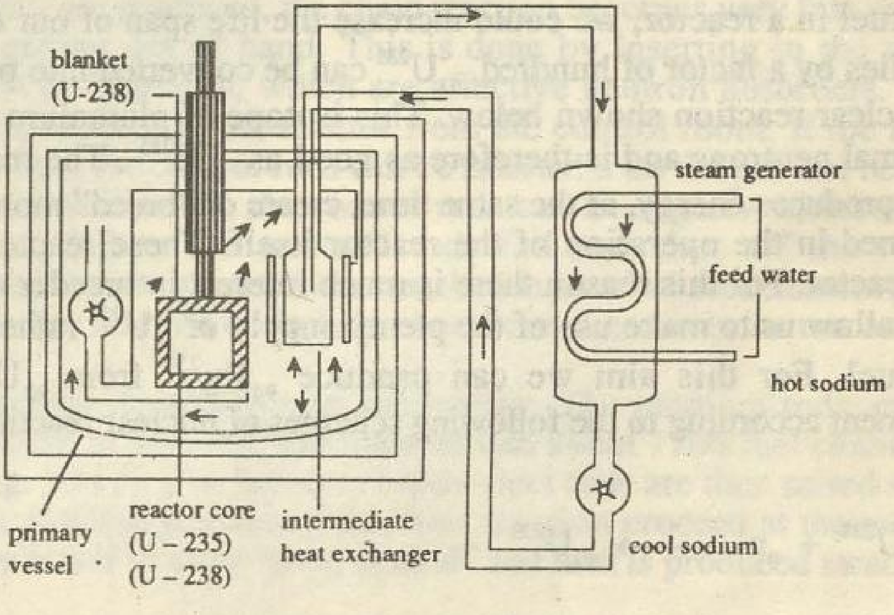

blanket
(U-238)
steam generator
feed water
hot sodium
primary
vessel
reactor core
(U – 235)
(U – 238)
intermediate
heat exchanger
cool sodium

*Fig. 19.12 Diagram of LMFBR*

## Benefits and Risks of Breeder Reactors

No energy source seems to be without its advantages and disadvantages, so we will discuss briefly the benefits and risks of the reactors.

### Advantages :

The advantages of LMFBR are

(1) ${}_{92}U^{238}$ can be substituted for ${}_{92}U^{235}$ as the basic fuel source, and hence increase the fuel life time of nuclear fission reactors by a factor of 100.

(2) Enrichment left over already on hand from the present generation of ${}_{92}U^{235}$ reactors could be used for the ${}_{92}U^{238}$ blanket, eliminating the need for mining other ores.

(3) Ores with very low concentration of ${}_{92}U^{235}$, which would be useless for conventional reactors, would also be useful as uranium sources for ${}_{92}U^{238}$ blanket in LMFBR.

(4) The cost of fuel would have almost no effect on the cost of electricity produced by LMFBR.

**Disadvantages :**

For LMFBR the disadvantages and dangers are commensurate with the great advantages of the secure, long term supply of energy. However, the disadvantages are

(1) Sodium is used as a coolant in LMFBR and the heat exchanger is opaque. It is therefore not possible to see what is going on inside the core during refueling and maintenance.

(2) Sodium is highly reactive and burns instantly on contact with air or water.

(3) Neutron bombardment of $Na^{23}$ produces ${}_{11}Na^{24}$ which emits energetic gamma rays. The half-life of ${}_{11}Na^{24}$ is 15 hours. Twenty half-lives is 300 hours or 12.5 days, hence at least this length of time is required before access to the system after shut down for maintenance.

(4) The above difficulty has a significant impact on the cost and reliability of LMFBR.

(5) There is a greater likelihood of a serious accident with LMFBR because of the more highly enriched sodium coolant. Morever, if a serious accident should occur, there is the danger that ${}_{94}Pu^{239}$ would be emitted into atmosphere.

The above difficulties with LMFBR have led to stress the advantages of a water moderated slow neutron breeder reactor using ${}_{90}Th^{232}$ to breed ${}_{92}U^{233}$ in spite of a lower breeding ratio. Such a breeder would be safer, since it could use water as a moderator and have a less concentrated core, thus avoiding safety problems with the LMFBR.

### 19.6.2 Nuclear fusion and Nuclear Reaction in the sun

From the binding energy curve in section 19.2.5 we can note that nuclei with mass number A less than 30 or 40 have smaller binding energies per nucleon than heavier nuclei. This suggests that, in principle, a process inverse to fission is energetically possible for the lighter nuclei. Hence, the process of forming heavier nuclei from two or more lighter nuclei is catagorized as nuclear fusion. The energy released in the fusion of light nuclei into heavier nuclei is called thermonuclear fusion, specially when fusion takes place under our control. Most promising fusion reactions as terrestrial energy source are the direct combination of two deuterons in either of the following reactions :

$${}_1H^2 + {}_1H^2 \longrightarrow {}_2He^3 + {}_0n^1 + 3.3\text{ MeV}$$

$${}_1H^2 + {}_1H^2 \longrightarrow {}_1H^3 + {}_1H^1 + 4.0\text{ MeV}$$

Another source may be the direct combination of a deuteron and a triton to form a heavier alpha particle i.e.,

$$\underset{\text{(Triton)}}{{}_1H^3} + \underset{\text{(Deuteron)}}{{}_1H^2} \longrightarrow \underset{(\alpha\text{-particle})}{{}_2He^4} + {}_0n^1 + 17.6\ \text{MeV}$$

Fusion reactions of this type require an abundant, as well as, cheap source of deuterium. Such a natural source is the oceans and seas of the world which contain about 0.015% deuterium.

Extra terrestrial sources such as stars including sun derive their energy from nuclear fusion. One set of fusion reaction which is supposed to be the source of solar and stellar energy is as following

$${}_1H^1 + {}_1H^1 \longrightarrow {}_1H^2 + \beta^+ + \upsilon \text{ (neutrino)}$$

$${}_1H^1 + {}_1H^2 \longrightarrow {}_2He^3 + \gamma \quad \text{(gamma-ray)}$$

$${}_1H^1 + {}_2He^3 \longrightarrow {}_2He^4 + \beta^+ + \upsilon \text{ (neutrino)}$$

This means that the net effect of this proton-proton cycle of three fusion reactions is the conversion of four protons (${}_1H^1$) into an alpha particle (${}_2He^4$), two positrons ($\beta^+$), two neutrinos ($\upsilon$), and a gamma ray photon.

In the fusion of four protons in the above cycle about 25 MeV of energy is released. This can be calculated by the sum of the above three fusion reaction, i.e.,

$$4\ {}_1H^1 \longrightarrow {}_2He^4 + 2\beta^+ + 2\upsilon + \gamma$$

The energy released in the net fusion reaction will be the Q-value of the reaction i.e.,

$$Q = (\text{Mass of } 4\ {}_1H^1 - \text{Mass } {}_2He^4 - \text{Mass } 4\ \beta^+)\, c^2$$

In this expression it is to be noted that the mass of the four hydrogen atoms includes that of two electrons, which are not needed for helium. These two $\beta^-$ masses plus two $\beta^+$ mass do not contribute to the Q-value for the fusion reaction although they would annihilate (destroy) and produce gamma ray photons. Thus we have

$$Q = (4\times1.00\,7825 - 4.002603 - 4\times0.000549) \times 931.5\ \text{MeV}$$

1 atomic mass unit = 931.5 MeV

i.e., $Q = (0.0265 \times 931.5)\ \text{MeV} = 24.7\ \text{MeV} \cong 25\ \text{MeV}$

There is another set of fusion reaction that are supposed to occur in our sun and known as carbon cycle. In this cycle also, four protons are converted into an alpha particle, but in this cycle carbon acts as a catalytic agent. This cycle is expected to occur only in stars that are hot enough to form elements upto $Z = 8$ by fusion process. The steps of the carbon cycle are

$${}_6C^{12} + {}_1H^1 \longrightarrow ({}_7N^{13})^* + \gamma \text{ (gamma ray photon)}$$

$$({}_7N^{13})^* \longrightarrow {}_6C^{13} + \beta^+ + \upsilon \text{ (neutrino)}$$

$${}_6C^{13} + {}_1H^1 \longrightarrow {}_7N^{14} + \gamma \text{ (gamma)}$$

$${}_7N^{14} + {}_1H^1 \longrightarrow {}_8O^{15} + \gamma \text{ (gamma)}$$

$${}_8O^{15} \longrightarrow {}_7N^{15} + \beta^+ + \upsilon \text{ (neutrino)}$$

$${}_7N^{15} + {}_1H^1 \longrightarrow {}_6C^{12} + {}_2He^4 \text{ (alpha)}$$

Controlled fusion reactors or thermonuclear power plants are probably the promising sources of energy for the future. We have an abundant supply of hydrogen and fusion is a relatively safer and cleaner process compared with the fission reactions. Temperature required to induce fusion reaction can be achieved by bombarding the fuel with high energy particles or with a high power laser.

## SUMMARY

***Nucleus :*** Based on the concepts of modern physics, atoms of the known elements possess a structure consisting of a central core of the atom called the nucleus. The dimensions of the nucleus are of the order of $10^{-15}$ m, fundamental particle neutron and proton reside inside the nucleus. The atomic region surrounding the nucleus is of the order of $10^{-10}$ m.

The number of electrons and the protons is the same and designated by the atomic number 'Z' of the atom. The number of neutrons 'N' is equal to the difference of mass number 'A' and the atomic number 'Z', i.e., $N = A - Z$. Any element X is therefore written as ${}_ZX^A$

***Isotopes and Isobars :*** Nuclei of the same element having the same Z but different values of N are called isotopes of the concerned element. Nuclei of different elemetns with the same mass number A are called isobars. The isobars have different Z and different N, but the mass number $A = Z + N$ is the same.

***Radioactivity :*** Radioactivity is the phenomenon of spontaneous disintegration of one or more atoms of naturally occuring elements of higher atomic masses. As a result of such a self disrupting activity three different type of radiation known as alpha, beta and gamma rays are emitted. Alpha rays are positively charged helium nuclei. Beta rays are negatively or positively charged particles, electrons ($\beta^-$) or positron ($\beta^+$), respectively, whose source is the nucleus. The gamma rays are the electromagnetic radiation (mass less photons).

***Half-Life :*** Half-life of a radioactive substance is defined as the time required for the element to decay to one half of its original number.

***Mass energy relation :*** The change in mass of an object, when multiplied by the square of the speed of light is equal to the energy of the object. Thus mass-energy are interconvertable and obey the Einstein's relation $E = (\Delta m)c^2$. The change in mass ($\Delta m$) is referred to be the mass defect or mass deficit.

***Nuclear Reaction :*** If nuclei are brought close enough so that strong (nuclear) interaction can come into action a nuclear reaction is said to occur. As a result of a nuclear reaction new combinations or rearrangements of nuclei may take place.

***Nuclear Fission :*** A nuclear reaction in which a heavy nucleus splits into fragments is called nuclear fission. In the fission process, in addition to bigger fragments some smaller particles are also produced. The most important aspect of the fission is the release of energetic neutrons which may further induce fission in the form of a chain reaction resulting in the sudden release of a large amount of nuclear energy.

***Nuclear Fusion :*** A process inverse to the fission is called nuclear fusion. The heavier nuclei are formed from two or more lighter nuclei. The energy released in the fusion of lighter nuclei into heaveir nuclei is called thermonuclear fusion.

***Nuclear Reactors :*** A nuclear reactor is a device to extract nuclear energy with easier means and with out any harm to environemt and human beings and utilize the energy for fruitful purposes in everyday life and work. There are different types of nuclear reactors, namely fission reactors, and the breeder reactors.

***Breeder Reactor :*** Starting with an abundant (99.3%) nonfissile ${}_{92}U^{238}$ it is possible to breed a fissile nucleus ${}_{94}Pu^{239}$, which can be used in a nuclear reactor to produce roughly the same amount of energy as in the fission of the less abdundant (0.7%) fissile nucleus ${}_{92}U^{235}$. Reactors capable of doing this job are called Breeder Reactors. Development of breeder reactors can increase the life span of the existing uranium fuel supplies by a factor of hundred. Due to this reason the technology of breeders occupies an important position in the modern world.

# QUESTIONS

**19.1** In radioactive decay, if the probability of decay per unit time is doubled, the half-life will be doubled. Comment.

**19.2** After four half-lives what percent of a sample remains?

**19.3** The binding energy per nucleon is 7.1 MeV for $He^4$ and 2.2 MeV for $H^2$. What percentage of rest mass converts into energy in a thermo-nuclear process that converts deuterium to helium.

**19.4** A slow neutron will cause fission in ${}_{92}U^{234}$ ,${}_{92}U^{235}$, ${}_{92}U^{238}$

**19.5** Natural uranium is a low grade fuel. It is due to –

a) High % of ${}_{92}U^{234}$

b) High % of ${}_{92}U^{235}$

c) High % of ${}_{92}U^{238}$

**19.6** It is said that nuclear energy is raw heat energy. How heat is produced in a nuclear reactor?

**19.7** Nuclear energy released under fusion reaction is greater than what is obtained under fission reaction. Although release of nuclear energy per fission is 200 MeV as compared to 25 MeV per fusion reaction. Comment.

**19.8** The purpose of LMFBR is to convert - into fissable material $U_{92}$

a) ${}_{92}U^{238}$

b) ${}_{92}U^{235}$

c) ${}_{92}U^{234}$

**19.9** The core of the LMFBR is heated up if water is used as coolant instead of liquid Sodium. Comment.

**19.10** A nuclear transmutation equation is :

$${}_{83}Bi^{210} \longrightarrow {}_{84}Po^{210} + \beta + Q$$

**a)** Q is the ..... released in this reactions.

**b)** Q comes from the ..... in masses of ..... and final particles.

**c)** Velocity of $\beta$-particles is measured by a .......

**d)** Radioactive disintegration by $\beta$-particle emission is sometimes known as..............

**e)** Mass number $A = Z + N$, the neutron number of the product nucleus equal ..... in the $\beta$-decay.

**f)** A disintegration by the emission of a $\beta$-particle, that is, an electron, the atomic number.....but the mass number A remains......

# PROBLEMS

**19.1** An alpha particle on ejection from the radioactive radium atom is 5.86 MeV. Determine

**(i)** the velocity of particle,

**(ii)** the total energy released during the process of escape of the particle. Whre the amu of Ra is 226.025402 and the amu of Rn is 222.01757.

[(i) Ans. $1.680 \times 10^7$ ms$^{-1}$ (ii) Ans. 5.95 MeV]

**19.2** Find the isotopes produced from

**a)** ${}_{92}U^{238}$ after two $\alpha$-decays and two $\beta$-decays.

[Ans. ${}_{90}Th^{230}$]

**b)** ${}_{90}Th^{232}$ after three $\alpha$-decays and four $\beta$-decays.

[Ans. ${}_{88}Ra^{220}$ ]

**c)** the radio-active isotope ${}_{3}Li^{8}$ after two $\beta$-decays and one $\alpha$ decays.

[Ans. ${}_{3}Li^{4}$ ]

**d)** the radio-active isotope of antimony ${}_{51}Sb^{133}$ after three $\beta$-decays.

[Ans. ${}_{54}Xe^{133}$ ]

**19.3** A - particle while escaping from the nucleus of ${}_{84}Po^{214}$ is of 7.58 MeV during radio-active decay process. Find the velocity of the particle.

[Ans. $1.91 \times 10^7$ ms ]

**19.4** In what ratio nuclear binding energies of ${}_1H^3$ and ${}_2He^3$ occur and which of these nuclei is more stable and why? Take masses of ${}_1H^3$ and ${}_2He^3$ as 3.01700 amu and 3.01699 amu respectively.

(The nucleus of $H^3$ is more stable due to high B.E.).

[Ans. Ratio = 1.103 MeV]

**19.5** Find the energy released during following nuclear reactions.

**a)** ${}_3Li^7 + {}_1H^1 \longrightarrow 2\,{}_2He^4$ [Ans. 17.3 MeV]

**b)** ${}_7N^{14} + {}_2He^4 \longrightarrow {}_1H^1 + {}_8O^{17}$ [Ans. 1.18 MeV]

**c)** ${}_1H^2 + {}_1H^2 \longrightarrow {}_1H^1 + {}_1H^3$ [Ans. 4.04 MeV]

**d)** ${}_1H^2 + {}_1H^2 \longrightarrow {}_2He^3 + {}_0n^1$ [Ans. 3.26 MeV]

**e)** ${}_3Li^6 + {}_1H^1 \longrightarrow {}_2He^3 + {}_2He^4$ [Ans. 4.20 MeV]

Take masses of the isotopes involved in the above reactions as ${}_3Li^7$ (7.01823 amu) ${}_1H^1$ (1.00814 amu), ${}_7N^{14}$ (14.00752 amu), ${}_2He^4$ (4.00388 amu), ${}_1H^2$ (2.01474 amu) ${}_1H^3$ (3.016 amu) and ${}_3Li^6$ (6.01703 amu)

**19.6** Find the mass of water which can be heated from 0°C to its boiling point when supplied all the heat released in $_3Li^7$ $(p,\alpha)$ reaction. Take one gram of lithium and assuming its 100% disintegration in the above mentioned reaction.

[Ans. $5.7 \times 10^5$ kg]

**19.7** $_3Li^6$ is bombarded by deuterons giving two particles alongwith release of energy equal to 22.3 MeV. Knowing masses of a deuteron and an $\alpha$ particle, determine mass of lithium isotope $_3Li^6$.

[Ans. 6.017 amu]

**19.8** Radons $_{86}Rn^{222}$ is radioactive gas produced when radium $_{88}Ra^{226}$ (an element usually used to make the dial of a watch to glow in dark) undergoes $\alpha$-decay. There is a growing concern about radon as a health hazard because it can get trapped in houses entering through walls and floors and in the drinking water.

[ Ans. $N = 1.1 \times 10^5$ ;
Activity = 63 and
Activity = 0.23 Bq.]

**19.9** Find the energy released when $\beta^-$ decay changes $_{90}Th^{234}$ into $_{91}Pa^{234}$ .

[Ans. 0.27 MeV]

give masses of $_{90}Th^{234}$ × $_{91}Pa^{234}$ in amu

# 20

# NUCLEAR RADIATION

We have discussed that atoms of radioactive materials disintegrate with the emission of three different types of radiation. In nuclear fission reaction the release of neutrons has also been studied. We shall now discuss their interaction with matter. The interaction with matter depends primarily on three factors: the mass of the particle, its energy, and its charge. An $\alpha$-particle thus interacts more strongly than an electron travelling at the same speed when it collides with an atom. Moreover, the $\alpha$-particle has a charge of +2e compared with the charge of electron; it, therefore, exerts a larger coulomb force on nearby charges than electron does. For these reasons, an $\alpha$-particle ionizes atoms along its path much more frequently than an electron of the same energy.

The amount of ionization produced in a material can be used to detect the radiation and their type because the relative ionization strength of different radiation is different.

In this chapter, we shall discuss the effects of interaction of nuclear radiations with matter, some devices used to detect nuclear radiations and a few out of the many applications of nuclear radiations in our present day world.

## 20.1 INTERACTION OF ALPHA PARTICLE WITH MATTER

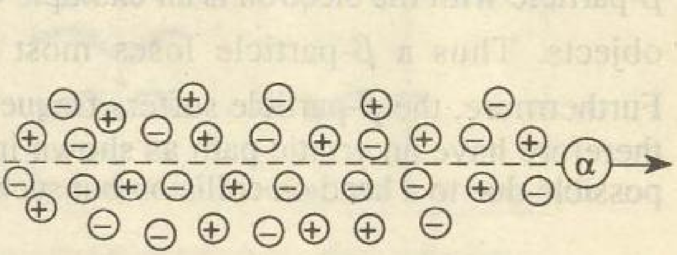


*Fig. 20.1 Ionization produced by an $\alpha$ – particle along a straight path.*

When an α-particle encounters with atoms or molecules of a gas it knocks out electrons from the gas atoms and thereby ionizes them . The charged particle, such as an $\alpha$-particle, need not actually hit an electron or atom to cause ionization. The coulomb force exerted on the electron by the charged particle is often strong enough to cause ionization even if the particle only passes close to the atom. Since an α-particle is much more massive than an electron, it is not significantly deflected during such ionization encounters. Thus the $\alpha$-particle continues producing intense ionization along its straight path as shown in Fig. 20.1. In doing so, $\alpha$-particle continuously loses part of its energy and as a result its velocity decreases. The process of ionization continues till the $\alpha$ particle loses all its energy and comes almost to rest. An average of about 35 eV energy is used up to produce one ion pair. Therefore, a 7.7 MeV $\alpha$-particle emitted from Radium-C produces as many as

$2 \times 10^5$ ion pairs before stopping. Finally, it acquires two electrons from the surrounding gas and becomes a neutral helium atom.

The $\alpha$-particle does not travel far enough in air due to intense ionization it produces in air. The average distance travelled from the source to the point where it stops in the medium is called its range. The range of 7.7 MeV $\alpha$-particle in air is about 7 cm which reduces further in a denser medium. The same $\alpha$-particle will penetrate only 0.04 mm thickness in an aluminium foil. $\alpha$-particles produce fluorescence or glow in substances like zinc sulphide or barium platinocyanide. High energy $\alpha$-particles can also produce disintegration in the nuclei of some atoms.

## 20.2 INTERACTION OF BETA PARTICLES WITH MATTER

Beta$\beta$--particles knock out electrons from the atoms due to electrostatic force of repulsion. The $\beta$-particles are relatively light and only a small amount of ionization is produced in air as compared to that due to $\alpha$-particles. Actually, the collision of a $\beta$-particle with the electron is an example of elastic collision between two identical objects. Thus a $\beta$-particle loses most of its energy in a single encounter. Furthermore, the $\beta$-particle suffers frequent deflections during its encounters and therefore have an erratic path as shown in Fig. 20.2. Large angle of deflection is possible due to a head on collision but such events are very rare.

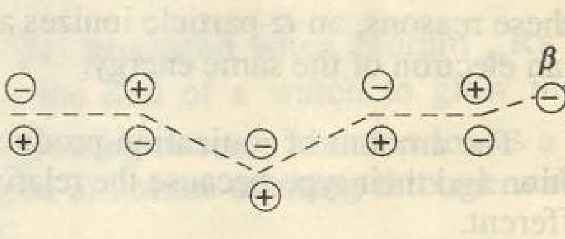


*Fig. 20.2 A zigzag path of$\beta$-particle producing ionization.*

Because of lesser ionization encounters, a $\beta$-particle can penetrate much deeper into matter than can an $\alpha$-particle of the same energy. Its range in air or penetration power in a medium has been found about 100 times more than that of an $\alpha$-particle of the same energy. For example, a 3 MeV $\beta$-particle can pass through about 6.5 mm thick aluminium sheet. $\beta$-particles also produce fluorescence in some substances such as barium platino-cyanide and calcium tungstate etc.

## 20.3 INTERACTION OF GAMMA RAYS WITH MATTER

Gamma rays are not easily stopped by matter because they have neither charge nor rest mass. They lose energy as they penetrate materials mainly by three processes: the photo-electric effect, Compton effect, and pair production. In photoelectric effect, the $\gamma$-ray is completely absorbed with the emission of electrons. It usually takes place when the energy of $\gamma$-rays is less than about 0.1 MeV.

In Compton effect, the $\gamma$-ray is deflected by an atomic electron transferring some of its energy to the electron. The possibility of Compton effect increases when the energy of $\gamma$-rays lies in the range 0.1 MeV to 1.0 MeV. In pair-production, the $\gamma$-ray photon, in the vicinity of a nucleus, is converted into a pair of electron and a positron. Thus an annihiliation of $\gamma$-rays photon takes place. This process dominates when the energy of $\gamma$-rays is greater than 1.02 MeV.

Gamma radiation does not have a specific range in a medium. Different radiation have different specific ranges. Some are absorbed in a shorter distance, where as, some may travel long distances before stopping. Therefore, the range of gamma radiation is estimated from the reduction of the beam intensity after passing through matter. Gamma radiation is considerbly more penetrating than either $\alpha$ or $\beta$ radiation. A comparison of the penetration powers of various radiation is shown in Fig. 20.3.

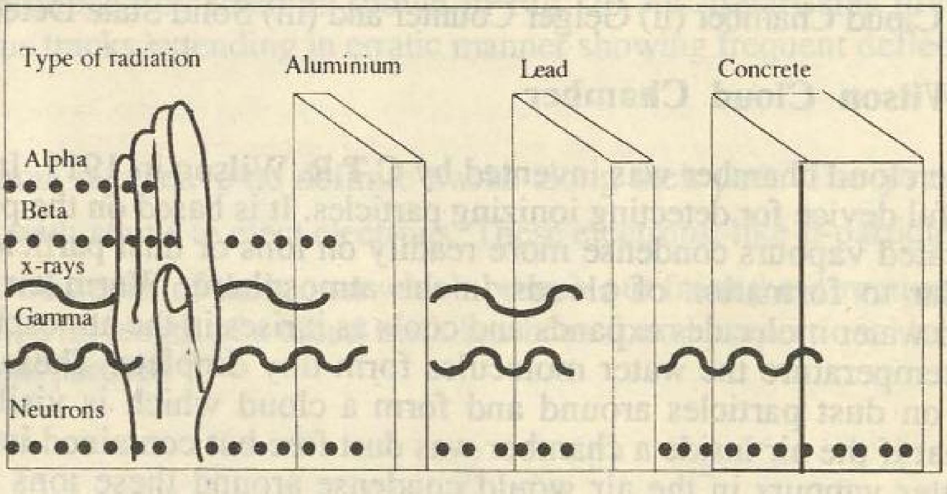


*Fig. 20.3 A rough comparison of penetration powers of various radiations*

Materials having large number of electrons per unit volume are more effective absorber of $\gamma$-radiation. Thus lead is usually used to shield $\gamma$-radiation for safety purpose.

Gamma radiation also produces fluorescence on striking some substances such as barium platinocyanide coated screen but of lower intensity than charged particles.

## 20.4 INTERACTION OF NEUTRONS WITH MATTER

Neutrons which have no charge, are extremely penetrating particles. To be stopped or slowed down, a neutron must undergo a direct collision with a nucleus or some other particle that has a mass comparable to that of neutron. Neutron can enter easily in the nucleus of an atom and may transfer its energy to the nucleus to initiate a neutron induced disintegration. The capture of a neutron by a nucleus often results in the formation of a radio isotope. In case of a heavy nucleus, the neutron can also cause fission.

Neutron encounters rarely with the atomic electrons due to the absence of Coulomb force. Some ionization is produced by neutrons in material that contains a large fraction of hydrogen, for example, body tissues. In this case, the neutron interact with the nuclear protons of the hydrogen atoms. These knocked out protons are the particles that produce almost all of the ionization in such materials.

## 20.5 RADIATION DETECTORS

Several types of radiation detectors are used in conjunction with various electronic circuits. They are used to measure the energy and intensity and also to distinguish between different types of radiations.

The radiations emitted by a radioactive material are invisible and thus has to be detected by the effects they produce. The working of most detectors is based on the ionization that results directly or indirectly from the passage of a charged particle or a photon through matter. Here we shall discuss the following
(i) Wilson Cloud Chamber (ii) Geiger Counter and (iii) Solid State Detectors.

### 20.5.1 Wilson Cloud Chamber

The cloud chamber was invented by C.T.R. Wilson in 1911. It has proved a very useful device for detecting ionizing particles. It is based on the principle that supersaturated vapours condense more readily on ions or dust particles. It is in a way similar to formation of clouds in the atmosphere. Warm air containing evaporated water molecules expands and cools as it rises in the atmosphere. Below a certain temperature the water molecules form tiny droplets. These drops then condense on dust particles around and form a cloud which is visible. Wilson showed that if the air inside a chamber was dust free but contained ions, then the excess water vapours in the air would condense around these ions on suitably cooling the air. The droplets of water round the ions reflect light and can be photographed. In this way a visible track of the path of ionizing particle can be obtained.

The device, being used now a days, consists of a base metal plate A (Fig. 20.4), circular in shape, with hole in the centre. From this hole a tube leads to a vacuum pump. Fixed just above A is another metal plate B and on top of this is a black felt pad so that the white vapour tracks may be seen easily. A perspex or glass dome fits over plate A, and the source holder S passes through the top of the dome. A metal ring C is attached inside the dome, and the electric field is applied between C and A to sweep out all the ions from the chamber to make it ready for use. Usually this process is repeated antomatically after every snap shot. Before the dome is placed on plate A, the black felt pad is placed on plate A, the black felt pad is soaked with alcohol, so that the chamber contains a mixture of vapour and air. When the pump is given a sharp pull, the pressure in the chamber is released, consequently the air and vapours expand. The resulting cooling causes the vapours to become supersaturated and white tracks of condensed vapours can be seen emanating from the source.

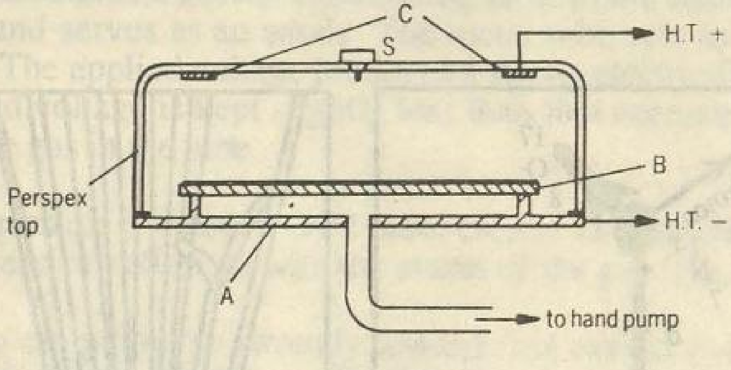


*Fig. 20.4 Wilson cloud chamber*

The $\alpha$-particles leave dense, straight and continuous tracks due to intense ionization produced by them as shown in Fig. 20.5,a. $\beta$-particles give thin and discontinuous tracks extending in erratic manner showing frequent deflections (Fig. 20.5,b).

Gamma rays leave no definite tracks along their path. The $\gamma$-rays photons can interact with atoms to eject electrons. These electrons, like $\beta$-particles, produce irregular cloud tracks of their own which branch out from the direction of $\gamma$-rays. (Fig. 20.5,c). The length of a cloud track has been found proportional to the energy of the incident particle.

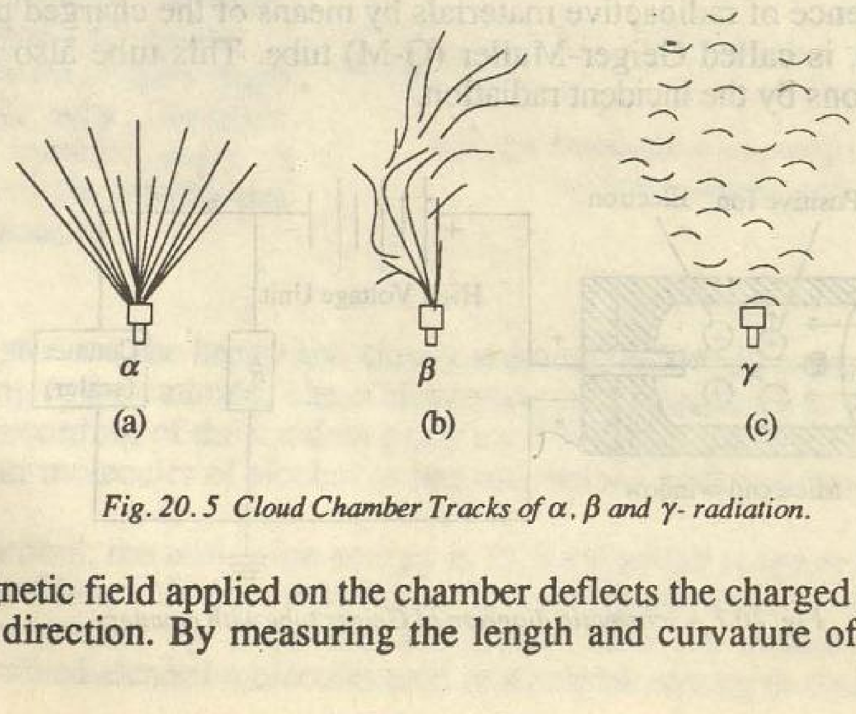
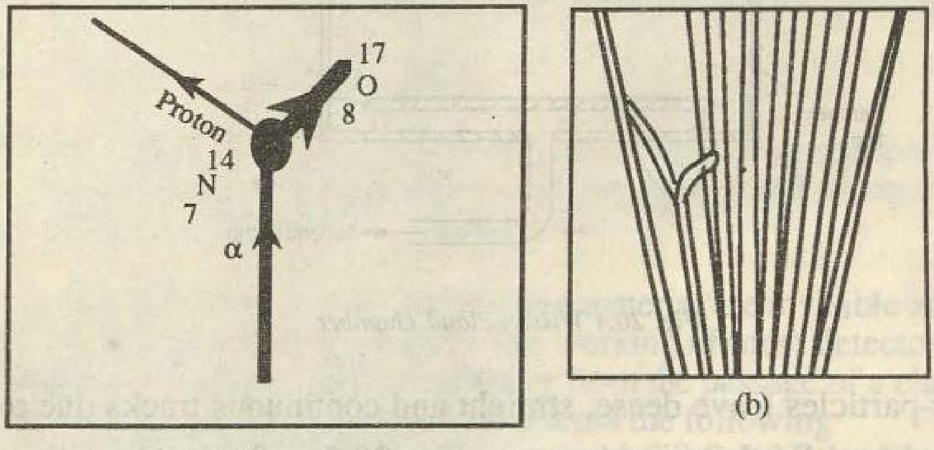


*Fig. 20. 5 Cloud Chamber Tracks of α, β and γ- radiation.*

A magnetic field applied on the chamber deflects the charged particles from their original direction. By measuring the length and curvature of the path, the charge to mass ratio of the particle can be determined. This has also helped in the discovery of new particles.

The chamber can be used to study some of the nuclear reactions and disintegrations. For example, when $\alpha$-particles are made to interact with nitrogen atoms present in the chamber, the result of the interaction can be observed as shown in Fig. 20.6.



(a) (b)

Fig. 20.6 *(a) Tracks showing the interaction of $\alpha$-particles with Nitrogen (b) An actual cloud chamber tracks of $\alpha$-N $^{14}$ interaction showing long tracks (high energy particles) and short tracks (lower energy particles).*

## 20.5.2 Geiger Counter

The Geiger counter is widely used as a detector of radiations. It is often used in radioactivity experiments in laboratories. Geiger, a student of Lord Rutherford, designed this detector working with Muller. The tube which is used to detect the presence of radioactive materials by means of the charged particles or $\gamma$-rays they emit, is called Geiger-Muller (G-M) tube. This tube also relies on the production of ions by the incident radiation.

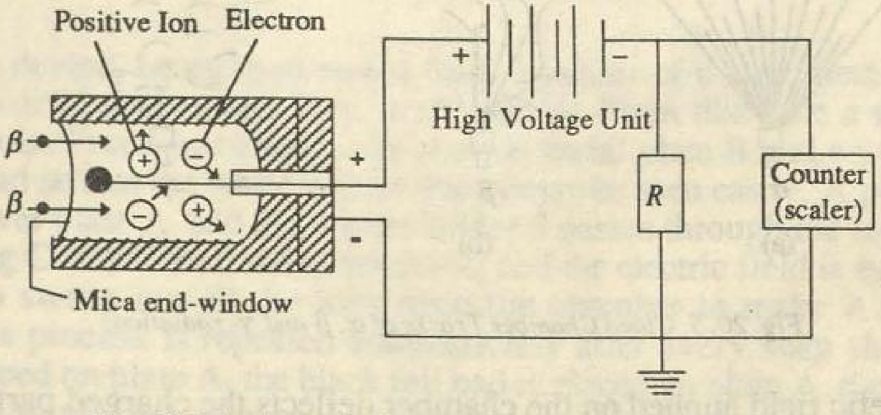


*Fig. 20.7 A schematic diagram of Geiger tube with counter*

It consists of a small thin-walled closed metal tube (Fig. 20.7) containing an inert gas such as argon at a pressure of about 10 cm of mercury, together with a small quantity of alcohol. The other type contains a little halogen gas such as bromine mixed with neon. Different gases and pressures may be used according to the radiation to be examined.

A thin wire, insulated from the surrounding tube, in the centre of the tube is positively charged and serves as an anode. The metal tube acts as cathode and is negatively charged. The applied voltage produces a strong electric field in the tube. However, the applied voltage is kept slightly less than that necessary to produce a discharge through the gas in the tube.

An ionizing particle if enters a G-M tube, creates ions and free electrons in the gas through successive collisions with the atoms of the gas (Fig. 20.8).

The electrons are pulled so strongly towards the central positive wire that they further strike gas atoms, knocking off other electrons. Thus an avalanche of electrons is formed, which when reaches the positive wire, creates a detectible current pulse through the counting unit such as scaler. For each particle passing through the tube a pulse is recorded. In this way Geiger counter is able to count the particles that come through it.

The gas must be made non-conducting immediately in order to register the entry of the next particle. This is called quenching of discharge. The conduction is terminated by an external unit which momentarily drops the voltage on the anode below the critical value for ionization. The tube thus becomes ready to register the next incoming particle. Likewise a momentary current or pulse is recorded corresponding to each single particle entering the tube. However, the size of the current pulse is independent of both the energy and type of incident radiations.

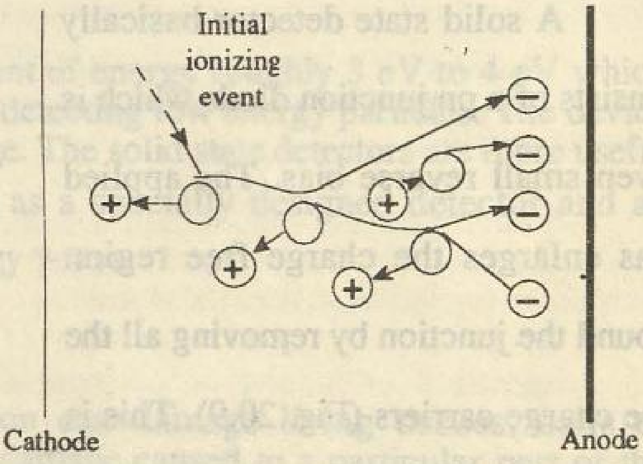


*Fig. 20.8 Production of ions and free electrons by incident radiation*

During the process, the heavy and slowly moving positive ions may release electrons on reaching at the cathode. These electrons could then travel to the anode and may upset the recording of the incident particles. This situation is prevented by the quenching vapour molecules of alcohol or bromine mixed with principal gas.

In case of alcohol, the ionization energy is 11.3 eV which is lower than that of argon (15.7 eV) and hence the ions arriving at the cathode are alcohol ions. They become excited molecules by acquiring electrons from the cathode. After neutralization, the excited alcohol molecules tend to dissipate energy in dissociation of the molecules rather than to release electrons from the cathode. Even if the photons are emitted, they are unable to eject electrons due to their insufficient energies and are usually reabsorbed by the alcohol molecules.

In case of bromine gas, the molecules when interact with positive ions split into individual atoms. The positive ions thus lose energy in this process and so arrive at the cathode with insufficient energy to cause any further electron emission. Thus the probability of a current pulse through the external resistance other than that of incident radiation is extremely small.

The Geiger counter can be used to determine the penetration power of β-particles and γ-rays. The reduction in the count rate by inserting aluminium or lead plates of various thicknesses between the source and the tube helps to estimate the penetration power of the incident radiation.

Geiger counter is not suitable for fast counting. Scintillation counter or solid state detectors are fast enough, more efficient and accurate.

### 20.5.3 Solid State Detector

A solid state detector basically consists of a pn junction diode which is given small reverse bias. The applied bias enlarges the charge free region around the junction by removing all the free charge carriers (Fig. 20.9). This is called depletion region.

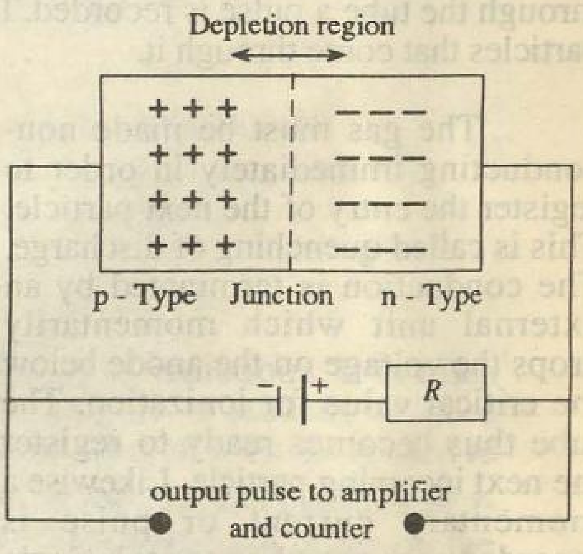


*Fig. 20.9 A reverse bias enlarges the charge free region in a pn Junction.*

When an incident particle enters the n-type side, it is absorbed in the depletion region and electron-hole pairs are produced. These charge carriers move under the influence of biasing potential. The electrons move towards the side of the junction connected to the positive terminal of the battery and the holes are swept towards the negative side. The arrival of these charges at the two layers produces a potential drop across the junction. This results in a current pulse, proportional to the energy of the incident particle. The current pulse is amplified and recorded by an electronic counter.

After the electrons and holes reach at the specific ends, the pn junction again becomes non-conducting until another incident particle produces electron-hole pairs. A common type of silicon pn junction detector is shown in Fig. 20.10.

The back of the detector consists of thick layer of p-type material connected to the negative terminal of the battery. The front surface is made of very thin layer of n-type material over which a thin layer of gold is coated which serves as positive electrode. The combined thickness of n-type and gold absorbs so less energy of the incident particle that the junction may be assumed to be situated at the front surface. The incident radiation is absorbed almost instantaneously resulting in the production of electron hole pairs.

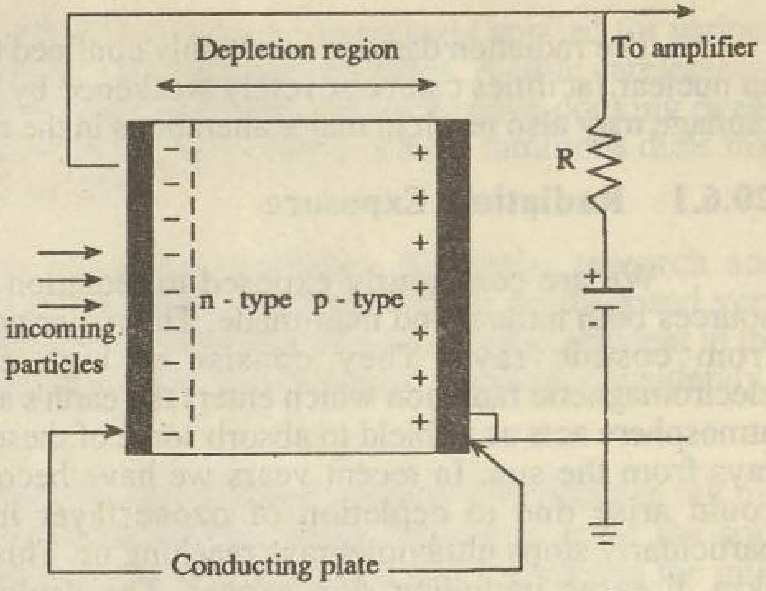


*Fig. 20.10 A typical silicon pn junction Detector*

Their production requires a small amount of energy roughly 3 eV to 4 eV which makes the device particularly useful for detecting low energy particles. The device can count fast and operates at low voltage. The solid state detectors are more useful for detecting $\alpha$ and $\beta$ particles where as a specially designed detector and an amplifier can also be used for high energy $\gamma$-rays.

## 20.6 RADIATION DAMAGE

Excessive exposure to radiation can damage living tissues, cells or organism. The degree and the kind of damage caused to a particular part of the whole body depends on the type, energy and dose of radiation received. There appears to be no limit below which radiation damage does not occur. Sometimes, exposure to even one ionizing particle may damage or change the working of some complex molecules resulting in the making of a wrong substance.

The problem with the nuclear radiation is that one can receive a severe dose of radiation without being aware of it. Even a number of small doses recieved over a long period build up commulatively in some parts of the body, and may eventually lead to fatal consequences later in life.

Radiation damage to living organism is primarily due to ionization effects in the cells. The cell is the basic unit of life. Its normal function may be disrupted as a result of interaction with the ionizing radiation. Excessive radiation dose may even cause death of the cell. The cell that do survive radiation may become defective. The cell has some capacity to repair certain harmful alterations provided the dose rate is not excessive. However, a gene mutation is irreparable and therefore cumulative. Sometimes the cell start reproducing themselves wildly leading to condition called cancer.

The radiation damage is not only confined to living systems. The metal used in nuclear facilities can be severely weakened by highly penetrating neutrons. The damage may also result in major alterations in the metal properties.

### 20.6.1 Radiation Exposure

We are continously exposed to radiation in our environment from many sources both natural and man-made. The major portion of natural radiation comes from cosmic rays. They consist of high energy charged particles and electromagnetic radiation which enters the earth's atmosphere from outer space. The atmosphere acts as a shield to absorb some of these radiations as well as ultra violet rays from the sun. In recent years we have become aware that a serious hazard could arise due to depletion of ozone layer in the upper atmosphere which particularly stops ultraviolet rays reaching us. This may result in increased eye and skin diseases including skin cancer. The depletion is suspected to be caused due to excessive release of some chemicals in the atmosphere particularly chloroflourocarbons (CFC) used in refrigeration, aerosol spray and plastic foam industry.

The high energy particles interact with the constituents of the earth's atmosphere producing secondary lower energy particles which produce ionization in the atmosphere.

Some rocks and soil contain traces of naturally occurring radioactive series based on uranium and thorium. Thus we receive some radiation from the building materials extracted from rocks and soil.

Ingestion or inhalation of naturally occurring radio nuclides provides another source of exposure and more serious consequences due to local and continuous irradiation of vital organs inside the body. They mainly enter the body through the food chain. For example, potassium—40 and carbon—14 are accumulated in the diet and make their way to the body where they emit $\beta$-particles.

Radioactive radon gas is also present in the air in small quantity. It decays into a solid that is deposited usually on dust particles. As we breathe some of these radioactive dust particles stick to the lining of lungs. Thus lungs receive more radiation than the rest of the body.

Tobacco leaves also collect radioactivity while drying. A cigerette smoker is not only prone to the toxic smoke but also the hazardous radiation.

It may be difficult to protect ourselves from natural radiation but we can certainly do something to reduce irradiation caused by our own activities. One significant factor is the medical radiation exposures which come from diagnostic and therapeutic uses of x-rays and radioisotopes. The irradiation should be kept as low as commensurate with medical needs in view of the damage risk to the individual and the possible hereditery effects on future generations.

Radio isotopes and radiation techniques are increasingly applied for various useful purposes in agriculture, industry and in other processes. A little carelessness may lead to irreparable loss. Colour television sets and microwave cooking ovens also emit x-rays in small quantities. Some wrist watches have luminous dials that contain radioactive substance and so forth.

The radioactive waste from nuclear facilities, hospitals, research and industrial establishments pose a great threat to safety. It must be disposed very carefully at some proper places. In the unlikely event of a catasrophic accident in the nuclear facility or at the waste disposal unit, the exposure could be considerably higher and fatal.

Some toxic nuclides such as caesium-137 and strontium-90 occur in fall out from nuclear explosions. They contaminate the plants directly or enter the soil. Cattles eating grass may accumulate them, which are then concentrated in the milk and meat to be ingested by us or contaminated vegetables and fruits may be eaten directly as part of human diet. Since strontium-90 behaves biologically in much the same way as calcium, it follows a similar metabolic path in the body and may replace calcium in the bone and teeth resulting in the cancer of bone and blood or other serious disorders. A simple food chain is shown in Fig. 20.11.

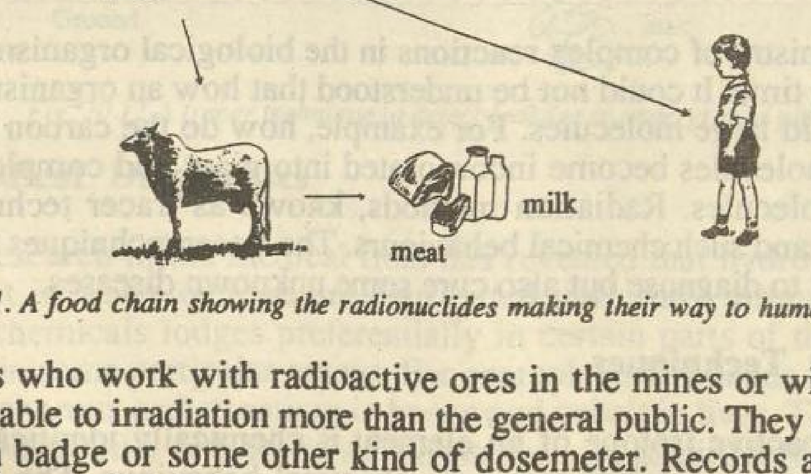


*Fig 20.11. A food chain showing the radionuclides making their way to human body.*

Persons who work with radioactive ores in the mines or with radioactive processes are liable to irradiation more than the general public. They are monitored, wearing a film badge or some other kind of dosemeter. Records are kept of the doses received by individuals in order to see that radiation dose limits are not exceeded.

There is no safe limit for a body exposure to radiation. Higher levels of exposures are certainly damaging. We cannot avoid exposure to radiation, nevertheless, it is very important to limit the amount to which people are exposed to the value lowest possible within reason.

### 20.6.2 EFFECT OF RADIATION

When a living tissue is exposed to radiation, the energy absorbed may produce chemical changes in the cells which affect cell metabolism. Large molecules may be broken up or free radicals, capable of initiating chemical changes, may be produced. These changes may be sufficiently damaging to lead to the death of individual cells, chromosome abnormalities or mutations, resulting in a characteristic not contained in the parent cell.

The biological effects are generally of two types - somatic and genetic. Somatic effects affect an individual directly. Some of the common somatic effects of radiation are skin burns, loss of hair, ulceration, stiffening of the lungs, an induction of contracts in the eye, and a drop in the white blood cells which is followed by a sickness pattern of diarrhoea, vomiting and fever known as radiation sickness. Recovery is possible from small acute doses but shortening of life span is inevitable. The most feared somatic effect of radiation is the induction of many types of cancer. High levels of radiation covering the whole body may disrupt the blood cells so seriously that life cannot be maintained and death is likely to occur.

The genetic effects may become apparent after a long time. Radiation can alter the chemistry of the genes and may cause mutations, which are changes in the genetic messages with the cells. Even very low radiation doses reaching the reproductive organs of the body are potentially dangerous. A chromosome may be affected by a single radiation impact. As a result the subsequent cells may be abnormal, and the genetic effects arising may be passed on to the future generations.

## 20.7 BIOLOGICAL AND MEDICAL USES OF RADIATION

The chemistry of complex reactions in the biological organism remained a mystery for long time. It could not be understood that how an organism uses small molecules to build large molecules. For example, how do the carbon atoms in the carbondioxide molecules become incorporated into giant and complex protein or carbohydrate molecules. Radiation methods, known as tracer techniques, have helped to understand such chemical behaviours. The tracer techniques have made it possible not only to diagnose but also cure some unknown diseases.

### 20.7.1 Tracer Techniques

The radioactive isotope of an element is chemically identical to a stable isotope and therefore, exhibits the same chemical behaviour. The location and concentration of radioactive isotope, however, can be detected easily by measuring the radiation emitted. A radioactive isotope thus acts as a tracer that makes it possible to follow the course of a chemical or biological process. The technique is to substitute radioactive atoms for stable atoms of the same kind in certain molecules and then to follow the "tagged" atoms with radiation detectors in the process. For example, radioactive carbon (C—14) is one of the useful tracer because of the occurrence of carbon in all organic molecules. The investigation using this isotope has made it possible to understand more elaborately the complex process of photosynthesis. The technique has provided detailed information on the series of chemical reactions that leads from carbon dioxide to carbohydrate in the plants.

Similarly, the exact position of an underground pipe can be located if a small quantity of a radioisotope is added to the liquid being carried by the pipe. This also allows leaks to be detected. A short lived radioisotope such as Na-24 or Iodine-131 is used as tracer to avoid prolonged contamination of the soil or water. A schematic diagram of the technique is shown in Fig. 20.12. A similar technique was used to identify faults in the fountains system of the histroical Shalimar gardens of Lahore by the scientist of pakistan Atomic Energy Commission.

Radioactive tracers are also employed to follow the path that various chemicals or food constituents take in the human body or in animals and plants. This helps to ascertain a chemical to a specific point within the body for highly efficient localized action. This technique is proving very useful to medical and biological research.

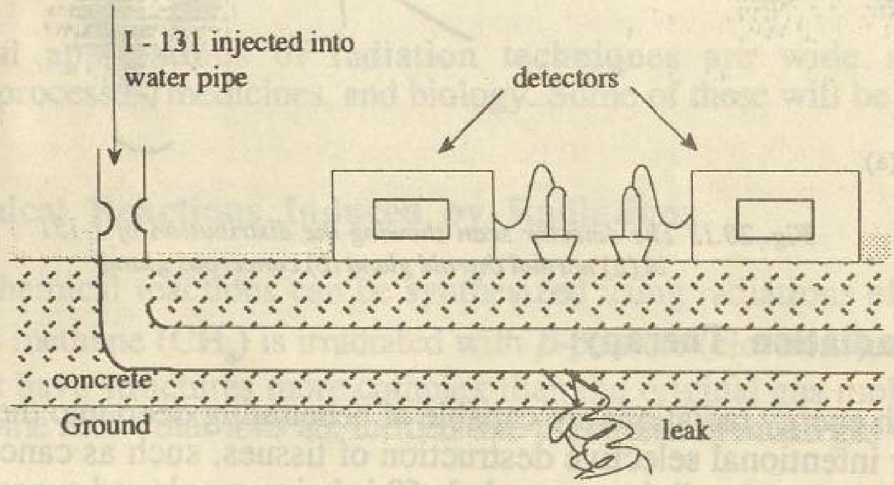


*Fig. 20.12 A tracer technique to detect leakage in underground pipes.*

### 20.7.2 Medical Diagnostics

The research in the medical field has revealed that hydrogen and sodium atoms present in water or food are distributed uniformly throughout the body. Many other chemicals lodges preferentially in certain parts of the body and are concentrated in some particular organs. For example, iodine tends to concentrate in thyroid, phosphorous and strontium in bones and cobalt in liver. They can serve as tracers. Small quantities of low-activity substances are administered by injection or otherwise to patients, and their passage through the body and location in diseased tissue may be ascertained by means of radiation detectors.

It has also been known that cancerous tissues of some kinds of cancer absorb more radioactive atoms than the surrounding healthy tissues. This not only furnishes a clue to the location of cancerous tissue but also makes it possible to treat them through irradiation. For example, the thyroid gland plays a major role in the distribution of iodine throughout the body. In order to evaluate the performance of the thyroid, the patient is given to drink a very small amount of radioactive sodium iodide and a radiation detector positioned near the thyroid gland is monitored. A normal thyroid gland will take up less than about 20% of the ingested iodine during the first hour, but a hyperactive gland will accumulate more than twice this amount.

Cancerous tissues in the thyroid gland can also be detected by obtaining a pattern of distribution of radio iodine in gland as shwon in Fig.20.13.

The use of positron emitting radio isotopes such as technetium-99 has given rise to a growing field of positron emission tomograplay. The annihilation radiation of positron can be easily detected and localised.

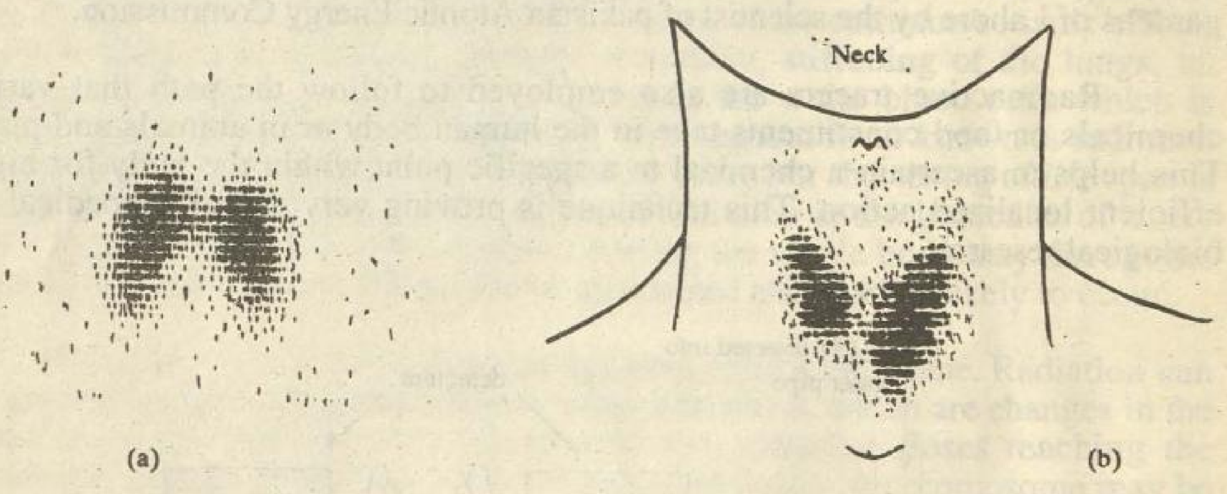


*Fig. 20.13 The detector scan showing the distribution of I-131 in (a) normal thyroid gland (b) cancerous gland*

### 20.7.3 Radiation Therapy:

High energy radiations are capable of penetrating deep into the body and can be used for intentional selective destruction of tissues, such as cancerous tumour. Artificially produced radioisotope cobalt-60 is being employed successfully for the treatment of various kinds of cancer. This isotope emit $\beta$-particles and high energy $\gamma$-rays. The malignant growths or tumors are irradiated by $\gamma$-rays from all directions carefully shielding the healthy tissues. Certain other types of tumour cannot be effectively attacked by $\gamma$-rays. They are sometimes treated with neutron therapy.

Some radioisotopes are taken internally where they are selectively absorbed by certain organs and thus concentrate the radiation where it is most needed. For example, radiation from Iodine-131 is used to treat thyroid cancer. When food containing iodine 131 is ingested by the patients, much of the iodine localizes in the thyroid gland. The radiation emitted by Iodine-131 tends to destroy the thyroid tissues, both normal and cancerous. Since cancerous tissue takes up iodine at a rate that is greater than that for the normal tissue, more radiation is absorbed by the abnormal tissues and more of the cancerous cells are destroyed.

Sometimes small pellets of a radioisotope are planted inside the body close to the tumour, after the radiation destroys the growth, the pellets are removed.

Similarly radioactive radon gas enclosed in miniature gold capsules, known as radon seeds, is employed. The half life of radon is only about four days, thus they soon become uneffective after doing their job and their presence in the body do no harm.

## 20.8 RADIATION TECHNIQUES IN VARIOUS FIELDS

Radiation techniques are widely employed for different purposes in various fields and the results are very encouraging. The use of radioactive phosphorus or nitrogen as a tracer in agricultural research has provided information about the best fertilizer to supply to a particular crop and soil. The technique also helps to adopt a better mode of fertilizer supply to the plants.

Through the use of radiation-induced mutations, improved varities of certain crops, such as rice, chickpea, wheat and cotton have been developed. They have improved plant structure. The plants have shown more resistance to disease and pest, and give better yield and grain quality.

Practical applications of radiation techniques are wide spread in manufacturing processes, medicines, and biology. Some of these will be discussed here briefly.

### 20.8.1 Chemical Reactions Induced by Radiations

Some chemical reactions can be synthesized using radiations effects. For example, when methane ($CH_4$) is irradiated with $\beta$-particle (electrons), molecules are formed that have structures more complex than the original gas molecules. In the first step, some molecular ions are formed due to ionization in the gas

$$CH_4 \xrightarrow{\beta\text{-particle}} CH_4^+ + \bar{e}$$

The molecular ions are further broken apart on collision to produce such fragments as $CH_3^+$, $CH_2^+$, $CH^+$ even $H^+$ and $C^+$. These ions are chemically active and can combine with methane molecules to build larger molecules, that is,

$$CH_3^+ + CH_4^+ \longrightarrow C_2H_5^+ + H_2$$

A $C_2H_5^+$ ion can react with a hydrogen atom to produce a molecule of ethane ($C_2H_6$) or in case of combination with $CH_3^+$ to produce a molecule of propane ($C_3H_8$).

When the irradiation is terminated, the ions present during irradiation capture electrons and become neutral molecules. On analyzing the composition of the gas, a host of new molecules, ranging from hydrogen to long chain hydrocarbons are found. Similar techniques are widely used in polymerisation. Irradiation also initiates reactions of nitrogen oxides and sulphur dioxide with ammonia which is added to purify industrial gases from sulphur or nitrogenous impurities. The ammonia salts produced can then be trapped with an electrical precipatator.

### 20.8.2 Radio Processes in Space

The new range of sensitive detectors has made it possible to analyze in details the cosmic rays entering the Earth's atmosphere from outer space. The ions, detected in the cosmic rays range from protons to that of ions,protons being in abundance. This supports the hypothesis that most of the cosmos consists of hydrogen.

Atoms and molecules can be identified by the characteristic radiations they emit. Radio signals emanating from the gas and dust in the space between the stars reflect radiations of such frequencies that indicate the presence of particular types of molecules. They include carbon monooxide (CO), cynagen (CN), ammonia ($NH_3$), methyle alcohol ($CH_3OH$) and formaldehyde ($CH_2O$).Organic compounds such as aminoacids have also been identified in the meteorites that have crashed into the earth. Amino acids are the basic building blocks of the complex molecules of living matter. The presence of aminoacids in space is perhaps due to the fact that the elements of organic matter, hydrogen, carbon, nitrogen and oxygen, are among the most abundant elements in the Universe. When these atoms mix in space and are exposed to various radiations, they can be joined into complex molecules eventually leading to the formation of amino acids.

### 20.8.3 Polymerisation

The irradiation process in Polymerisation has helped to evolve new plastic materials, pesticides and some other complex compounds. For example, when ethelene gas is irradiated with γ-rays, it is changed to polyethelene which is a kind of plastic and is used to produce soft and flexible products.

The irradiation of some kinds of plastics has resulted in enhanced strengths, increased melting point and more tolerance to heat.

Another product of irradiation process is wood-plastic which is a combination of wood and plastic. The product is stronger, better tolerant to heat and in other characteristics is also better than its constituents. The process involves the injection into wood of a liquid that consists of small organic molecules called monomers. The irradiation causes these monomers to unite and to become connected into a network of large molecules or polymers, resulting in a stronger material. Similarly when wood treated with vinyle acetate is irradiated with $\gamma$-rays, becomes not only stronger but also proves resistant to moisture effects and termite.

### 20.8.4 Sterilization of Medical Supplies

A beam of electrons ($\beta$-particles) or $\gamma$-rays can destroy germs and is thus helpful to sterlise surgical instruments and other medical supplies such as sutures and bandages.

The schematic arrangement of one sterlization process for small packages of medical supplies is shown in Fig. 20.14.

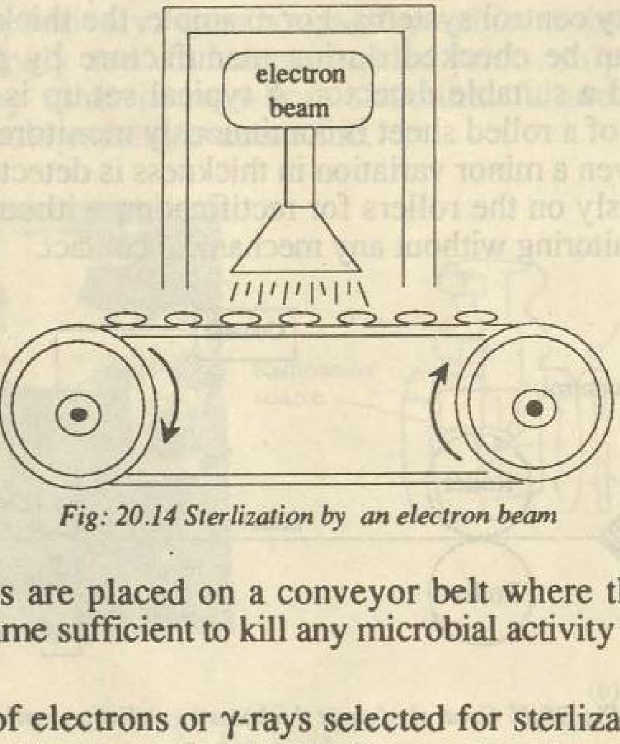


*Fig: 20.14 Sterlization by an electron beam*

The packages are placed on a conveyor belt where they pass through an electron beam for a time sufficient to kill any microbial activity with in the package.

The energy of electrons or $\gamma$-rays selected for sterlization purpose is such that irradiation cannot induce radioactivity. Hence, there is no danger of irradiated material to become radioactive.

### 20.8.5 Food Preservation

The length of time that a food material can be stored depends on the microbial activity that attacks the food and causes it to spoil. Radiation treatment have shown better results for preservation and storage of food stuff. For example, low level irradiation has been found effective in limiting the microbial activity and inhibiting sprouting in potato, onion and garlic, thus enhancing their storage life. Radiation has also been found useful in prolonging shelf lives of fish and some fruits.

Fungal infestation on grain reduces germination. The fungi also produces toxins that are health hazard. Radiation process have not only isolated over one hundred species of fungi infesting the grain and flour but also effectively used for disinfestation.

Small doses of radiation could also be applied to certain food stuffs to kill the bacteria that cause a disease characterized by diarrhoea and vomiting.

Radiation treatment of food material needs intensive care and further research to keep it safe for human consumption. A little more radiation dose or exposure time can produce undesireable and harmful effects in the food material.

### 20.8.6 Gauging and Control

Some applications use the fact that the extent to which radiation is absorbed when passing through matter depends on its thickness and density. This can help to devise precise quality control systems. For example, the thickness of paper, plastic or a metal sheet can be checked during manufacture by passing it between a radiation source and a suitable detector. A typical set up is shown in Fig. 20.15 where the thickness of a rolled sheet is continuously monitored and controlled by a radiation method. Even a minor variation in thickness is detected and the message is passed instantaneously on the rollers for rectification without stopping. The main advantage is the monitoring without any mechanical contact.

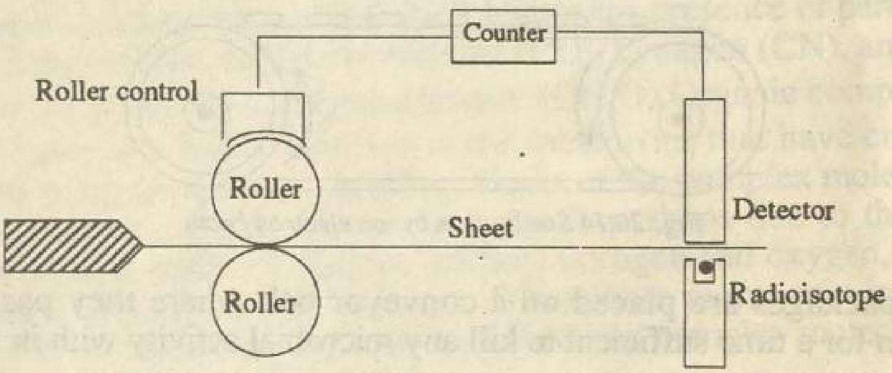


*Fig. 20.15 Control of sheet thickness by radiation method*

A contact technique may sometime damage a thin and fragile sheet. Gamma gauges are particularly useful for high density or thicker materials such as steel, aluminium, asbestos, rubber and celluloid etc. The source usually used is cobalt-60 which emit $\gamma$-rays of 1.17 MeV energy.

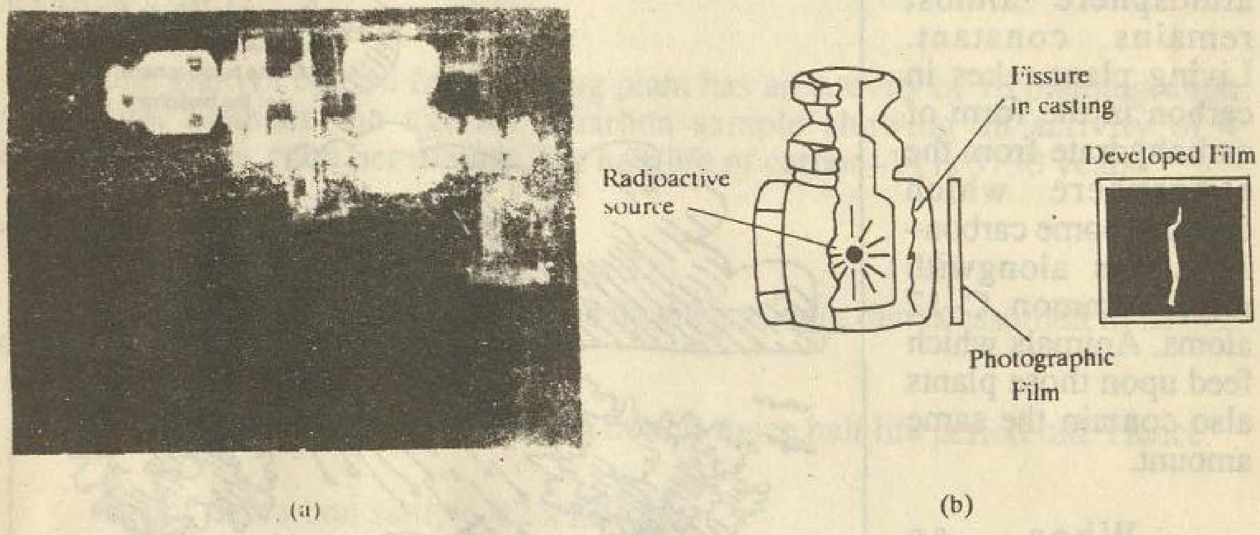
Beta gauges are used for thin sheets and low density materials such a cardboard, paper and thin coats of paints. Strontium-90 is used as a $\beta$-source which emits 0.54 MeV $\beta$-particles.

In lithographic printing press, the flow of ink on the rollers is controlled by plating the rollers with radioactive nickel (Ni–63) which emits $\beta$-particles of specific energy. The Geiger counter is used to monitor the flow of ink on the rollers.

Some level indicaters work on a similar principle to check whether the sealed cylinders are full of a liquid gas or not. In some pharmaceutical units, the capsule filling can also be controlled by radiation gauging. The control device is adjusted to accept capsules only if the radiation count rate fall within a certain narrow range. All the faulty capsules are automatically rejected.

### 20.8.7 Radiography

The possible fault within the heavy machinery of a plant or vessel can be detected using γ-rays by producing radiographs. The presence of any void or fissure in the metal casting, cracks in metal plates or welding faults become apparent on the film just like the x-rays of the teeth. The radiograph gives a clear insight picture of the metal parts as shown of a drill machine and a valve in Fig. 20.16(a) and (b) respectively. Similar radiographic techniques employing radioisotopes are routinely used to inspect welded joints, ship paints, jet engine components, and many other items whose structural failure could necessitate expensive repair and could endanger human life.



(a) (b)

*Fig. 20.16*
Radiograph of (a) drill machine (b) a valve

The γ-rays radiography has also shown better results in some medical diognostics such as internal imaging of the brain to determine precisely the size and location of a tumour.

### 20.8.8 Radiation Methods and Archaeology

Measurement of the relative amount of radio isotope and stable isotope provides a fairly accurate method of dating the remains of ancient times. For example, the activity of a given mass of carbon taken from an ancient piece of wood, linen or charcoal can be compared with that of an equal mass of carbon from a living plant. The comparison helps to estimate the time that has passed since the carbonaceous material was part of a living plant.

The radioisotope carbon–14 is constantly produced from Nitrogen–14 atoms present in the atmosphere by the reaction,

$$^{14}_{7}N + ^{1}_{0}n \longrightarrow ^{14}_{6}C + ^{1}_{1}H$$

the neutrons being produced by the interaction of cosmic rays with some atomospheric nuclei.

Although the amount of radioactive carbon in the atmosphere is very small but it has been found that the ratio of C–14 to C–12 in the atmosphere almost remains constant. Living plant takes in carbon in the form of carbohydrate from the atmosphere which contains some carbon–14 atoms alongwith more common C–12 atoms. Animals which feed upon those plants also contain the same amount.

When an organism dies, it ceases to take in atmospheric carbon. Its carbon–14 contents gradually decreases due to the fact that half life of carbon - 14 is about 5730 year.

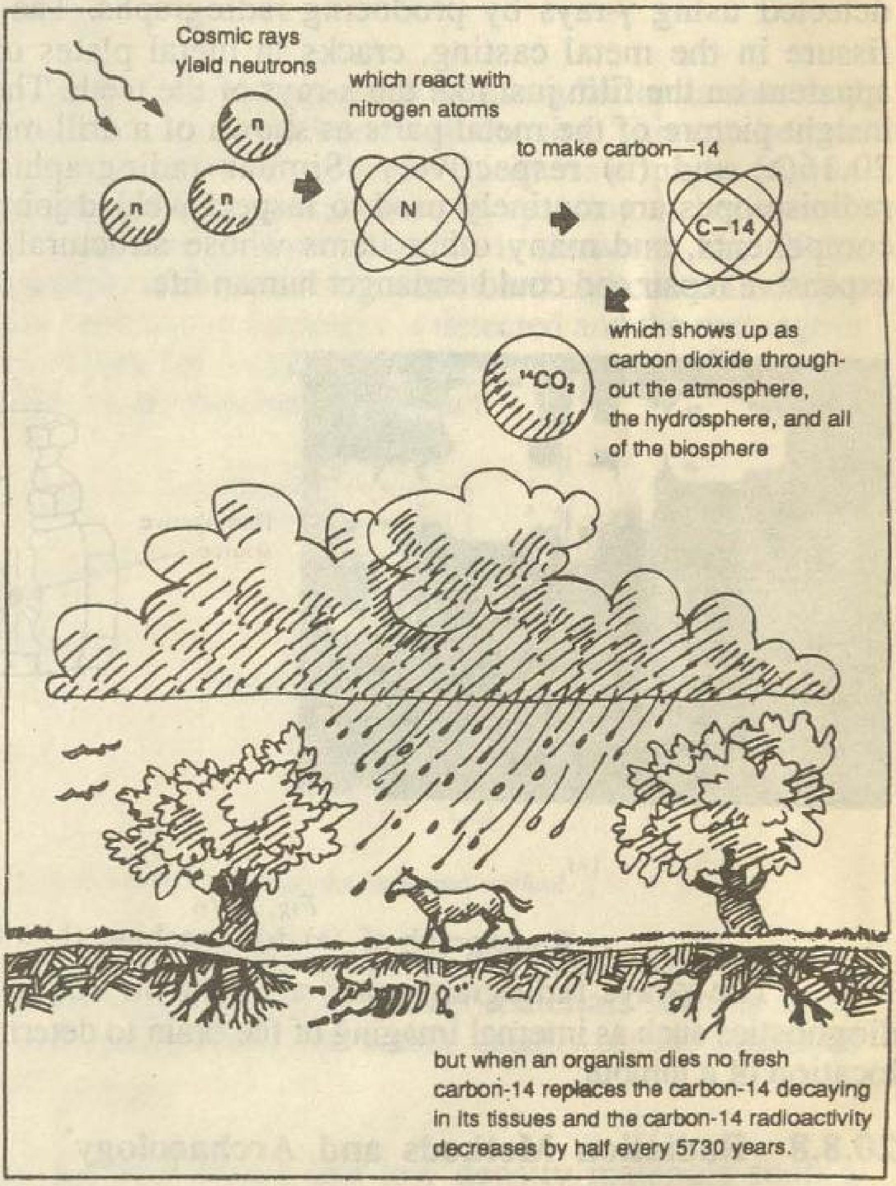

Cosmic rays yield neutrons
which react with nitrogen atoms
to make carbon—14
n
n
n
n
N
C–14
which shows up as carbon dioxide throughout the atmosphere, the hydrosphere, and all of the biosphere
$^{14}CO_2$
but when an organism dies no fresh carbon-14 replaces the carbon-14 decaying in its tissues and the carbon-14 radioactivity decreases by half every 5730 years.

*Fig. 20.17. An illustration showing the process of carbon dating*

The carbon–12 contents, on the other hand, stays constant and therefore, from the moment of death, the ratio of C–14 to C–12 decreases whereas in living matter, this ratio remains constant. As some C–14 nuclei decay, other are absorbed. In dead matter, no renewal of the decayed nuclei can take place. Thus the ratio of C–14 to C–12 found in dead matter is a measure of the time elapsed since death. Using this technique, the grains, wood and linen found in the Egyption Pyramids has helped to estimate the age of these remains to be 4500 years.

For a longer time scale, potassium–40 to argon–40 ratio has been found useful. Radioactive potassium is also found in all living matter. It decays into two modes, about 89% potassium decays to calcium–40 by emitting $\beta$-particles and 11% is converted to argon–40 by absorbing electrons. The half life of argon–40 is 1.3 billion years. By measuring the ratio between $^{40}_{19}K$ and $^{40}_{18}Ar$, the dates as back as one million years can be estimated.

The presence of uranium–238 in some rocks allows estimation of their ages to be made from the ratio of lead–206 to uranium–238. Lead is the stable end product of the natural uranium series. uranium–238 is the parent of the series and therefore, for every uranium atom that has decayed since the rock containing uranium was formed, one atom of Pb – 206 would have been produced. The ratio of $^{206}_{82}Pb$ to $^{238}_{92}u$ can be used to determine the age of the rock.

**Example 20.1**

One gram of carbon from a living plant has an activity of 16 disintegration per minute. Estimate the age for a carbon sample showing an activity of 4 disintegrations per gram per minute. The half life of carbon–14 is 5730 years.

**Solution**

A sample showing an activity of 8 disintegration/g.min is assumed to be one half life period (5730 years) old.

A sample with 4 disintegrations/g.min. is twice half life period old. Hence

Age of the carbon sample = 2 × half life

$$= 2 \times 5730 = 11460 \text{ years}$$

### 20.8.9 Activation Analysis:

Neutron activation analysis is an analytical technique to determine the concentration in a sample of an element of interest. When a sample is bombarded with neutrons, some nuclei are excited by absorbing neutrons. The $\beta$-particles emission from the excited nuclei is usually followed by gamma radiation. Measurement of the $\gamma$-photon energies permits the identification and calculation of the concentration in the sample of the nuclei of interest, even if the quantity is extremely minute. For example, a sample is suspected to have extemely small trace of silicon. When irradiated with neutrons, the silicon atoms, if present, absorb neutrons resulting in silicon–31 of half life 2.6h

$$^{30}_{14}Si + ^{1}_{0}n \longrightarrow ^{31}_{14}Si^{*}$$

The β-decay of silicon–31 to phosphorus–31 produces a 1.27 MeV gamma photon.

$$^{31}_{14}Si^{*} \longrightarrow ^{31}_{15}P^{*} + ^{0}_{-1}e$$

and

$$^{31}_{15}P^{*} \longrightarrow ^{31}_{15}P + \gamma \text{ (1.27 MeV Photon)}$$

The measurement of half life and the energy of the emitted radiation helps to identify the radioactive isotope.

One interesting application of activation analysis is the investigation to ascertain the cause of Napolean's death. In 1961, pieces of his hair were separately irradiated with neutrons and their arsenic contents analyzed to a precesion that there was a corelation between the variation in arsenic content along one hair and the daily medical report on Napoleon's health while in exile on the island of St. Helena. The results confirmed that arsenic was fed to him irregularly over the period and the Emperor did not die a natural death.

In criminology, a suspect may be linked to the scene of a crime by activation analysis of a speck of material or dust found in his clothings.

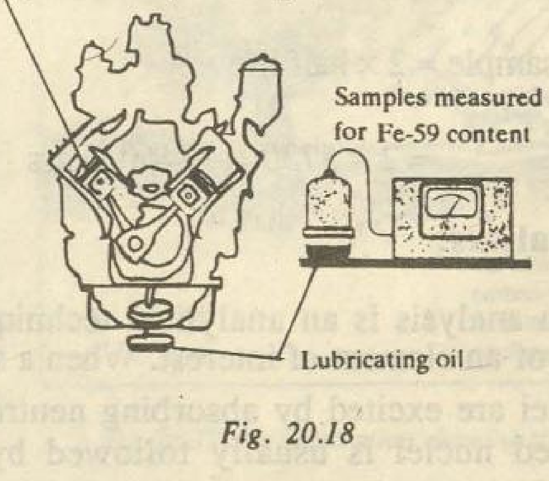


*Fig. 20.18*

The corrosion and wear of machinery such as turbine blade, bearing or piston rings in an automobile engine can be studied with this technique. For example, the upper layers of the ring piston, as shown in Fig. 20.18, are made radioactive through irradiation with neutrons. The rate of wear can be calculated by measuring the activity of the wear away material carried away in the lubricating oil.

# SUMMARY

### Interaction of radiations with matter

It is ionization produced in matter by the nuclear radiations that make them useful in various situations. The alpha particles produce intense ionization along their paths and hence do not travel far enough in air. Beta particles suffer lesser ionization encounters and thus can penetrate deeper into matter than can an α-particle of the same energy. The ionization produced by gamma rays is due to electrons released through photoelectric effect. Gamma radiation is considerably more penetrating than either alpha or beta radiation. Neutron can enter easily in the nucleus of an atom and can initiate a neutron induced reaction.

### Radiation detectors

Detectors of radiations include Wilson cloud chamber, Geiger counter and semi conductor detectors. The cloud chamber is based on the principle that super saturated vapour condenses preferentialy on ions or dust particles. The discharge in the Geiger tube results from the ionization produced by the incident radiation. The solid state detector is basically a reversed baised pn Junction in which electron–hole pairs are produced by the incident radiation to cause a pulse to flow through the external circuit.

### Radiation effects

Nuclear radiations can cause damage to biological tissues because they ionize the atoms and molecules of the cells.

### Uses of radiation

Radioactive tracers and imaging devices help in the understanding, diognosis and treatment of some diseases. Radiation techniques are also applied in polymerisation, sterilization of medical supplies, guaging and control devices, activation analysis and for dating the age of many materials.

# QUESTIONS

**20.1** Give a brief account of interaction of various types of radiation with matter.

**20.2** Explain how $\alpha$ and $\beta$-particles may ionize an atom without directly hitting the electrons. What is the difference in the action of the two particles?

**20.3** Why is lead a better shield against gamma radiation than the same quantity of water?

**20.4** Water is a better shield against neutrons than lead. Explain why?

**20.5** Describe two methods by which $\alpha$ and $\beta$-particles can be identified on the basis of the following characteristics:

(i) ionization (ii) penetration power

**20.6** A particle which produces more ionization is less penetrating. Explain why?

**20.7** Describe the principle and working of a Wilson Cloud Chamber for detecting nuclear radiations. What information is revealed by the length and shape of the tracks of an incident particle?

**20.8** Explain the working of a Geiger Counter. How an incident particle produces an electric pulse through the external circuit?

**20.9** For what purpose, alcohol or bromine is mixed with principal gas in Geiger tube?

**20.10** What is meant by depletion region? Explain the principle and operation of a solid state detector with the help of a labelled diagram.

**20.11** What are the harmful effects of nuclear radiations? What measures can be adopted to safeguard overselves from the nuclear hazards?

**20.12** What is meant by nuclear exposure? What are the major factors contributing towards nuclear exposure? Is there any safe limit for exposure?

**20.13** Give an account of the uses of nuclear radiations in medical diagnostics and treatment.

**20.14** Write an application of tracer technique each in medicine, agriculture, and industry.

**20.15** Explain how radioactive iodine can be used to diagnose the thyroid cancer. Is the same technique can be utilized for treatment purpose? Explain.

**20.16** Which of $\alpha$, $\beta$ and $\gamma$-radiation would you advise and why for?

(i) treatment of skin of a patient,

(ii) treatment of flesh just under skin,

(iii) treatment of an infection in the interior of the body.

**20.17** How could a radioisotope be used to determine the effectiveness of a fertilizer?

**20.18** Explain how radiation methods could be used to gauge the level to which bottles are filled in a soft drink bottling plant?

**20.19** What is meant by radioactive dating? The half life of radioactive carbon being 5730 years, one gram of carbon from a living plant has an activity of 16 disintegrations per minute. Estimate the age of carbon sample showing an activity of 2 disintegrations per gram per minute.

**20.20** What is meant by activation analysis? How could the amount of wear that pistons and piston rings experience in automobile engines be made using irradiation method?

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## The Quaid Said,

"You must devote yourself wholeheartedly to your studies, for that is your first obligation to yourselves, your parents and to the state."

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smoking
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